

A Latin Hypercube Sampling Utility: with an application to an Integrated Assessment Model

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Table of Contents

- 1 Latin Hypercube Sampling: An Introduction
- 2 Application of LHS to an Integrated Assessment Model
- 3 Results
- 4 Conclusions and references

Introduction

- Stratified form of Monte Carlo sampling
- Objective is to reduce sample size yet cover entire sampling domain
- Allows for targeting of correlations across two or more variables
- LHS tool is a compiled executable coded in C/C++
- Original FORTRAN program developed at Sandia Labs ([Swiler and Wyss \(2004\)](#))
- New version includes more output options and distributions
- Similar functionality exists in Excel's @Risk add-in

Sampling

The inverse transform method. Example using the Logistic CDF:

$$F(x) = \frac{1}{1 + e^{-(x-\mu)/s}}$$

Leads to the quantile function (F^{-1}):

$$x = Q(p) = \mu + s \ln \left(\frac{p}{1-p} \right)$$

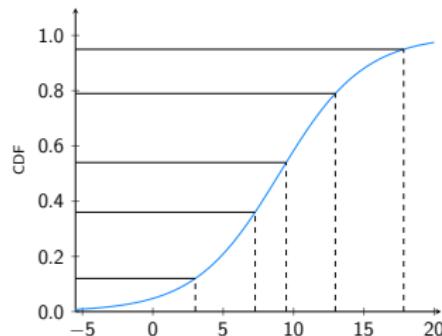
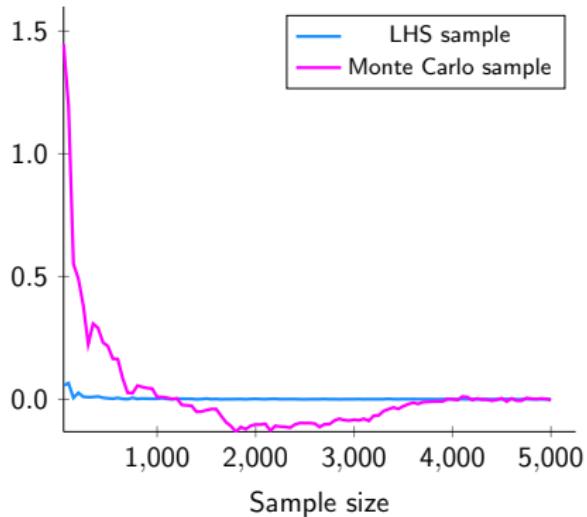


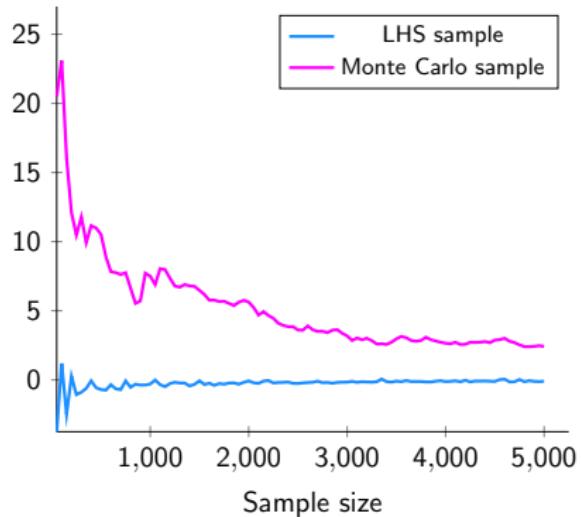
Figure: The Logistic distribution CDF and selecting random deviates

LHS—stratified sampling

- Sample in each $1/n$ interval



(a) Deviation from the expected mean (0)



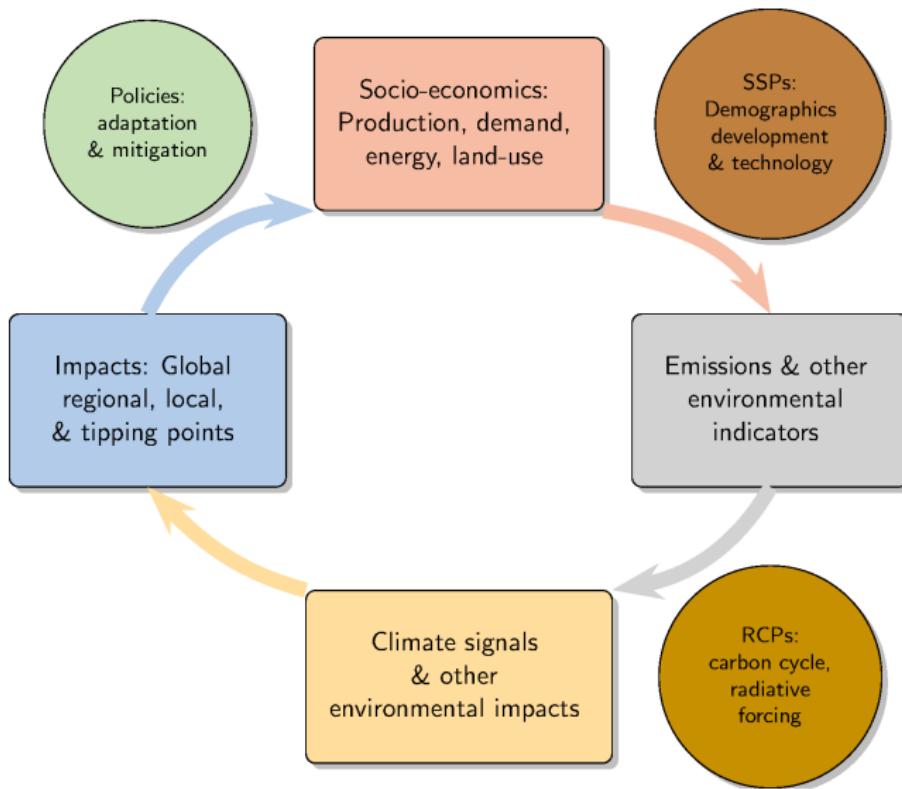
(b) Deviation from expected standard deviation, %

Figure: Comparison of LHS versus Monte Carlo sampling for a $\text{Logistic}(0,2)$ distribution

LHS Utility

- Compiled program—easy to integrate into a workflow
- Output options—CSV, XML, GMS, GDX and 'classic'
- Around 40 distributions Distributions
- User-specified correlations—default is 0

Integrated Assessment Models (IAM)



META 21 Model

- META 21 Model is an integrated assessment model (IAM) focused on bio-physical tipping points ([Dietz et al. \(2021\)](#))
- Simple economics—though downscaled to country-level
- Uses a relatively new simple climate model (FaIR)
- Change in global temperature down-scaled to country level
- Uses country-specific impacts caused by changing temperature, sea-level rise and non-market damages
- Captures changes in the planet's albedo from the change of ice cover—leading to adjustments in the change in the global mean surface temperature.
- Interaction effects across the tipping points.
- Designed for uncertainty analysis
- Original version in Excel. GAMS version is described in [van der Mensbrugghe \(2023b\)](#).

Tipping points

- The thawing of permafrost (PFC), which leads to additional carbon and methane emissions
- The dissolution of methane hydrates (OMH) which leads to additional methane emissions
- The Amazon dieback (AMAZ), which leads to a jump in carbon emissions
- Ice sheet melting, which impacts sea-level rise. There are four sources of sea-level rise: thermal expansion, small ice-sheets and glaciers, the Greenland ice-sheet (GIS) and the West Antarctic ice-sheet (WAIS)
- Modification of the Atlantic Meridional Overturning Circulation (AMOC), also called the thermohaline circulation. AMOC affects the change in country-level average temperatures
- The weakening of the Indian summer monsoon (ISM), which leads to an additional damage effect for India.

Energy Balance Model (EBM)

The EBM in FaIR has the same functional form as the EBM in DICE, though with a different parameterization. It is based on a two-box model: atmosphere and shallow ocean and deep ocean.¹

$$T_t = A \cdot T_{t-1} + M \cdot RF_t$$

$$A = \begin{pmatrix} -(\lambda + \kappa_2)/C_1 & \kappa_2/C_1 & 0 \\ \kappa_2/C_2 & -(\kappa_2 + \varepsilon\kappa_3)/C_2 & \varepsilon\kappa_3/C_2 \\ 0 & \kappa_3/C_3 & -\kappa_3/C_3 \end{pmatrix}$$

$$M = \begin{pmatrix} 1/C_1 \\ 0 \\ 0 \end{pmatrix}$$

¹FaIR 2.0 has three boxes: atmosphere and shallow and deep oceans.

Sampling strategy

Table: Sampling assumptions for the forcing and EBM parameters

Parameter	LHS	Distribution	Expected		Sample	
			<i>mu</i>	σ	<i>mu</i>	σ
$1/C_1$	xi_1	Pareto(5.9,0.116)	0.14	0.029	0.14	0.029
C_2	C_0	Pareto(1.7,53.0)	128.0	∞	126.6	259.6
κ_2	xi_3	Triangular(0.5,0.5,1.24)	0.75	0.174	0.75	0.174
$f2xco2$	f2xco2	Normal(3.46,0.437)	3.46	0.437	3.46	0.437
ECS	t2xco2	Normal(3.25,0.800)	3.25	0.800	3.25	0.800

Note: $\lambda = f2xco2/ECS$

Table: Targeted correlations for the energy balance model parameters

	xi_1	C_0	xi_3	f2xco2	t2xco2
xi_1	1				
C_0	-0.0445	1			
xi_3	-0.4372	-0.1198	1		
f2xco2	0.0139	-0.0397	-0.4623	1	
t2xco2	-0.1934	-0.0802	0.0655	0.6512	1

Table: Sampled correlations for the energy balance model parameters

	xi_1	C_0	xi_3	f2xco2	t2xco2
xi_1	1				
C_0	-0.0184	1			
xi_3	-0.3383	-0.0528	1		
f2xco2	0.0095	-0.0064	-0.4505	1	
t2xco2	-0.1676	-0.0175	0.0623	0.6510	1

Uncertain events: Ocean methane hydrates

$$p_t^{OMH} = 1 - \exp\left(-b^{OMH} T_t^{atm}\right)$$

$$I_t^{OMH} = \begin{cases} 1 & \text{if } I_{t-1}^{OMH} = 1 \\ \text{randBinomial}(1, p_t^{OMH}) & \text{if } I_{t-1}^{OMH} = 0 \end{cases}$$

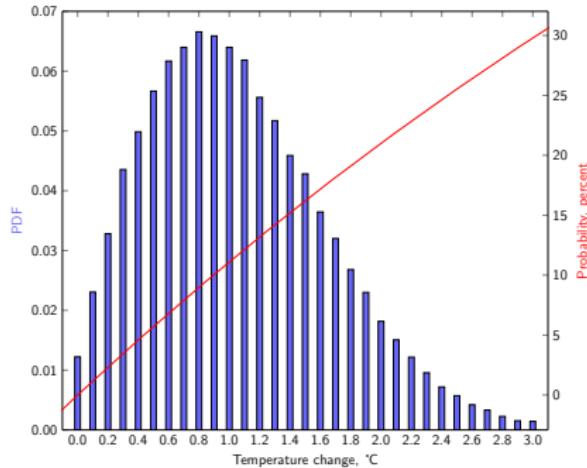


Figure: Sampling for OMH, where $b^{omh} = 0.118$

Snippet of LHS input file

Input file for 582 random variables has over 75,000 lines

```
titl Sampling for the META 21 Model
out Meta21LHS.gdx
msg Meta21.lst
seed 9328
obs 10000
reps 1
dataset:
xi_1      Pareto      5.907      0.11628
c_0       ParetoAlt   1.7062     53.0
xi_3      Triangular  0.5        0.5          1.23723
f2xco2    Normal      3.45938    0.43674
t2xco2    Normal      3.25312    0.80031
CORRELATE xi_1  c_0    -0.0445077056186261
CORRELATE xi_1  xi_3   -0.437158290210702
CORRELATE xi_1  f2xco2 0.0139180621139756
CORRELATE xi_1  t2xco2 -0.193425755823774
CORRELATE c_0   xi_3   -0.119776142709625
CORRELATE c_0   f2xco2 -0.0396551663115062
CORRELATE c_0   t2xco2 -0.0801614029197139
CORRELATE xi_3  f2xco2 -0.462276744415161
CORRELATE xi_3  t2xco2 0.0654874733489397
CORRELATE f2xco2 t2xco2 0.651224938074659
```

A few results: Social Cost of Carbon

The median value of the SCC with tipping points is \$60, and without tipping points is \$48.

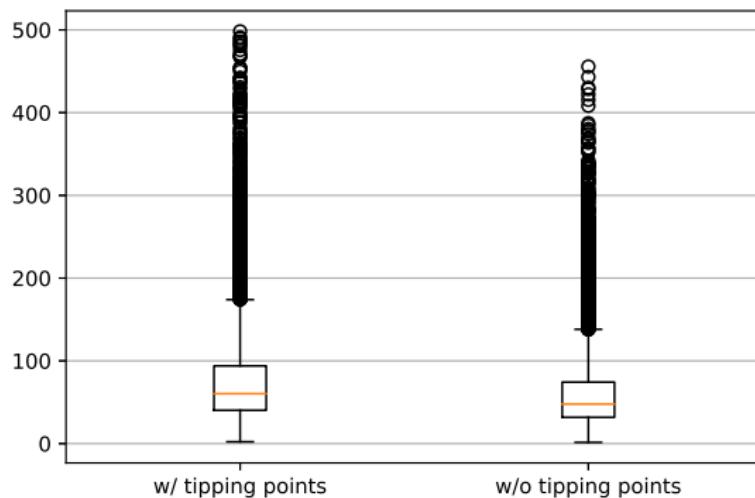
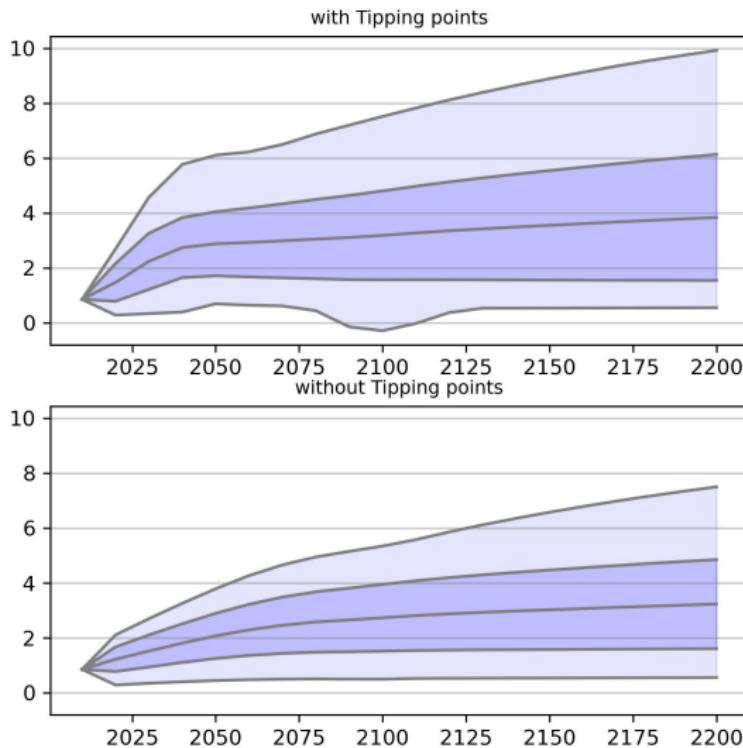


Figure: Distribution of Social Cost of Carbon

Temperature distribution



Damages avoided when excluding tipping points

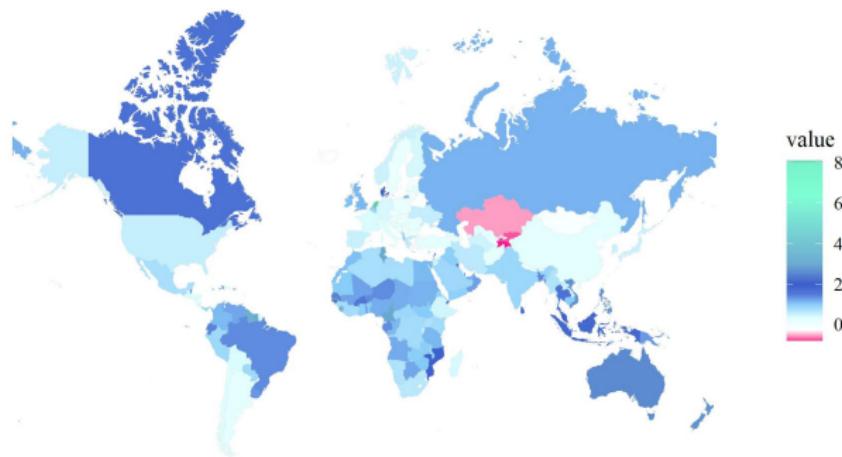


Figure: Average change in per capita consumption in 2100 when excluding tipping points, relative to the full damage scenario, percent

Concluding remarks

- The LHS utility can generate a stratified sample for Monte Carlo simulations that also includes assumptions on specific correlations. User guide is provided in [van der Mensbrugghe \(2023c\)](#)
- Coupled with parallelization techniques ([van der Mensbrugghe \(2023d\)](#)), it is an avenue for exploring the uncertainty space for large models
- Gaussian Quadrature is useful in some contexts, but becomes unwieldy with a large number of uncertain parameters and for assessing higher order moments
- Many parts of META 21 can be readily incorporated in CGE models that generate greenhouse gas emissions
- See also [van der Mensbrugghe \(2023a\)](#)

References

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- van der Mensbrugghe, D. 2023d. "Using Python for Parallelization." Global Trade Analysis Project (GTAP), Department of Agricultural Economics, Purdue University, West Lafayette, IN, GTAP Working Paper No. 93. https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=6826.

Distributions

Continuous	Continuous other	Discrete
Uniform	Beta	Binomial
Loguniform	Cauchy	Geometric
Triangular	Dagum	Hypergeometric
	Exponential	Negbinomial
Normal	Frechet	Poisson
Boundnormal	Gamma	
Truncnormal	Gompertz	<u>User-defined</u>
Lognormal (lognormalalt)	Gumbel	Contlin
Boundlognormal	Invgauss	Contlog
Trunclognormal	Kumaraswamy	Contfreq
Lognormaln	Laplace	Discrcum
Boundlognormaln	Levy	Discrhist
Trunclognormaln	Logistic	
	Maxentropy	
	Pareto (paretoalt)	
	Rayleigh	
	Weibull	