

# Structural Estimation and Solution of International Trade Models with Heterogeneous Firms

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## Abstract

We present an empirical implementation of a general-equilibrium model of international trade with heterogeneous manufacturing firms. The theory underlying our model is consistent with Melitz (2003). A nonlinear structural estimation procedure identifies a set of core parameters and unobserved firm-level trade frictions which best fit the geographic pattern of trade. Once the parameters are identified, we utilize a decomposition technique for computing general-equilibrium counterfactuals. We illustrate this technique using trade and protection data from the Global Trade Analysis Project (GTAP). We first assess the economic effects of reductions in measured tariffs. Taking the simple-average welfare change across regions the Melitz structure indicates welfare gains from liberalization that are nearly four times larger than in a standard policy simulation model. Furthermore, when we compare the economic impact of tariffs with reductions in estimated fixed trade costs we find that policy measures affecting the fixed costs of firm entry are of greater importance than conventional tariff barriers. (JEL C68,F12)

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# 1 Introduction

In canonical models of international trade, trade policy changes induce factors of production to move between industries. Recent empirical and theoretical developments suggest that *within*-industry factor movements may also be an important channel through which policy affects economic variables. Studies of plant level data document significant intra-industry differences in productivity levels.<sup>1</sup> The movement of factors from less-productive to more-productive plants is an important channel of productivity growth.<sup>2</sup> An empirical literature has begun linking trade to productivity growth via these channels.<sup>3</sup>

A recent theoretical model by Melitz (2003) rationalizes these and other empirical phenomena within a general equilibrium model of production and trade. A fixed entry cost and differentiated products allow equilibrium co-existence of firms with heterogeneous productivities. Fixed costs of trade ensure that only more productive firms export. Among these, a smaller subset of even more productive firms serve multiple export markets. Trade liberalization raises industry productivity by shifting market share away from low-productivity non-exporters, and toward high-productivity exporters. Trade liberalization also improves welfare by increasing the number of imported varieties available to domestic consumers.

These qualitative features of the model are important. Our purpose is to apply the theoretic innovations to a quantitative analysis of policy. The model relies upon parameters that are difficult to measure directly. Fixed costs of trade play an important role in this framework, but they are typically unobserved. Another of the model's key parameters, the implicit shape of the productivity distribution, is not observed directly. Even indirect estimates of this parameter rely on confidential access to plant-level data.

Recent papers in the geography-of-trade literature use structural assumptions and the

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<sup>1</sup>Bartelsman and Doms (2000) review this literature.

<sup>2</sup>See Foster et al. (2001), among others.

<sup>3</sup>Pavcnik (2002)

bilateral pattern of trade to make inferences about the size of trade costs. Anderson and van Wincoop (2003) develop a structural estimation technique that can be used to infer trade costs. Balistreri and Hillberry (forthcoming) develop an alternative estimation strategy for the same model, and argue that it can be extended to most general equilibrium models of trade. Balistreri and Hillberry (2005) link structural estimation techniques to established methods for calibrating general equilibrium models. We adapt these methods to calibrate a multi-sector, multi-region general equilibrium model in which the manufacturing sector has a Melitz-style market structure.

A key technical challenge in the development of this method is an algorithm for efficient solution of a multi-dimensional model with heterogeneity in plant level productivity. This is computationally difficult because the model requires joint solution of the trade equilibrium and a host of entry conditions identifying marginal firms. To solve this problem, we decompose the general equilibrium into an industry-specific module which determines the industrial organization and a general equilibrium model which evaluates relative prices, comparative advantage and the terms of trade. The full general equilibrium solution is achieved through an iterative procedure in which the exchange equilibrium is solved conditional on the industrial organization.

Our structural estimation procedure uses the equations defined by the numeric model as a series of side constraints on the econometric objective. The model is underidentified, but assumptions about a few key structural parameters allow the econometric procedure to complete the identification of both the production technology and average bilateral trade costs. Leaning heavily on our structural model, we attribute differences between observed and fitted bilateral trade in manufactured goods to unobserved fixed costs of trade. This interpretation allows us to generate a complete, exact calibration of the model, without any role for idiosyncratic preferences in manufacturing trade.

The key structural parameters of the model are the distance elasticity of ad valorem trade

costs and the parameter defining the shape of the Pareto distribution of firm level productivities. We estimate these under three different sets of identifying assumptions. Our preferred specification ties down the distance elasticity of trade costs, using unexplained variation in the trade pattern to fit the implied shape of the productivity distribution. Our estimate of this parameter is largely consistent with estimates from the confidential plant-level data.

With our general equilibrium system fully parameterized, we proceed to a quantitative assessment of the effect of trade policy changes. Using a standard Armington structure as the benchmark, we consider a 50 percent reduction in manufacturing tariffs. As expected, endogenous productivity changes and growth in the number of imported varieties lead to larger welfare changes in the Melitz model than in the Armington baseline. Taking the simple-average welfare change across regions, the Melitz structure indicates welfare gains from liberalization that are nearly four times larger than those from the baseline model. We also consider reductions in the inferred bilateral fixed costs, and find that the welfare gains from these changes are substantially larger. Joint reductions of tariffs and the inferred fixed costs of trade generate even larger welfare gains. When we reduce both tariffs and fixed border costs by 50% the average welfare gain is nearly 30 times larger than in the case of tariff cuts in the Armington structure.

In section 2 we provide a review of the relevant literature, with a focus on the empirical literatures on heterogeneous productivity and the geographic pattern of trade. Section 3 provides a brief review of the Melitz (2003) theory as it relates to our application. Section 4 details a practical method for numerically solving the heterogeneous-firms model. In Section 5 we outline the nonlinear estimation procedure, and estimate the structural parameters that allow the model to best fit the data. These parameters are used to calibrate an operational general equilibrium model, which we employ to conduct counterfactual analysis. The results of this analysis appear in section 6. In section 7 we discuss implications of our work and directions for further research.

## 2 Literature Review

Two broad areas of the empirical trade literature motivate the models like those proposed in Melitz (2003) or Bernard et al. (2007).

First, an extensive literature has documented heterogeneity across establishments in productivity, export behavior and responses to trade shocks. Important findings from this literature include a) there is wide variation in productivity levels among coexisting plants;<sup>4</sup> b) only a small fraction of establishments engage in exporting, and exporters tend to be larger and more productive than non-exporters;<sup>5</sup> c) there is considerable heterogeneity *among* exporters in the number of markets served per firm;<sup>6</sup> d) within-industry reallocation of market share from less-productive to more-productive establishments is an important component of aggregate productivity growth;<sup>7</sup> and e) productivity growth via shifting market shares (including the exit of the lowest productivity plants) is an important channel through which trade cost reductions induce aggregate productivity growth.<sup>8</sup> All these features can be modelled in the Melitz framework.

Second, several authors have noted that gravity models of bilateral trade do not adequately deal with the presence of zero observations.<sup>9</sup> An emerging literature links variation in aggregate trade flows to variation in the number of a) firms trading, b) commodities traded, and c) trading partners.<sup>10</sup> Of particular interest in this literature has been explaining

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<sup>4</sup>This is a robust feature of the data, as documented in the review of the literature by Bartelsman and Doms (2000). Bartelsman and Dhrymes (1998) show that productivity differentials persist over time, are therefore are unlikely to be attributable to data collection errors.

<sup>5</sup>See, for example, Bernard and Jensen (1999), Aw et al. (2000), Clerides et al. (1998), Roberts and Tybout (1997), and Bernard et al. (2003).

<sup>6</sup>Eaton et al. (2004) document this using French data.

<sup>7</sup>See Foster et al. (2001) and Aw et al. (2001), among others. An important component of the contribution of shifting market share is the exit of less-productive establishments.

<sup>8</sup>See, for example, Bernard et al. (2006) and Pavcnik (2002).

<sup>9</sup>See Haveman and Hummels (2004) and Helpman et al. (2007)

<sup>10</sup>Eaton et al. (2004) and Hillberry and Hummels (forthcoming) show that variation in the number of firms serving a market explains variation in exports to that market. Hummels and Klenow (2005) and Broda and Weinstein (2006) identify the extensive margin in terms of role of added commodities/trading partners.

trade growth via this *extensive* margin.<sup>11</sup> These observations, as well as trade-policy induced growth along the extensive margin, can also be represented in the Melitz-framework.<sup>12</sup>

In a critique of the performance of applied general equilibrium models commonly used in trade policy analysis, Kehoe (2005) argues that the models typically fail along two dimensions: they do not allow trade policy to affect aggregate productivity, and they do not allow trade policy to induce trade growth along the extensive margin. The absence of endogenous productivity gains is often noted by policymakers.<sup>13</sup> While some policy estimates include *ad hoc* productivity adjustments [Anderson et al. (2005)], these attempts do not typically specify the mechanism by which trade policy is meant to induce productivity growth. Policymakers typically do not criticize the absence of policy-induced trade growth along the extensive margin, but this shortcoming has been noted elsewhere in the academic literature.<sup>14</sup>

While the empirical literature has demonstrated the relevance of within-industry productivity heterogeneity and trade growth via the extensive margin, what has been lacking until recently is a sound theoretical structure that formalizes the insights from the empirical literature. Melitz-type models with firm heterogeneity and fixed trade costs offer a useful framework for addressing Kehoe's critique. Trade policy changes affect industry productivity by shifting market share away from low-productivity non-exporters, and toward high-productivity exporters. The model also allows for trade growth along the extensive margin, and provides a mechanism by which such trade growth can be linked to policy changes. What is lacking, to date, is a) a computational method for solving models of this type in a

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<sup>11</sup>See Kehoe and Ruhl (2002) and Evenett and Venables (2002). DeBaere and Mostashari (2006) find relatively little evidence that trade cost reductions induce trade growth via the extensive margin.

<sup>12</sup>We calculate trade-policy induced change in the number of foreign firms serving each market, as well as exits by the least productive firms.

<sup>13</sup>See, for example United States Trade Representative (2005). In its statement on the economic benefits of each agreement, USTR describes numeric estimates from applied general equilibrium models, but also adds that these models "fail to estimate or fully estimate dynamic or intermediate growth gains from trade liberalization."

<sup>14</sup>Hummels and Klenow (2005) note that Armington-type models fail to account for trade growth along the extensive margin. Romer (1994) notes that the welfare gains attributable to new varieties are likely to be large. Broda and Weinstein (2006) calculate relatively large welfare gains for the United States from increased import variety.

multicountry, multi-commodity world, and b) estimates of the model's structural parameters that would allow consistent calibration of the model to multidimensional data.

Calibration of the model requires estimates of structural parameters of the model. Calibrated policy models typically rely on the econometric literature to provide estimates of structural parameters. Unfortunately, the econometric literature to date is insufficient for this task.<sup>15</sup> We estimate the model's structural parameters using methods similar to those developed for estimation of the Anderson and van Wincoop (2003) model.<sup>16</sup>

### 3 Theory

Consumers have Cobb-Douglas utility over commodity bundles which are defined as constant-elasticity-of-substitution (CES) aggregates of differentiated products. Firms pay a fixed cost of entry. Entrants receive a random productivity draw. Firms with sufficiently low productivity draws exit, and the remaining firms produce with a technology exhibiting increasing returns to scale. Trade costs include *ad valorem* iceberg costs, revenue-generating tariffs, and a fixed cost of entering each market. Firms with higher levels of productivity will be able to profitably serve more markets. The model is simplified by isolating the characteristics and behavior of the average firm participating in each bilateral market. Melitz (2003) develops the critical links between the average and marginal firms, and how average firm characteristics relate to consumer utility.

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<sup>15</sup>Both Helpman et al. (2007) and Chaney (2006) conduct econometric work in order to *test* model implications. They do not provide estimates of the model's structural parameters. Bernard et al. (2007) provide an estimate of the productivity distribution using US data. Our estimates (of a common parameter across all countries) is roughly similar to this estimate.

<sup>16</sup>Our method is most similar to that described in Balistreri and Hillberry (forthcoming). Like Balistreri and Hillberry (2006) and Anderson and van Wincoop (2004), our interest is, in part, identifying unobserved trade costs.

### 3.1 Demand

Consumers in region  $s \in R$  are assumed to have Cobb-Douglas preferences over composites from different sectors,  $A_{ks}$ , where the sector is indexed by  $k$  and  $\alpha_k$  is the expenditure share;

$$U_s = \prod_k (A_{ks})^{\alpha_k}. \quad (1)$$

We drop the industry index at this point and isolate the Dixit-Stiglitz composite of manufactured goods consumed in region  $s$ ,

$$A_s = \left[ \sum_r \int_{\omega_r \in \Omega_r} q_s(\omega_r)^\rho d\omega_r \right]^{\frac{1}{\rho}}, \quad (2)$$

where  $\omega_r$  indexes the differentiated products sourced from region  $r \in R$ . Substitution is indicated by  $\rho = 1 - 1/\sigma$ , where  $\sigma$  is the constant elasticity of substitution. The dual price index,  $P_s$ , is given by

$$P_s = \left[ \sum_r \int_{\omega_r \in \Omega_r} p_s(\omega_r)^{1-\sigma} d\omega_r \right]^{\frac{1}{1-\sigma}}. \quad (3)$$

Defining this in terms of the average variety's price,  $\tilde{p}_{rs}$ , we have

$$P_s = \left[ \sum_r N_{rs} \tilde{p}_{rs}^{1-\sigma} \right]^{1/(1-\sigma)} \quad (4)$$

where  $N_{rs}$  is the number of varieties shipped from  $r$  to  $s$ . Melitz (2003) obtains this simplification by noting that  $\tilde{p}_{rs}$  is the price set by a small firm with the CES weighted average productivity  $\tilde{\varphi}_{rs}$ .<sup>17</sup> Demand for the average variety to be shipped from  $r$  to  $s$  at a gross of

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<sup>17</sup>The weighted average productivity is given by

$$\tilde{\varphi}_{rs} = \left[ \int_0^\infty \varphi_{rs}^{\sigma-1} \mu_{rs}(\varphi_{rs}) d\varphi_{rs} \right]^{\frac{1}{\sigma-1}},$$

where  $\mu_{rs}(\varphi_{rs})$  is the distribution of productivities of each of the  $N_{rs}$  firms.



trade and tax price of  $\tilde{p}_{rs}$  is

$$\tilde{q}_{rs} = \frac{\alpha E_s}{P_s} \left( \frac{P_s}{\tilde{p}_{rs}} \right)^\sigma \quad (5)$$

where  $E_s$  is the value of total expenditures in region  $s$ .<sup>18</sup>

### 3.2 Firm-level environment

We assume a single composite input price,  $c_r$ , associated with all fixed or marginal costs of manufacturing in region  $r$ . In application, we adopt an upstream Cobb-Douglas technology for generating the composite input. This is represented by a cost function of the form

$$c_r = (P_r^E)^{\beta_r^E} \prod_j (w_{jr})^{\beta_{jr}}, \quad (6)$$

where the  $w_{jr}$  are the prices of the factor inputs and  $P_r^E$  is the price of the composite intermediate input. Constant returns in the technology for forming the composite input indicates that the sum of the share parameters, the  $\beta$ , equals one.

Operating firms in a given market use the composite input to cover both fixed-operating and marginal costs, but firms also face an *entry* cost. The entry cost entitles the firm to a productivity draw. If the productivity draw is sufficiently high the firm will operate profitably. Let  $f_r^e$  indicate the entry cost (in composite-input units), and let  $M_r$  denote the number of entered firms in region  $r$ . Then each of the  $M_r$  firms incur the nominal entry payment  $c_r f_r^e$ , although this payment is spread across time (as there is a nonzero probability that the firm will survive beyond the current period).

Now consider the input technology for a firm from region  $r$  that finds it profitable to sell into market  $s$ . Let  $f_{rs}$  indicate the recurring fixed cost of operating on the  $r$ - $s$  link, and let  $\varphi$

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<sup>18</sup>One problem we face in reconciling the empirical model with the established theory is the discrepancy between gross expenditures and value added, because of intermediate inputs. To simplify we assume that intermediate inputs are purchases of the aggregate consumption commodity. Gross expenditures,  $E_s$ , less the value of intermediate inputs (to all industries) equals regional income.

represent the firm-specific measure of productivity. A firm supplying  $q$  units to  $s$  uses

$$f_{rs} + \frac{q}{\varphi}$$

units of inputs. Higher productivity (higher  $\varphi$ ) indicates lower marginal cost.

Once a firm incurs the *entry* cost,  $f_r^e$  it is sunk and has no bearing on the firm's decision to operate in a given bilateral market. The profits earned by infra-marginal firms in the bilateral markets do, however, give firms the incentive to incur the entry cost in the first place. There is no restriction on the markets that can be served by a given member of  $M_r$ . If a firm's productivity is high enough such that it is profitable to operate in multiple markets it can replicate itself maintaining the same marginal cost but incurring the fixed operating cost,  $f_{rs}$ , for each of the  $s$  markets it serves.

The small firms, facing constant-elasticity demand for their differentiated products, follow the usual optimal markup rule. Let  $\tau_{rs}$  indicate the *iceberg* transport-cost factor, and let  $t_{rs}$  indicate the tariff. Focusing on the average firm (with productivity draw  $\tilde{\varphi}_{rs}$ ) shipping from  $r$  to  $s$ , optimal (gross) pricing is given by

$$\tilde{p}_{rs} = \frac{c_r \tau_{rs} (1 + t_{rs})}{\rho \tilde{\varphi}_{rs}}. \quad (7)$$

### 3.3 Operation, Entry, and the Average Firm

We assume that each of the  $M_r$  firms choosing to incur the entry cost receive their firm-specific productivity draw  $\varphi$  from a Pareto distribution with probability density

$$g(\varphi) = \frac{a}{\varphi} \left( \frac{b}{\varphi} \right)^a; \quad (8)$$

and cumulative distribution

$$G(\varphi) = 1 - \left(\frac{b}{\varphi}\right)^a, \quad (9)$$

where  $a$  is the shape parameter and  $b$  is the minimum productivity.

Considering the fixed cost of operating,  $f_{rs}$ , on the  $r$ - $s$  link there will be some level of productivity,  $\varphi_{rs}^*$ , at which operating profits are zero. All firms drawing a  $\varphi$  above  $\varphi_{rs}^*$  will serve the  $s$  market, and firms drawing a  $\varphi$  below  $\varphi_{rs}^*$  will not. A firm drawing  $\varphi_{rs}^*$  is the marginal firm from  $r$  supplying region  $s$ . This leads us to the fundamental condition which determines the number of operating firms in a given market,  $N_{rs}$ . Let  $r(\varphi) = p(\varphi)q(\varphi)$  indicate the gross-of-tariff firm revenues as a function of the draw  $\varphi$ . Zero profits for the marginal firm requires

$$c_r f_{rs} = \frac{r(\varphi_{rs}^*)}{\sigma(1 + t_{rs})}. \quad (10)$$

We would like, however, to define this condition in terms of the average firm rather than the marginal firm.

Following Melitz (2003) we define  $\tilde{\varphi}$  as the productivity of a firm pricing at  $\tilde{p}$ , such that our simplification in equation (4) is consistent. The probability that a firm will operate is  $1 - G(\varphi^*)$ , so we find the CES weighted average productivity,

$$\tilde{\varphi}_{rs} = \left[ \frac{1}{1 - G(\varphi_{rs}^*)} \int_{\varphi_{rs}^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}. \quad (11)$$

Using the Pareto distribution this becomes

$$\tilde{\varphi}_{rs} = \left[ \frac{a}{a + 1 - \sigma} \right]^{\frac{1}{\sigma-1}} \varphi_{rs}^*. \quad (12)$$

Again, following Melitz (2003) optimal firm pricing and the input technology ( $f_{rs} + q/\varphi$ ) we

establish the relationship between the revenues of firms with different productivity draws:

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left( \frac{\varphi_1}{\varphi_2} \right)^{\sigma-1}. \quad (13)$$

Using (12) and (13) to simplify (10) we derive the zero cutoff profit condition in terms of average-firm revenues and the parameters:

$$c_r f_{rs} + \pi_{rs}^c = \frac{\tilde{p}_{rs} \tilde{q}_{rs} (a + 1 - \sigma)}{(1 + t_{rs}) a \sigma}. \quad (14)$$

The variable  $\pi_{rs}^c$  is introduced to track any extra profits that are generated when each of the  $M_r$  firms operate in a market. We term these profits *capacity rents*. The value of  $\pi_{rs}^c$  must be zero in a steady-state, but if  $M_r$  is sticky a policy shock might lead to  $N_{rs} = M_r$  indicating rents.<sup>19</sup>

Next we turn to the entry condition which determines the mass of firms,  $M_r$ . Firm entry requires a one-time payment of  $f_r^e$ , and entered firms face a probability  $\delta$  in each future period of a *bad* shock, which forces exit. In a steady-state equilibrium  $\delta M_r$  firms are lost in a given period so total entry payments in that period must be  $c_r \delta M_r f_r^e$ . From an individual firm's perspective the *annualized* flow of entry payments is  $c_r \delta f_r^e$ .

Assuming risk neutrality and no discounting firms enter to the point that expected operating profits equal the entry payment. A firm from  $r$  operating in market  $s$  can expect to earn the average profit in that market:

$$\tilde{\pi}_{rs} = \frac{\tilde{p}_{rs} \tilde{q}_{rs}}{\sigma(1 + t_{rs})} - c_r f_{rs}. \quad (15)$$

Using the zero cutoff profit condition to substitute out the operating fixed cost this reduces

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<sup>19</sup>The value of  $\pi_{rs}^c$  is determined by the variational-inequality presented in the next section, equation (20). We are only concerned with steady-state equilibria in this study, but we found that the computational model performed better with the extended condition, which avoids numeric moves where  $N_{rs} > M_r$ .

to

$$\tilde{\pi}_{rs} = \frac{\tilde{p}_{rs} \tilde{q}_{rs}}{(1 + t_{rs})} \frac{(\sigma - 1)}{a\sigma}. \quad (16)$$

The probability that a firm in  $r$  will service the  $s$  market is simply given by the ratio of  $N_{rs}/M_r$ .<sup>20</sup> Setting the firm-level entry-payment flow equal to the expected profits from each potential market gives us the free entry condition

$$c_r \delta f_r^e = \sum_s \frac{N_{rs}}{M_r} \frac{\tilde{p}_{rs} \tilde{q}_{rs}}{(1 + t_{rs})} \frac{(\sigma - 1)}{a\sigma} \quad (17)$$

which determines the mass of firms,  $M_r$ .

The final equilibrium conditions establishes the marginal productivity as a function of the fraction of operating firms,  $N_{rs}/M_r = 1 - G(\varphi^*)$ . Applying the Pareto distribution and inverting we have

$$\varphi^* = \frac{b}{\left(\frac{N_{rs}}{M_r}\right)^{1/a}}. \quad (18)$$

In the following section we formalize a computational model based on these fundamental equilibrium conditions.

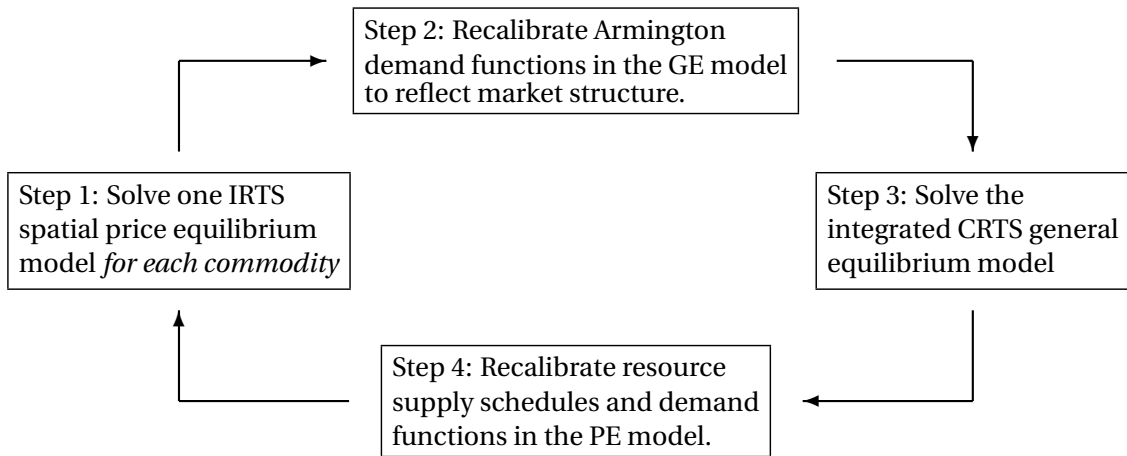
## 4 Solution Method

We represent the policy analysis model on the basis of two related equilibrium problems. The first is a partial equilibrium (PE) model which captures the heterogeneous-firms industrial organization in manufacturing and the associated impact on productivity and prices. The PE model takes aggregate income levels and supply schedules as given. The second module is a *constant-returns* general equilibrium (GE) model of global trade in all products. The GE model takes industrial structure as given and determines relative prices, comparative

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<sup>20</sup>In Melitz (2003) the probability that a firm will operate, which equals the fraction of operating firms in equilibrium, is presented as  $1 - G(\varphi^*)$ .

Figure 1: A Decomposition Algorithm



advantage and terms of trade. We iterate between these two models in policy simulations, letting the first module determine industrial structure and the second module establish regional incomes and relative costs. Industrial structure (numbers of firms operating within and across borders) are passed from the first module to the second whereas the structure of aggregate demand (income levels and supply prices) are passed back from the GE module to the PE module. Once the models are mutually consistent we have a solution to the multi-region general equilibrium with heterogeneous manufacturing firms. The four steps involved in the solution algorithm are depicted in Figure 1.

In most policy modeling exercises, applied economists prefer to work with integrated equilibrium models formulated as systems of equations in which prices and quantities are determined simultaneously. Indeed, the mixed complementarity format, in which we solve both the GE and PE modules, is particularly attractive as an integrated framework in which complementary slackness conditions, e.g. activity analysis, can be readily incorporated along with conventional neoclassical production functions. In the present application, however, dimensionality and non-convexities argue strongly in favor of decomposition. When we solve the industrial organization model on a market by market basis, we avoid dealing with

excessively high dimensionalities which otherwise arise when there are large numbers of both goods and markets. In addition, we find that decomposition leads to a significant improvement in robustness of the solution method.

The Melitz model incorporates two types of non-convexity. The first is the conventional interaction of prices, quantities and incomes. Income effects are the source of most of the difficulties in proving convergence for the complementarity algorithms. [Mathiesen (1987)]. The second non-convexity is associated with the Dixit-Stiglitz aggregation and productivity effects. While it is possible to solve general equilibrium models including Dixit-Stiglitz effects [Markusen (2002)], it is well known that even small instances of the problem class can be extremely difficult. Our decomposition approach seems to avoid these computational difficulties by a “divide and conquer” strategy in which income effects are handled in one submodule and productivity effects in a second module.

#### 4.1 Partial Equilibrium Module

The exogenous links that make the PE module operational are the expenditure levels in each region,  $\bar{E}_r$ , (which establish demand for manufactured goods) and the prices,  $\bar{c}_r$ , and quantities,  $\bar{Y}_r$ , of the composite inputs to manufacturing. The model needs some flexibility to react to shocks, however, so we assume a constant-elasticity input-supply function centered (each iteration) on the quantity of inputs used by the sector in the general equilibrium ( $\bar{Y}_r$ ). Input supply is thus  $\bar{Y}_r (c_r / \bar{c}_r)^\eta$ , where  $\eta > 0$  is the elasticity. If the PE model is consistent with the general equilibrium  $c_r = \bar{c}_r$ , where  $c_r$  satisfies the equilibrium conditions in both modules.

Table 1 summarizes the nonlinear conditions in the PE module and establishes the complementarity between equations and associated variable. In addition to the conditions developed in the previous section we add the input-market clearance condition (which deter-

Table 1: PE module; multiregion heterogeneous-firms partial-equilibrium

Equilibrium Condition	(Equation)	Associated Variable	Dimensions
Zero cutoff profits (ZCP)	(14)	$N_{rs}$ : Number of operating firms	$R \times R$
Free entry (FE)	(17)	$M_r$ : Mass of firms taking a draw	$R$
Dixit-Stiglitz preferences	(4)	$P_r$ : Price index	$R$
Firm-level demand	(5)	$\tilde{q}_{rs}$ : Average-firm quantity	$R \times R$
Firm-level pricing	(7)	$\tilde{p}_{rs}$ : Average-firm price	$R \times R$
CES wtd. Average $\varphi$	(11)	$\tilde{\varphi}_{rs}$ : Average-firm productivity	$R \times R$
Pareto dist. Marginal $\varphi$	(18)	$\varphi_{rs}^*$ : Marginal-firm productivity	$R \times R$
Input-market clearance	(19)	$c_r$ : Composite-input price	$R$
Capacity constraint	(20)	$\pi_{rs}^c$ : Capacity rents	$R \times R$
<b>Total Dimensions:</b>			$3R + 6R^2$

mines  $c_r$ )

$$\bar{Y}_r \left( \frac{c_r}{\bar{c}_r} \right)^\eta = \delta f_r^e M_r + \sum_s N_{rs} \left( f_{rs} + \frac{\tilde{q}_{rs} \tau_{rs}}{\tilde{\varphi}_{rs}} \right), \quad (19)$$

and the complementary-slack condition for determining capacity rents ( $\pi_{rs}^c$ )

$$M_r - N_{rs} \geq 0; \quad \pi_{rs}^c \geq 0; \quad \pi_{rs}^c (M_r - N_{rs}) = 0. \quad (20)$$

As noted above, in a steady-state equilibrium  $\pi_{rs}^c$  will equal zero, but the computational model benefits from an explicit constraint that prevented numeric moves where  $N_{rs} > M_r$ .<sup>21</sup>

## 4.2 General Equilibrium Module

The General Equilibrium Module (GE) is formulated as a standard constant-returns model of world trade in all products. Consumers have preferences over goods differentiated by region of origin (the Armington assumption). Consider the unit expenditure function associated with region- $s$  purchases of goods of type  $k$  (we reintroduce the commodity index,  $k \in K$ , in

<sup>21</sup>This also indicates how the model might be extended into an intertemporal context where  $M_r$  cannot adjust instantaneously.



Table 2: GE module: multiregion constant-returns general equilibrium

Equilibrium Condition	(Equation)	Associated Variable	Dimensions
<b>Optimality conditions:</b>			
Expenditure function	(22)	$E_r$ : Total Expenditures	$R$
Zero-profits Armington Activity	(21)	$A_{kr}$ : Armington activity level	$K \times R$
Zero-profits Production	(6)	$Y_{kr}$ : Sectoral Output	$K \times R$
<b>Market-clearance conditions:</b>			
Input markets	(23)	$w_{jr}$ : Factor price by type	$J \times R$
Product markets	(24)	$c_{kr}$ : Composite-input price	$K \times R$
Armington-Composite markets	(25)	$P_{kr}$ : Price index by commodity	$K \times R$
Gross supply and demand	(26)	$P_r^E$ : Aggregate price index	$R$
<b>Income balance:</b>			
Final demand	(27)	$W_r$ : Hicks welfare index	$R$
Factor income + tariff revenue	(28)	$I_r$ : Nominal income	$R$
<b>Total Dimensions</b>			$4R$ $+4(K \times R)$ $+(J \times R)$

the general equilibrium):

$$P_{ks} = \psi_{ks} \left[ \sum_r \xi_{krs} [(1 + t_{krs}) c_{kr}]^{1-\sigma} \right]^{1/(1-\sigma)} \quad (21)$$

Notice that we define preferences directly over the composite manufacturing inputs from region  $r$ , which trade at a price of  $c_{kr}$ . The total and relative productivity parameter  $\psi_{ks}$  and  $\xi_{krs}$  control the functional calibration. These are the instruments through which the GE module is affected by the PE solution.

Table 2 summarizes the full set of equilibrium conditions in the GE module. First we define the aggregate Cobb-Douglas expenditure function;

$$P_r^E = \prod_k (P_{kr})^{\alpha_k}. \quad (22)$$

The remaining transformation technologies are given by our characterization of the commodity-specific price index, (21), and the composite-input cost function, (6).

Each price (index) has an associated market. Let  $\bar{e}_{jr}$  be the exogenous endowment of factor  $j$  in region  $r$ . This will equal the quantity demanded;

$$\bar{e}_{jr} = \sum_k \frac{\beta_{jkr} c_{kr} Y_{kr}}{w_{jr}}. \quad (23)$$

In turn the supply of the composite-input activity will equal demand (as derived from the Armington activity):

$$Y_{kr} = \sum_s \xi_{krs} \psi_{ks} A_{ks} \left( \frac{P_{ks}}{(1 + t_{krs}) c_{kr}} \right)^\sigma. \quad (24)$$

Supply of the Armington composite equals gross demand:

$$A_{kr} = \frac{\alpha_k E_r}{P_{kr}}. \quad (25)$$

Gross expenditures equal the value of final demand plus the value of intermediate use:

$$E_r = P_r^E W_r + \sum_k \beta_{kr}^E c_{kr} Y_{kr}. \quad (26)$$

The welfare index is calculated directly from ratio of income to the price of the aggregate commodity:

$$W_r = \frac{I_r}{P_r^E}. \quad (27)$$

Income in a region equals the value of factor endowments plus tariff revenues:

$$I_s = \sum_j w_{js} \bar{e}_{js} + \sum_k \sum_r t_{krs} \xi_{krs} \psi_{ks} A_{ks} \left( \frac{P_{ks}}{(1 + t_{krs}) c_{kr}} \right)^\sigma. \quad (28)$$

### 4.3 Full Solution

The challenge to arriving at a fully consistent general equilibrium is to adjust the  $\psi_{ks}$  and  $\xi_{krs}$  (where  $k = \text{Manufacturing}$ ) such that aggregate supply of the manufacturing composite

and relative demands for inputs are consistent with the PE solution. Changes in the number of firms will indicate total and relative productivity changes in the composite inputs. Once these productivity changes are incorporated the GE module can be solved to find a new set of gross expenditures, input prices, and input quantities to pass back to the PE module. At the global solution there are no additional adjustments in the  $\psi$  and  $\xi$ , and the common variables across the PE and GE modules have the same solution values.

In passing information from the PE module to the GE, we first establish total factor productivity by relating  $\psi_{ks}$  to the ratio of the price index and the total value of manufacturing expenditures:

$$\psi_{ks} = \frac{P_{ks}}{\alpha_{ks} E_r}. \quad (29)$$

An increase in the total number of varieties consumed in region  $s$ ,  $(\sum_r N_{rs})$ , indicate a decrease in the computed PE price index. Through equation (29) the Dixit-Stiglitz effect is carried over to the GE Armington technology. Finding the relative productivity changes involves finding the set of  $\xi_{krs}$  that are consistent with the value of input demands in the PE module. All payments are to the factors so we must have:

$$c_{kr} \xi_{krs} \psi_{ks} A_{ks} \left( \frac{P_{ks}}{(1+t_{krs})c_{kr}} \right)^\sigma = N_{krs} \tilde{p}_{krs} \tilde{q}_{krs}. \quad (30)$$

Solving for  $\xi_{krs}$ , and noting that  $A_{ks}$  is the inverse of the new  $\psi$ , we have

$$\xi_{krs} = \frac{N_{krs} \tilde{p}_{krs} \tilde{q}_{krs}}{c_{kr} \left( \frac{P_{ks}}{(1+t_{krs})c_{kr}} \right)^\sigma} \quad (31)$$

The recalibration of the constant returns GE ( $\psi_{ks}$  and  $\xi_{krs}$ ) based on the heterogeneous-firms PE solution, and the subsequent recalculation of the  $\bar{E}_r$ ,  $\bar{c}_r$ , and  $\bar{Y}_r$ , has proven to be a robust solution method. The iterative procedure stops at the point that all variables common to the PE and the GE are consistent and there is no further recalibration indicated.

## 5 Nonlinear Least-squares Estimation

### 5.1 Estimation strategy

Consider that the  $B = 3R + 6R^2$  nonlinear conditions in the PE module presented above might be written as  $F(\mathbf{x}, \boldsymbol{\gamma}) = 0$  which implicitly maps a set of exogenous parameters,  $\boldsymbol{\gamma} \in \mathbb{R}^A$ , to a vector of endogenous variables  $\mathbf{x} \in \mathbb{R}^B$ . Let  $\hat{\boldsymbol{\gamma}} \in \{\mathbb{R}^{\hat{A}} : \hat{A} \leq A\}$  denote a vector of core parameters to be estimated, and let  $\hat{\mathbf{x}} \in \{\mathbb{R}^{\hat{B}} : \hat{B} \leq B\}$  denote a key endogenous series (e.g., bilateral trade flows). Our estimation strategy is to find the  $\hat{\boldsymbol{\gamma}}$  that minimize the sum of the squared differences between the  $\log \hat{\mathbf{x}}$  and observed  $\log \mathbf{x}^0$  subject to  $F(\mathbf{x}, \boldsymbol{\gamma}) = 0$  and an additional  $A - \hat{A}$  direct assumptions about the values of the remaining parameters:

$$\begin{aligned} \min_{\{\hat{\boldsymbol{\gamma}}, \hat{\mathbf{x}}\}} \quad & (\log \hat{\mathbf{x}} - \log \mathbf{x}^0)^T (\log \hat{\mathbf{x}} - \log \mathbf{x}^0) \\ \text{subject to:} \quad & F(\mathbf{x}, \boldsymbol{\gamma}) = 0, \\ \text{and} \quad & \tilde{\boldsymbol{\gamma}} = k, \end{aligned}$$

where  $\tilde{\boldsymbol{\gamma}}$  are the assumed parameters and  $k$  is a vector of constants. We minimize the logged errors to be consistent with the empirical trade literature, which often assumes a log-linear form of the trade equation.

We utilize data that is commonly employed in gravity estimations. The economic data includes gross manufacturing output by region, bilateral trade flows, and measured tariffs. Because we are interested in fitting a complete general equilibrium (including various non-manufacturing sectors), we take these data from the Global Trade Analysis Project (GTAP) [Dimaranan (2006)], a data set commonly employed in general equilibrium simulations of trade policy changes.<sup>22</sup> The GTAP data has been balanced for use in general equilibrium studies (household income equals expenditure, for example). The data are aggregated to include nine regions,

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<sup>22</sup>We supplement this with data on international distances from Head and Mayer (2002).

CHN China	NAF North America	LAM Latin America
EUR Europe	EER Eastern Europe and FSU	JKT Jpn, Korea, and Taiwan
ROA Rest of Asia	ANZ Australia and N. Zel.	ROW Rest of World ;

and seven aggregate sectors,

AGR Agriculture	MTL Mtls-related industry	EIS Other Energy Intensive
MFR Manufacturing	SER Services	ENG Energy
CGD Savings good .		

Presently we only estimate the heterogeneous-firms model over the aggregate manufacturing sector (the subscript  $k$  is thus suppressed in the remainder of our estimation description). The other sectors in the general equilibrium are assumed competitive and calibrated via the usual techniques.<sup>23</sup>

In addition to the economic data we utilize distances between regions to inform transportation costs. Consistent with the gravity literature we assume that iceberg trade costs take on the following form:

$$\tau_{rs} = d_{rs}^{\theta};$$

where  $\theta$  is an estimated distance elasticity.<sup>24</sup> In addition, we include rent generating ad valorem tariffs as measured in the GTAP data,  $t_{rs}$ . The c.i.f. import prices, thus, includes the variable trade costs  $(1 + t_{rs})\tau_{rs}$ .

Taking the GTAP data as given, and given our assumed structure of trade costs, we have the following candidates for inclusion in  $\hat{\gamma}$ :

- $\sigma$  : the inter-variety elasticity of substitution
- $\delta$  : the probability of firm death,
- $a$  : shape parameter for the Pareto distribution,
- $b$  : minimum productivity parameter for Pareto distribution,
- $\theta$  : distance elasticity of iceberg trade costs,
- $f_r^e$  : fixed entry cost,
- $f_{rs}$  : bilateral fixed cost of shipping from region  $r$  to region  $s$ .

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<sup>23</sup>We assume an Armington trade structure, with constant returns and perfect competition, in the sectors other than manufacturing.

<sup>24</sup>We scale distance such that  $\tau = 1$  on the shortest link.

Informing these parameters off observed bilateral flows is not meaningful unless we are willing to significantly reduce the parameter space (beyond our implicit assumptions that the core distribution, substitution, and transport cost parameters are identical across regions). With  $R$  regions there are potentially  $\hat{B} = R^2$  observable flows, but there are at least  $R^2 + R + 5$  parameters. We might eliminate  $\sigma$  from the list of parameters; noting that we are interested in identifying trade costs conditional on second-order curvature.<sup>25</sup> We assume that  $\sigma = 3.8$  throughout our analysis following the plant-level empirical analysis of Bernard et al. (2003). In addition, we directly assume the values  $\delta = 0.025$ ,  $f_r^e = 2$ , and  $b = 0.2$  following Bernard et al. (2007).<sup>26</sup>

The primary assumption that we employ to reduce the parameter space is to impose structure on the fixed costs. Let  $f_r^p$  be a fixed cost that is specific to goods produced in region  $r$ , and  $f_s^x$  be a fixed cost that is specific to goods exported to region  $s$ . Now consider decomposing the bilateral fixed costs as follows:

$$f_{rs} = f_r^p + f_s^x + f_{rs}^r.$$

When  $r = s$  the  $f_s^x$  term drops out reflecting the idea that  $f_s^x$  is an outward trade barrier. The  $f_{rs}^r$  are idiosyncratic residual bilateral costs. In the initial estimation  $f_{rs}^r$  is assumed to be zero. The number of parameters to be estimated is thus reduced to  $\hat{A} = 2n + 2$ . Essentially, we are assuming that the expected  $f_{rs}^r$  is zero. Once the core parameters are estimated, and locked down, the system can be used to calculate the matrix of residual  $f_{rs}^r$  which generate an exact fit on trade flows.<sup>27</sup>

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<sup>25</sup>Our approach is similar to Anderson and van Wincoop (2003) in that we estimate trade costs conditional on  $\sigma$ . Alternatively, Hummels (2001) uses direct measures of transportation costs to estimate  $\sigma$  in a gravity framework.

<sup>26</sup>Bernard et al. (2007) explain that changes in  $f^e$  rescales the mass of firms where as changes in  $\delta$  rescale the mass of entrants relative to the mass of firms. For consistency we simply adopt their values.

<sup>27</sup>Balistreri and Hillberry (2005) use a similar technique of stochastic estimation and subsequent exact-fit calibration. This entails a consideration that the econometric residuals might logically be interpreted as idiosyncratic calibration parameters (as opposed to measurement error).

## 5.2 Estimation Results

The primary purpose of our nonlinear estimation is to complete an exact calibration of the numerical model. This entails a complete enumeration of the structural parameters necessary to reconcile the structural model with observed data. The primary parameters of interest are those that are taken to be common across the world: the shape of the implied Pareto distribution of productivity draws,  $a$ , and the distance elasticity of trade costs,  $\theta$ . The model links both these parameters to the geographic pattern of trade.<sup>28</sup>

We conduct three econometric calibrations of the model. In the first, we allow both  $\theta$  and  $a$  to be free parameters; they take the values that minimize the econometric objective, subject to the constraints defined by the model and our choices of the parameters in  $\bar{\gamma}$ . Very good estimates of  $\theta$  appear in the literature, and our second set of estimates constrains the estimation procedure to replicate a commonly accepted value,  $\bar{\theta} = 0.27$ . As a sensitivity check, our third set of estimates imposes the constraint  $\bar{\theta} = 0.46$ .<sup>29</sup> Our estimates of key structural parameters appear in Table 3.

The interaction between  $\theta$  and  $a$  is a key point of interest. Conditional on the bilateral trade pattern, our procedure must assign responsibility for trade reductions to these two parameters (along with the fixed costs).<sup>30</sup> Our unconstrained estimate of  $\theta$  is  $\hat{\theta} = 0.139$ , while  $\hat{a} = 5.685$ . This is a relatively low estimated distance elasticity, and a somewhat high estimate for the Pareto distribution parameter.<sup>31</sup> As we constrain  $\bar{\theta}$  to higher values, the

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<sup>28</sup>The link between the geographic pattern of trade and  $\theta$  is straightforward. Chaney (2006) shows that  $a$  exerts a substantial influence on the geography of bilateral trade, via the extensive margin.

<sup>29</sup>These latter two estimates are taken from Hummels (2001), who estimates  $\theta$  directly off observed transportation cost margins. 0.27 is Hummels' central estimate, using data from 7 countries that report transport margins in their international trade statistics. 0.46 is the elasticity of air freight charges with respect to distance in U.S. data, and Hummels uses this as a plausible upper bound on  $\theta$ . Our unconstrained estimate of  $\theta$  lies well below Hummels' central estimate, and we treat this as a lower bound.

<sup>30</sup>As Chaney (2006) explains,  $a$  affects the trade volume via the extensive margin.  $\theta$  governs the degree to which prices rise over distance, leading consumers to reduce imports along the intensive margin of trade.

<sup>31</sup>As noted earlier, Hummels' central estimate of  $\theta$  is 0.27. Estimates of  $a$  - which are taken from distributions of plant/firm level market shares - vary, and are conditional on a choice of  $\sigma$ . Bernard et al. (2007) choose  $a = 3.4$ , and the estimates in Eaton et al. (2004) imply  $a = 4.2$  under our maintained assumption that  $\sigma = 3.8$ .

Table 3: Nonlinear estimation results: (dependent variable is log bilateral flows; core fixed parameters are  $\sigma = 3.8$ ,  $\delta f^e = 0.05$ , and  $b = 0.2$ )

		Specification		
		$\theta = \text{free}$	$\bar{\theta} = 0.27$	$\bar{\theta} = 0.46$
<b>Pareto shape parameter: <math>a</math></b>		5.685 (0.810)	4.753 (0.148)	3.671 (0.040)
<b>Distance elasticity: <math>\theta</math></b>		0.139 (0.037)	0.27	0.46
<b>Source-specific fixed cost: <math>f_r^p</math></b>				
CHN	China	0.837 (0.422)	0.365 (0.204)	5.668 (16.225)
NAF	North America	1.314 (0.630)	0.798 (0.302)	68.701 (73.782)
LAM	Latin America	0.792 (0.593)	0.139 (0.074)	0.080 (0.041)
EUR	Europe	0.792 (0.358)	0.316 (0.102)	2.261 (1.945)
EER	Eastern Europe and FSU	1.575 (0.575)	0.282 (0.060)	0.010 (0.006)
JKT	Jpn, Korea, and Taiwan	0.153 (0.206)	0.068 (0.038)	0.039 (0.067)
ROA	Rest of Asia	0.125 (0.170)	0.024 (0.009)	0.012 (0.005)
ANZ	Australia and N. Zel.	0.045 (0.066)	0.031 (0.003)	0.018 (0.030)
ROW	Rest of World	3.577 (1.765)	1.539 (0.465)	0.128 (0.050)
<b>Destination-specific fixed cost: <math>f_r^x</math></b>				
CHN	China	4.134 (2.744)	0.551 (0.594)	1.976 (15.066)
NAF	North America	0.830 (0.856)	0.018 (0.028)	<b>0.0 bound</b> (0.021)
LAM	Latin America	14.789 (13.309)	3.985 (4.822)	33.463 (14.460)
EUR	Europe	1.046 (0.928)	0.062 (0.059)	<b>0.0 bound</b> (0.005)
EER	Eastern Europe and FSU	68.505 (29.806)	51.883 (5.905)	48.791 (2.426)
JKT	Jpn, Korea, and Taiwan	0.138 (0.432)	0.096 (0.314)	0.414 (4.605)
ROA	Rest of Asia	0.704 (0.668)	0.147 (0.254)	11.814 (19.627)
ANZ	Australia and N. Zel.	0.684 (0.484)	0.773 (0.504)	5.146 (17.296)
ROW	Rest of World	67.956 (53.776)	97.096 (33.536)	83.177 (5.567)



Table 4: Heterogeneity in the productivity distribution:  $\frac{\varphi}{b}$  for selected values of  $a$

	Percentiles			
	50th	75th	90th	95th
<b>Fitted values of <math>a</math></b>				
$\hat{a}=5.685$	1.130	1.276	1.499	1.694
$\hat{a}=4.753$	1.157	1.339	1.623	1.878
$\hat{a}=3.671$	1.208	1.459	1.872	2.262
<b>Implied estimate from Eaton et al. (2004)</b>				
$a=4.2$	1.179	1.391	1.730	2.041
<b>Value used in Bernard et al. (2007)</b>				
$a=3.4$	1.226	1.503	1.968	2.414

estimated value of  $a$  falls. For  $\bar{\theta} = 0.27$ ,  $\hat{a} = 4.753$ , and for  $\bar{\theta} = 0.46$ ,  $\hat{a} = 3.671$ .

The lower values of  $a$  that occur in our restricted estimates imply greater heterogeneity in firm productivities. Table 4 illustrates some features of the productivity distributions implied by different values of  $a$ . For our unconstrained estimate of  $a$ , a firm with a productivity draw at the median of the distribution would be 1.130 times as productive as a firm with the minimum draw. As  $a$  falls, the productivity distribution flattens out. In our subsequent counterfactual scenarios, we will be employing the constrained estimate  $a = 4.753$ , the estimate corresponding to  $\bar{\theta} = 0.27$ . In this case, the median productivity draw is 1.157 times the size of the minimum draw.

The structural estimation results in Table 3 also contain estimated values of source- and destination-specific fixed costs. In the data, regions will differ in the share of manufacturing output that they export.<sup>32</sup> Structural estimation interprets such variation largely through variation in the source-specific fixed cost  $f_r^p$ . The lowest of these costs appear to be in Rest of Asia (ROA) and Australia and New Zealand (ANZ). Both relative and absolute estimates of fixed production costs vary with  $\theta$  and  $a$ .<sup>33</sup>

<sup>32</sup>Some of the observed variation in this statistic will arise because regions vary in their non-manufacturing sectors' use of manufacturing output. Our structural estimation procedure maintains these idiosyncrasies throughout calibration, so variation of this sort does not pollute the estimates of  $f_r^p$ .

<sup>33</sup>Standard errors for the estimated fixed costs are generally tighter when the distance elasticity is fixed. Fixing  $\theta$  allows  $a$  to be more precisely estimated (exploiting any unexplained variation in the distance elasticity of trade, for example). More precise estimates of  $a$  allow more precise estimates of the fixed costs in the model.

Table 5: Residual Bilateral Fixed Trade Costs ( $f_{rs}^r$ )

Source	Destination								
	CHN	NAF	LAM	EUR	EER	JKT	ROA	ANZ	ROW
CHN	-0.055	-0.259	-1.629	0.461	-15.612	0.243	1.785	0.249	6.163
NAF	-0.364	0.371	0.003	0.460	135.899	-0.346	-0.531	-0.826	60.143
LAM	-0.275	0.185	-0.035	0.133	-46.115	0.076	-0.146	1.628	-44.404
EUR	-0.193	0.160	-0.941	0.312	-32.652	0.226	-0.025	-0.773	-3.190
EER	0.214	0.940	6.883	-0.018	-0.090	0.677	1.058	1.131	42.623
JKT	-0.047	0.063	-3.003	0.331	5.381	-0.011	0.060	-0.311	-38.246
ROA	0.572	-0.005	-1.556	0.077	-10.019	0.094	6.9E-4	-0.146	-43.248
ANZ	-0.183	0.022	-2.091	-0.034	-37.023	-0.097	-0.010	-2.3E-4	-76.522
ROW	4.378	-0.837	26.601	-0.078	5.801	1.007	1.068	1.535	0.182

The fitting procedure also defines fitted destination-specific fixed costs,  $f_r^x$ . These parameters would largely be identified off of observed *home bias* in fitted trade flows, conditional on the other structural parameters. The estimates reported here suggest relatively low fixed costs of importing to the most developed regions in the data. Recall that the GTAP tariff data are included in the calibration, so these are best interpreted as implicit non-tariff barriers to trade (averaged across origins).<sup>34</sup>

Once the core parameters reported in Table 3 are established we can freeze these at their point estimates and find a set of residual bilateral costs,  $f_{rs}^r$ , that give us perfect consistency with observed trade flows. We report these estimates for the constrained estimates ( $\bar{\theta} = 0.27$ ) in Table 5. From the perspective of the nonlinear estimation these are effectively econometric residuals—they allow the structure to fit the data exactly. Alternatively, from the perspective performing theory consistent counterfactual analysis they are idiosyncratic calibration parameters.<sup>35</sup>

<sup>34</sup>These and subsequent estimates take the rather extreme view that any unexplained reduction in fitted trade volumes should be attributed to trade costs of one type or another. This interpretation is in keeping with much of the geography-of-trade literature. The trade policy literature typically focuses only on *known* trade barriers, and attributes unexplained variation in trade flows to Armington distribution parameters. Balistreri and Hillberry (2005) explain that these two approaches are best understood as different identification strategies for fitting the trade pattern.

<sup>35</sup>Hillberry et al. (2005) show the usefulness of framing standard general-equilibrium calibration exercises

Table 6: Total Bilateral Fixed Trade Costs ( $f_{rs}$ )

Source	Destination								
	CHN	NAF	LAM	EUR	EER	JKT	ROA	ANZ	ROW
CHN	0.310	0.123	2.721	0.888	36.636	0.704	2.297	1.387	103.624
NAF	0.986	1.169	4.785	1.320	188.581	0.549	0.414	0.745	158.037
LAM	0.415	0.341	0.104	0.333	5.906	0.310	0.140	2.539	52.831
EUR	0.675	0.493	3.360	0.628	19.547	0.638	0.438	0.316	94.222
EER	1.047	1.240	11.150	0.326	0.193	1.055	1.487	2.186	140.001
JKT	0.572	0.148	1.049	0.460	57.331	0.057	0.274	0.530	58.917
ROA	1.147	0.037	2.452	0.163	41.888	0.214	0.024	0.650	53.872
ANZ	0.399	0.070	1.924	0.058	14.891	0.030	0.168	0.031	20.605
ROW	6.468	0.719	32.124	1.523	59.223	2.642	2.754	3.846	1.721

Table 6 shows the full matrix of total bilateral fixed costs including the residual plus the source and destination charges. So, for example, we might consider that import penetration into EER is difficult, but it is particularly difficult for North American firms. On this particular link the residual fixed cost of 188.6 is more than three times as large as the base destination charge to get into EER, 51.9. The fixed costs into ROW are also large, but this is attributable to aggregation bias, as this region represents many small economies.<sup>36</sup>

## 6 Counterfactual Simulations

We analyze four scenarios that compare the impacts of tariff and fixed cost reductions:

- (A) Armington constant-returns formulation with a 50% reduction in manufacturing tariffs;
- (B) Heterogeneous-firms model with a 50% reduction in manufacturing tariffs;
- (C) Heterogeneous-firms model with a 50% reduction in fixed trade costs; and
- (D) Heterogeneous-firms model with both the tariff and fixed cost cuts.

as the systematic identification of idiosyncratic residual parameters. As in any standard econometric exercise these residual parameters are useful indicators of model fit.

<sup>36</sup>When we assume that the ROW aggregate is a large integrated market, large fixed costs are needed to explain the relatively low volume of trade.

Table 7: Counterfactual Welfare Impacts (% Equivalent Variation)

	Scenario			
	A	B	C	D
	CRTS-Tariff	Tariff	Fix Cost	Both
<b>Region</b>				
CHN	0.3	1.3	3.3	5.2
NAF	-0.0	0.0	0.7	0.8
LAM	0.1	0.5	1.6	2.4
EUR	0.1	0.2	1.3	1.7
EER	-0.1	-0.3	4.3	4.6
JKT	0.1	0.3	1.9	2.2
ROA	0.3	1.1	5.3	6.5
ANZ	0.4	1.4	2.2	4.3
ROW	-0.2	-0.7	2.4	1.9

Scenario (A) is a reference case where we assume a standard Armington trade structure and constant-returns production.<sup>37</sup> Table 7 shows welfare changes induced by the tariff cut. Although most regions gain from the tariff cuts, three regions suffer welfare losses (NAF, EER, and ROW).<sup>38</sup> Examining the same tariff cuts in the heterogeneous-firms model (Scenario B) indicates substantially greater gains. Taking the simple-average welfare change across regions the heterogeneous-firms structure indicates welfare gains from liberalization that are nearly four times larger than in the baseline case. The simple-average welfare gain of 0.4% in Scenario B may not seem particularly impressive, but consider the following statistics from our aggregation of the GTAP data: gross manufacturing output is only 26% of world gross output, only 15% of manufacturing output is traded to another region, and the simple average benchmark tariff on these flows is only 9.3%. So the typical tariff cut is less than 5%, and applies to less than 4% of gross output. In this context, an average welfare gain of 0.4% seems quite large. With the exception of the ROW and EER regions, tariff cuts in the heterogeneous-firms structure produce larger net welfare gains than the constant returns benchmark.

<sup>37</sup>We simply run the tariff cut on the GE module without making the iterative productivity adjustments. This gives us a perfectly comparable constant-returns benchmark to judge the performance of the new theory.

<sup>38</sup>This is not particularly surprising. At low substitution elasticities (3.8 in this case) the Armington structure implies high optimal tariffs, so the benchmark tariff structure is likely to benefit some regions.

Table 8: Domestic-firm Productivity Growth (% Change)

Region	Scenario		
	B	C	D
	Tariff	Fix Cost	Both
CHN	0.8	1.5	2.7
NAF	0.3	1.1	1.6
LAM	1.1	1.5	3.2
EUR	0.4	1.7	2.3
EER	0.8	3.1	4.2
JKT	0.6	1.6	2.6
ROA	1.3	3.4	5.5
ANZ	2.1	3.1	6.5
ROW	0.6	1.6	2.5

In Scenario (C) we examine a 50% cut in the fixed costs associated with non-domestic trade links. This generates important gains across the board. The results are consistent with a recent trade literature focussing on the relative importance of unobserved (non-tariff) barriers and tariffs.<sup>39</sup> In Scenario (D) both the tariff and fixed cost reductions are combined. There are considerable increases in welfare under Scenario (D) considering that Manufacturing is the only sector being liberalized. The simple-average welfare gain under Scenario (D) is nearly 30 times larger than in the Armington reference case. Notice also that fixed-cost (or non-tariff barrier) reductions often complement tariff cuts. Absolute welfare increases of 1% to 6% are considerably larger than most computational estimates.<sup>40</sup>

As noted above, one of the key critiques of current policy simulation models is that they fail to account for the productivity growth associated with liberalization. Table 8 indicates the simulated gains in average productivity across firms active in their respective domestic markets. Consistent with the arguments put forward by the proponents of the heterogeneous-firms model, our simulations show productivity gains due to liberalization. Increased exposure to external markets, whether induced by a reduction in tariff or non-tariff barriers,

<sup>39</sup>Anderson and van Wincoop (2004)

<sup>40</sup>See Rutherford and Tarr (2002).

Table 9: Manufacturing Price Index,  $P_r$  (% Change)

	Scenario		
	B	C	D
	Tariff	Fix Cost	Both
<b>Region</b>			
CHN	0.8	-0.9	-0.9
NAF	-0.2	-1.7	-2.2
LAM	-0.7	-2.4	-3.5
EUR	0.2	-1.3	-1.5
EER	-0.9	-3.8	-5.0
JKT	0.3	0.4	-0.1
ROA	-0.5	-2.0	-3.3
ANZ	0.4	-2.6	-3.3
ROW	-2.7	-3.5	-5.6

induces productivity growth.

The other key component of the model is that trade policy affects the extensive margin. The number of foreign varieties increases when trade costs fall. The threshold for import penetration falls and more foreign firms find it profitable to enter a given market. In contrast, the effect of changes in trade costs on the number of domestic varieties is not clear. As Melitz (2003) explains there are two mechanisms by which the distribution of operational firms in a given country changes with trade. First, the number of exporting firms increases and the profits of all exporting firms increase, which induces entry of new varieties. The increased activity of these firms, however, bid up the input price. Thus, the second effect acts to induce exit of varieties with low productivity realizations.

On net, however, consumers will likely benefit from lost domestic varieties because factor returns increase and the remaining domestic varieties are less expensive. More productive firms optimally price lower, so eliminating low productivity firms depresses the average price. All of the variety and price effects can be summarized in the solution price index on manufactured goods,  $P_r$ . Table 9 presents the percentage change in the price index across the scenarios. Further, we break out the variety effects in Table 10. Although the number of

Table 10: Changes in the Number of Operating Firms,  $N_{r,s}$  (% Change)

	<b>Scenario</b>		
	B	C	D
	Tariff	Fix Cost	Both
<b>Imported Varieties</b>			
<b>(extensive margin):</b>			
CHN	30.4	200.3	287.1
NAF	12.5	162.3	198.4
LAM	29.1	167.0	248.8
EUR	16.7	190.4	237.1
EER	13.9	175.6	215.8
JKT	39.3	229.7	346.2
ROA	22.9	203.2	264.4
ANZ	28.5	179.4	247.7
ROW	12.8	157.5	198.1
<b>Domestic Varieties:</b>			
CHN	-2.2	-5.1	-8.3
NAF	-1.2	-5.1	-6.9
LAM	-4.1	-6.6	-12.2
EUR	-1.6	-7.1	-9.2
EER	-3.3	-9.9	-13.8
JKT	-2.3	-7.3	-10.7
ROA	-4.6	-12.6	-18.9
ANZ	-7.3	-14.1	-22.3
ROW	-2.8	-5.7	-9.6
<b>Total Varieties:</b>			
CHN	-0.3	6.9	9.0
NAF	6.5	89.3	108.9
LAM	-3.8	-5.5	-10.5
EUR	2.0	31.2	38.5
EER	-3.3	-9.3	-13.1
JKT	-1.4	-1.9	-2.5
ROA	-4.0	-8.0	-12.9
ANZ	-6.5	-9.8	-16.4
ROW	-2.7	-5.0	-8.7

overall varieties falls for many regions, trade growth on the extensive margin combined with lower domestic prices result in lower overall price indexes.

## 7 Conclusion

A broad body of empirical literature documents persistent differences in plant-level productivity. This literature has also shown that the reallocation of production activities, from less- to more-productive plants, is an important part of aggregate productivity growth. These basic characteristics of industrial organization have important implications for international trade and commercial policy. The unifying theory proposed by Melitz (2003) offers insights into these implications. Our contribution is to present a quantitative assessment of the effects of this new richer structure on simulated policy analysis. We illustrate that relatively modest liberalization generates substantial gains due to the predicted endogenous productivity improvements.

In the case of a 50% reduction in tariffs on traded manufactured goods the simple-average welfare gains are on the order of 4 times greater when we consider the new theory. These gains are complemented, and compounded, by reductions in the fixed costs. When we add a 50% reduction in cross-border fixed costs to the tariff cuts the welfare gains grow to roughly 30 times what is measured in the constant-returns reference liberalization.

Some truth in advertising is in order for our results. First, we employ a novel, if not radical, method for measuring unobserved fixed costs. We depart from the econometric literature by employing a nonlinear estimation that includes the *extensive-form* general equilibrium conditions as side constraints. Our focus is on arriving at fitted values, while maintaining complete consistency between the econometric and simulation models. Our estimation method is also a stark departure from traditional calibration methods used to fit simulation models; we do not allow preference-bias parameters to drive trade. The onus of explaining



the observed pattern of trade is on the theory and the standard parameters that appear in the theory, not on added preference-bias parameters. The very large fixed costs that we estimate are open to criticism, and we view them as crude indicators of how big the barriers may be.<sup>41</sup> It is generally accepted by economists that unobserved trade costs are an important component of the world trade equilibrium. We follow one of the few paths available, which is to accept the structure fully and use it to inform unobservables from the observables.

The second major caveat that we place on our results involves the data. We accept the GTAP data as given and further aggregate it. This is useful in terms of reducing computational complexity and in allowing us to efficiently summarize reports. The GTAP data are balanced; they have already been fitted to a set of fundamental accounting identities. The data are consistent with general-equilibrium adding-up restrictions, but the original fitting procedure weakens the validity of any statistical inference that one might draw from our estimation.

The usual aggregation biases abound in our data, and we have additional concerns given the theory's focus on firm-level behavior. Our aggregate manufacturing sector is not a satisfying definition of an industry or product. Regional aggregation is also problematic. The aggregate rest-of-world region is actually numerous small disjoint markets rather than a large integrated market. We probably overstate the fixed costs of entering the rest-of-world region because these are necessary for explaining the relative lack of trade with what appears to be a large region.

We thus present our estimates conditional on the particular aggregation of the data, the assumed structure, and our maintained hypotheses about key structural parameters. We see important extensions in the area of regional and industry disaggregation. Future research will also need to tackle the issue of consistent structural estimation. We are somewhat

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<sup>41</sup>These estimates are not open the critique leveled by Balistreri and Hillberry (2006), because the fixed costs measured here impinge on missing trade not existing trade. Firms choked out of a market due to fixed costs do not incur the fixed costs. The payment of these costs is therefore not observed.

unique in our development of an econometric method that facilitates directly, and fully consistent, welfare analysis of policy. Others may find this departure from standard regression analysis useful and relevant. We are firmly within the empirical-trade tradition, which places theory, not established statistical methods, as the foundation for analysis. Given the rich nature of contemporary theory we hope our empirical welfare analysis encourages others to continue developing the literature in this direction.

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