Representative Firm Exposition of the Firm Heterogeneity Model*

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JEL codes: F12, F14.

1 Introduction

There is a lively debate in the recent trade literature about the value added of firm heterogeneity in trade models. Arkolakis, et al. (2012) show that the welfare gains from trade can be expressed with two sufficient statistics, the domestic spending share and the trade elasticity. This holds in the Armington model, the Ricardian Eaton-Kortum model, the equal firms monopolistic competition Ethier-Krugman model and the firm heterogeneity Melitz model. The only difference is the interpretation of the trade elasticity. In Armington and Ethier-Krugman the trade elasticity is determined by the substitution elasticity between varieties, whereas in Eaton-Kortum and Melitz it is determined by productivity dispersion. Melitz and Redding (2013) instead show that trade cost reductions generate larger welfare gains in the Melitz firm heterogeneity model than it is equivalent with homogeneous firms, the Ethier-Krugman model.

Firm heterogeneity has not been incorporated in a comprehensive way in multisector CGE models. Most important work in this respect is Balistreri (2012), who have included firm heterogeneity in one sector in a CGE model with other sectors characterized by an Armington setup. Allowing for firm heterogeneity in all sectors might be useful for various reasons. First, it can shed light on the discussion about the value added of firm heterogeneity in trade models by exploring the differences in modelling outcomes with other models. Second, various realistic microeconomic features can be modelled like the distinction of welfare effects into an intensive and extensive margin effect. Third, CGE models contain a large degree of sectoral detail, but are sometimes somewhat outdated in terms of modelling setup. With the incorporation of firm heterogeneity in all sectors, this drawback would disappear.

In this paper we map out a parsimonious representation of firm heterogeneity enabling incorporation in multisector CGE models. In particular, we show that both the Ethier-Krugman and the Melitz model can be defined as an Armington model by generalizing the expressions for iceberg trade costs and for marginal costs and by allowing for a demand externality in the Melitz model. In Ethier-Krugman generalized marginal costs are a function of the number of input bundles leading to so-called variety scaling (Francois (2013)). Variety scaling also props up in the Melitz model, but on top of that generalized marginal costs are also a function of the price of input bundles. The reason is that the extensive margin is affected by the price of input bundles. For the same reason there is a demand externality in the Melitz model: in a larger market with a higher price index more firms can survive, raising the extensive margin. Generalized iceberg trade costs are a function of both fixed and iceberg trade costs and of the Armington CES shifter. We show theoretically that the Ethier-Krugman model is a special version of the Melitz model if the firm size distribution becomes granular. Granularity corresponds with a trade elasticity in Melitz equal to the substitution elasticity minus one.

For illustration, we implement the parsimonious representation of the different models in a one sector model employing GTAP trade and gross output data. In particular we compare the effects of reductions in iceberg trade costs in the three models for various parameter values. It is shown that ...

Costinot and Rodriguez-Clare (2013) compare the welfare effects of trade and trade liberalization in the different trade models in different setups. They show that the expression for the price index in the most general model, the firm heterogeneity model, nests the expressions in the Armington and Ethier-Krugman model. Their exposition is different in several respects. First, they concentrate on welfare and thus only derive an expression for the price index. Second, they do not write the different models as special versions of an Armington economy with generalized marginal costs, generalized trade costs and a demand side externality. Third, they use exact hat algebra to derive their results on the welfare effects of trade liberalization.

2 Model

2.1 General Setup

2.1.1 Demand

Consider an economy with J countries. Agents in country j have identical utility q_j^e with CES preferences over quantities of representative goods q_{ij} from all trading partners i:

$$q_j^e = e_j q_j = e_j \left(\sum_{i=1}^{J} \alpha_{ij} q_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

e_j is a demand side externality playing a role in the firm heterogeneity version of the model and q_j is utility without the demand side externality. α_{ij} is an Armington shifter for goods shipped from i to j. Demand for q_{ij} can be written as:

$$q_{ij} = \alpha_{ij}^{\sigma} p_{ij}^{-\sigma} P_j^{\sigma-1} E_j \tag{2}$$

p_{ij} is the price of the representative good. q_{ij} and p_{ij} are defined as the quantity and price of the representative good not taking into account the possible demand externality in the firm heterogeneity model.

P_j is the price index corresponding to q_j and defined as:

$$P_j^e = \frac{1}{e_j} P_j = \frac{1}{e_j} \left(\sum_{i=1}^{J} \alpha_{ij}^{\sigma} p_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3}$$

P_j^e and P_j are respectively the price index with and without the demand externality. E_j is expenditure by all agents in country j.

2.1.2 Production

In exporter i input bundles Z_i can be transformed into representative output x_i at a marginal cost c_i, $x_i = \frac{L_i}{c_i}$. The price of representative output x_i, p_i^x, can thus be written as $p_i^x = c_i p_{Z_i}$ with p_{Z_i} the price of input bundles. To transform a unit of representative output in country i into a unit consumed in country j, generalized iceberg trade costs t_{ij} have to be paid, $q_{ij} = \frac{x_i}{t_{ij}}$. Taking into account ad valorem tariffs ta_{ij}, the price of the representative good p_{ij} can thus be written as:

$$p_{ij} = (1 + ta_{ij})\, t_{ij} c_i p_{Z_i} \tag{4}$$

The Armington model, the Krugman/Ethier model and the Melitz model can all be seen as special versions of the above structure, depending upon how the demand externality e_j in equation (1) and how generalized iceberg trade costs t_{ij} and marginal cost c_i in the price of the representative good in equation (4) are specified.

2.2 Armington Economy

Perfectly competitive firms in country i produce homogeneous country i varieties with marginal cost b_i. So, input bundles Z_i can be transformed into output x_i according to $x_i = \frac{L_i}{b_i}$. With marginal cost pricing the price of output in country i, p_i^x, is given by, $p_i^x = b_i p_{Z_i}$. Firms face iceberg trade cost τ_{ij}. There is no demand externality in the Armington economy, so $e_j = 1$. Therefore, the Armington economy is characterized by equations (1)-(4) with the following expressions for c_i, t_{ij} and e_j:

$$c_i = b_i \tag{5}$$

$$t_{ij} = \tau_{ij} \tag{6}$$

$$e_j = 1 \tag{7}$$

2.3 Ethier/Krugman Economy

In the Ethier/Krugman economy, preferences are characterized by love for variety over varieties ω produced in different countries. Utility q_j^e can thus be defined over physical quantities (output) $o(\omega)$ of varieties $\omega \in \Omega_{ij}$ shipped from all exporters i:

$$q_j^e = \left(\sum_{i=1}^{J} \alpha_{ij} \int_{\omega \in \Omega_{ij}} o(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{8}$$

The corresponding price index is defined over the prices of physical quantities of the varieties, $p^o(\omega)$:

$$P_j^e = \left(\sum_{i=1}^{J} \alpha_{ij}^{\sigma} \int_{\omega \in \Omega_{ij}} p^o(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{9}$$

Firms in country i produce with an identical increasing returns to scale technology with fixed cost a_i and marginal cost b_i implying that each firm produces a unique variety. Increasing returns in combination with love for variety implies also that a larger number of input bundles leads to a more than proportional increase in utility. To capture this externality, representative output in country i, x_i, is defined as variety scaled output in the Ethier/Krugman economy. Employing the expressions for markup pricing, the free entry condition and factor market closure, variety scaled output can be expressed as follows:[1]

$$x_i = \frac{\gamma_{ek}}{b_i a_i^{\frac{1}{\sigma-1}}} Z_i^{\frac{1}{\sigma-1}} Z_i \tag{10}$$

With γ_{ek} a function of the substitution elasticity σ:

$$\gamma_{ek} = \frac{\sigma-1}{\sigma}\sigma^{\frac{1}{1-\sigma}} \tag{11}$$

x_i can be transformed into q_{ij} accounting for the iceberg trade costs τ_{ij}. There is no demand externality in the Ethier/Krugman economy, so $e_j = 1$.

So, the Ethier/Krugman economy is characterized by equations (1)-(4) with the following expressions for c_i, t_{ij} and e_j:

$$c_i = \frac{b_i a_i^{\frac{1}{\sigma-1}}}{\gamma_{ek}} Z_i^{\frac{1}{1-\sigma}} \tag{12}$$

$$t_{ij} = \tau_{ij} \tag{13}$$

$$e_j = 1 \tag{14}$$

2.4 Melitz Economy

In the Melitz economy preferences are like in Ethier/Krugman characterized by love for variety over varieties produced by different firms from different countries as in equation (8)-(9). Goods are produced by firms with heterogeneous productivity. To start producing, firms can draw a productivity parameter φ from a distribution $G_i(\varphi)$ after paying a sunk entry cost en_i. The distribution of initial productivities is Pareto with a shape parameter θ and a size parameter κ_i:

$$G_i(\varphi) = 1 - \frac{\kappa_i^{\theta_i}}{\varphi^{\theta_i}} \tag{15}$$

[1]Derivations in Appendix A

A higher θ reduces the dispersion of the productivity distribution and a higher κ_i raises all initial productivity draws proportionally. We impose $\theta_i > \sigma - 1$ to guarantee that expected revenues are finite.

The productivity of firms stays fixed and firms face a fixed death probability δ in each period. Firms either decide to start producing for at least one of the markets or leave the market immediately. In equilibrium there is a steady state of entry and exit with a steady number of entrants drawing a productivity parameter, implying that the productivity distribution of producing firms is constant.

Firms produce with an increasing returns to scale technology with marginal cost equal to $\frac{1}{\varphi}$. Firms pay fixed costs f_{ij} for each market in which they sell. The fixed costs are paid partly in input bundles of the source country and partly in bundles of the destination country according to a Cobb Douglas specification with a fraction μ paid in source country input bundles.

Since preferences are characterized by love for variety and production occurs with increasing returns to scale, an increase in the number of input bundles leads to a more than proportional increase in utility. To account for this externality, representative output is like in the Ethier/Krugman economy defined as variety scaled output.

Since productivity is heterogeneous, variety scaled output is also affected by input costs. Following Head and Mayer (2013) changes in costs lead to an adjustment in output along three margins, an intensive margin, an extensive margin and a compositional margin. Lower costs lead to more sales of firms already in the market, the intensive margin. This is a price effect and hence does not affect variety scaled output. Lower costs also raises the mass of firms that can produce profitably, the extensive margin. This leads to a rise in variety scaled output. And finally, lower costs reduces the average productivity of firms in the market, as more firms can survive, the compositional margin. This margin also affects variety scaled output. Accounting for the latter two margins, variety scaled output can be written as:

$$x_i = \frac{\left(\gamma_m \frac{\kappa_i^\theta}{\delta e n_i}\right)^{\frac{1}{\sigma-1}}}{L_i^{\frac{1}{\sigma-1}}} p_{Z_i}^{-\left(\frac{\theta-\sigma+1}{\sigma-1}+\mu\frac{\theta-\sigma+1}{(\sigma-1)^2}\right)} Z_i \tag{16}$$

With γ_m a function of σ and θ and an additional conversion parameter ψ for later use set equal to 1:

$$\gamma_m = \psi \left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1} \tag{17}$$

The first coefficient on w_i represents the effect on variety scaled output through the extensive and compositional margin as a result of changes in marginal costs and the second coefficients the effect resulting from changes in fixed costs.

Equation (16) corresponds with the following expression for c_i:

$$c_i = \left(\gamma_m \frac{\kappa_i^\theta}{\delta e n_i}\right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\left(\frac{\theta-\sigma+1}{\sigma-1}+\mu\frac{\theta-\sigma+1}{(\sigma-1)^2}\right)} Z_i^{\frac{1}{1-\sigma}} \tag{18}$$

x_i can be transformed into q_{ij} accounting for generalized iceberg trade costs, which are a function of both the iceberg trade costs τ_{ij}, fixed trade costs f_{ij} and the Armington shifters α_{ij}. Iceberg and fixed trade costs affect the transformation in the same way through the extensive and compositional margin as the price of their input bundles p_{Z_i} in the transformation of input bundles into variety scaled output bundles in equation (16). A larger value of the Armington shifters raises demand and therefore raises variety scaled output through the extensive relative to the compositional margin and thus reduces generalized iceberg trade costs. According to the same logic, higher tariffs ta_{ij} raise generalized trade costs.[2] The following expression for generalized iceberg trade costs can be derived:

$$t_{ij} = \alpha_{ij}^{-\frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}} \left((1+ta_{ij})^{\frac{\theta-\sigma+1}{\sigma-1}+\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right) \tau_{ij} \tag{19}$$

The two terms between brackets represent the effect of iceberg and fixed trade costs through the extensive and compositional margin on converting fob variety scaled output into cif variety scaled output.

Finally, the demand externality does play a role under firm heterogeneity, again driven by the extensive and compositional margin. The following expression can be derived for the demand externality e_j:

$$e_j = \left(\frac{q_j P_j^\sigma}{p_{Z_j}{}^{1-\mu}}\right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tag{20}$$

Both a larger price index P_j and a larger market size E_j raise the extensive margin relative to the compositional margin and thus raise utility q_j. A lower price of input bundles p_{Z_i} in the destination country also raises utility, as it drives up welfare through the extensive margin relative to the compositional margin.

[2] Profits are calculated dividing revenues inclusive of tariffs by tariffs, $\pi = \frac{r}{1+ta} - cq - f$. Costinot and Rodriguez-Clare call this demand shifting. The alternative would be cost shifting with profits calculated as $\pi = r - c\,(1+ta)\,q - f$. This makes it impossible to find an expression for the mass of firms as a function of market size, a problem also occuring in the Ethier/Krugman model.

The Melitz economy is characterized by equations (1)-(4) with the expressions c_i, t_{ij} and e_j given in equations (18)-(20).

2.5 Nesting

From the expressions in the previous 3 subsections it follows directly that Krugman/Ethier is a special case of Melitz up to a constant and Armington is a special case of both.

Melitz can be converted into an Ethier/Krugman model by setting θ equal to $\sigma - 1$, the size parameter of the productivity distribution κ_i equal to the inverse of marginal cost $\frac{1}{b_i}$, sunk entry costs times the death probability δen_i equal to the fixed cost a_i and the conversion parameter ψ in equation (17) as follows:

$$\psi = \left(\frac{\sigma}{\sigma - 1}\right)^{\theta - \sigma + 2} \sigma^{\frac{\theta}{\sigma - 1}} (\theta - \sigma + 1) \tag{21}$$

$\theta = \sigma - 1$ implies that the demand externality e_j is 1. It can be easily verified that the expressions for c_i and t_{ij} in equations (18)-(19) become equal to the price of the representative good in the Ethier/Krugman economy in equations (12)-(13). Ethier/Krugman can be converted into Armington by setting the marginal cost parameter c_i equal to b_i and thus dropping the variety scaling.

The intuition for why $\theta = \sigma - 1$ implies that Melitz leads to Krugman/Ethier is the following. As pointed out above a change in trade costs generates a change in trade flows along three margins, an intensive margin of already exporting firms, an extensive margin representing an increase in the mass of varieties and a compositional margin representing the change in average productivity of firms exporting. If trade costs fall, trade rises with an elasticity of $\sigma - 1$ along the intensive margin and with an elasticity θ along the extensive margin. It falls along the compositional margin with an elasticity $\sigma - 1$. So, if $\theta = \sigma - 1$, the extensive and compositional margin cancel out and only the intensive margin remains. Therefore, the model with heterogeneous firms works out identically as a model with homogeneous firms.

The conversion factor ψ in moving from Melitz to Ethier/Krugman is necessary. Without this conversion factor utility would become infinite. The reason is that $\theta = \sigma - 1$ would imply that average productivity would become infinite. Still, when θ approaches $\sigma - 1$ the effect of changes in trade costs will be identical to the effect in an Ethier/Krugman economy. Therefore, when studying the effect of policy changes we can apply the conversion factor ψ without any consequences.

3 Simulations

3.1 One Sector Model without Intermediate Linkages

3.1.1 Setup

Without intermediates in production, the input bundle Z_i and its price p_{Z_i} will be equal to respectively factor input bundles L_i and its price w_i. Imposing the general equilibrium condition that output $w_i L_i$ is equal to the value of exports to all destination countries j, leads to:

$$w_i L_i = \sum_{j=1}^{J} \frac{\alpha_{ij}^{\sigma} \left(t_{ij} c_i w_i\right)^{1-\sigma}}{\sum\limits_{k=1}^{J} \alpha_{kj}^{\sigma} \left(t_{kj} c_k w_k\right)^{1-\sigma}} w_j L_j \tag{22}$$

We have used in equation (22) that the absence of tariffs and trade imbalances implies $E_j = w_j L_j$. Equation (22) shows that in a single sector setting without intermediate linkages the externalities on the destination side do not affect wages. In a setting with intermediate linkages instead the price index feeds back into the price of input bundles, implying that the destination externalities play a role in solving the model.

The price index can be calculated for each country based upon the solved wages w_i using the expressions for P_j^e, P_j and p_{ij} in equations (3)-(4). In the Melitz economy the externality should be written as a function of $E_j = w_j L_j$ and P_j:

$$e_j = \left(\frac{w_j L_j P_j^{\sigma-1}}{{p_{Z_j}}^{1-\mu}} \right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}$$

Substituting the expressions for t_{ij} and c_i in the Melitz economy in equations (19)-(18) gives:

$$w_i L_i = \sum_{j=1}^{J} \frac{\frac{\kappa_i^{\theta}}{\delta e n_i} L_i w_i^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)} \alpha_{ij}^{\frac{\sigma\theta}{(\sigma-1)}} \tau_{ij}^{-\theta} f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}}{\sum\limits_{k=1}^{J} \frac{\kappa_k^{\theta}}{\delta e n_k} L_k w_k^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)} \alpha_{kj}^{\frac{\sigma\theta}{(\sigma-1)}} \tau_{kj}^{-\theta} f_{kj}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}} w_j L_j \tag{23}$$

With J equations (23) the model can be solved for J unknown w_i. Equation (23) nests both the Ethier/Krugman and Melitz model. The Ethier/Krugman model follows by setting $\theta = \sigma - 1$ and the Armington model follows by eliminating the variety scaling terms $\frac{\kappa_i^{\theta}}{\delta e n_i}$.

3.1.2 Calibration

We use trade and gross output data from GTAP on the year 2007 on 109 countries on which distance data are available. Distance data come from sources on real shipping distance. Trade flows are regressed on distance and importer and exporter fixed effects. Fitted distance is used as a proxy for generalized trade costs t_{ij}, where domestic trade costs are normalized at 1. Equation (22) is solved for wages w_i and input bundles L_i employing the different specifications of c_i and imposing that $w_i L_i$ is equal to gross output in the data to generate a baseline. International trade costs are then reduced employing the baseline number of input bundles to compare the effects of trade cost reductions in the different models.

Following Bernard, et al. (2007), the substitution elasticity σ is set at 3.8 in the baseline, the Pareto shape parameter θ at 3.4, sunk entry costs en_i at 2, the death probability δ at 0.025 and the Pareto size parameter κ_i at 0.2. The fraction of fixed input bundles paid in the origin country is set at 0.5 to be as general as possible. To be able to show that Ethier-Krugmand and Melitz lead to the same results, fixed costs a_i in Ethier-Krugman are set at δen_i and marginal costs a_i at $1/\kappa_i$ and in one simulation θ is set at 2.8, implying granularity.

3.1.3 Simulation Results

First, we compare the wage levels in the baseline Melitz and Ethier-Krugman models in the different countries. Wages are normalized at 1 in Albania. σ is set at 3.8 and θ at 3.4 in one simulation and at 2.8 in another simulation. Setting θ at 2.8 generates exactly the same model outcomes as in the Ethier-Krugman model. Figures 1 and 2 shows the relation between simulated real wages and gross output (excluding the USA) both for the Ethier-Krugman model and the Melitz model (with θ equal to 3.4). Larger countries display higher real wage levels, because of agglomeration effects. This effect is similar for the Ethier-Krugman model and for the Melitz model. The correlation between the Melitz model wages and the Ethier-Krugman wages is very large, 0.9999. The correlation of both with gross output is about 0.80. Table 1 shows summary statistics for both nominal wages and real wages for the Ethier-Krugman model and for various values of θ. The table makes clear that outcomes are exactly identical in case of granularity. Moreover, the table makes clear that agglomeration effects are stronger in the Ethier-Krugman model than in the Melitz model. This follows from the fact that the standard deviation of both nominal and real wages is larger in the Ethier-Krugman model.

Second, we evaluate the effects of reductions in generalized iceberg trade costs t_{ij} for the

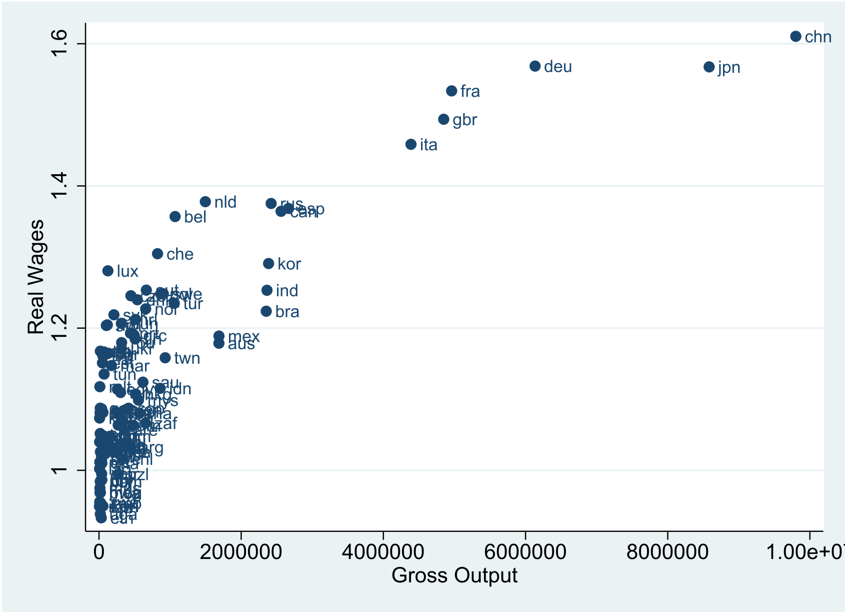

Figure 1: Wages and Gross Output in Ethier-Krugman Model

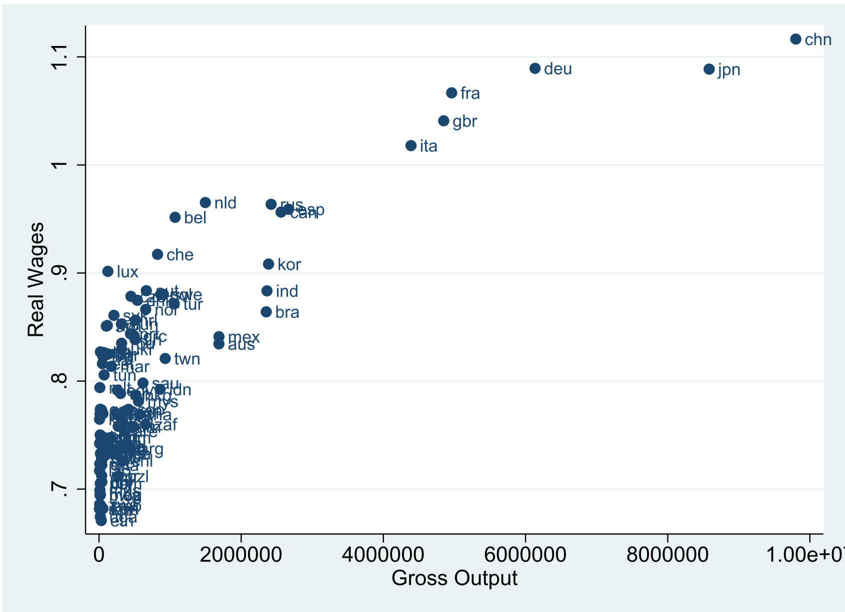

Figure 2: Wages and Gross Output in Melitz Model with $\theta = 3.4$

Table 1: Summary Statistics Wages and Real Wages

Variable	Trade Elasticity	Mean	Std. Dev.
Wages Melitz	4	0.99	0.042
Wages Melitz	3.4	0.989	0.049
Wages Melitz	2.8	0.987	0.059
Wages Ethier-Krugman	2.8	0.987	0.059
Real Wages Melitz	4	0.572	0.074
Real Wages Melitz	3.4	0.807	0.111
Real Wages Melitz	2.8	1.138	0.169
Real Wages Ethier-Krugman	2.8	1.138	0.169
N	109		

Ethier-Krugman model and for the Melitz model for various values of the trade elasticity θ. Figure 3 displays the evolution of the unweighted average real wage relative to the benchmark in all countries in the sample for different percentage trade cost reductions. The figure indicates that the impact of trade cost reductions is exactly identical in the Ethier-Krugman model and in the Melitz model with granularity, i.e. $\theta = \sigma - 1$. The figure also shows that the welfare gains from trade liberalization are larger in the granular economy and in the Ethier-Krugman economy. This effect is partially driven by the fact that we keep σ constant implying that granularity requires a lower trade elasticity in the Melitz model.

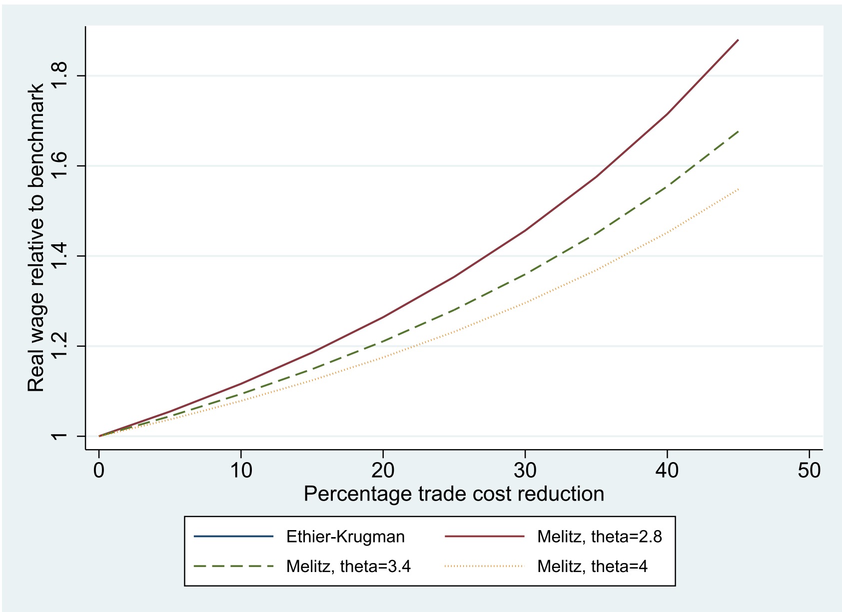

Figure 3: Effect of Trade Cost Reductions on Real Wages

3.2 Multiple Sector Model with Intermediate Linkages

To implement the three models in a CGE setting, we include a technical change parameter c_i on the supply side and a technical change parameter e_j on the demand side and we include generalized iceberg trade costs t_{ij}. The values for c_i, e_j and t_{ij} for the three economies are as specified in the equations in the subsections corresponding to these three economies.

4 Concluding Remarks

We have shown that both the Ethier-Krugman monopolistic competition model and the Melitz firm heterogeneity model can be defined as an Armington representative agent model. This representation of these two models also makes clear that the Melitz model generates the same equilibrium outcome as the Ethier-Krugman model when the firm size distribution is granular and specific values are chosen for parameters values like trade costs and sunk entry costs. This representation is in particular useful for implementation of the Melitz firm heterogeneity model in multisector CGE models.

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Appendix A Ethier/Krugman Economy

The goal of this section is to derive the expressions for c_i and t_{ij} in the main text in equations (12)-(13). We follow the dual exposition as it is clearer. Consumers in country j have CES preferences over physical quantities $o(\omega)$ of varieties ω from different countries. The quantity and price index are defined in equations (8)-(9). Demand for a variety ω shipped from i to j is equal to:

$$o_{ij}(\omega) = p_{ij}(\omega)^{\sigma-1} P_j^{\sigma-1} E_j \tag{A.1}$$

Varieties are produced by identical firms with an increasing returns to scale technology with fixed cost a_i and marginal cost b_i, implying that each firm produces a unique variety. As firms are identical, ω can be dropped in the remainder.

Firms face iceberg trade costs τ_{ij} and ad valorem tariffs ta_{ij} and use the following markup pricing rule:

$$p^o_{ij} = \frac{\sigma}{\sigma-1}(1+ta_{ij})\,\tau_{ij} b_i w_i \tag{A.2}$$

p^o_{ij} is the cif price of physical output o_{ij}. Firms do not face destination specific fixed costs and can enter all markets upon paying the fixed costs b_i. Profits from sales to all markets are thus equal to:

$$\pi_i(\varphi) = \sum_{j=1}^{J} \frac{\frac{p^o_{ij}}{1+ta_{ij}} o_{ij}}{\sigma} - a_i w_i \tag{A.3}$$

Total physical output by a firm from country i inclusive of iceberg trade costs, $\overline{o}_i$, can be defined as:

$$\overline{o}_i = \sum_{j=1}^{J} \tau_{ij} x_{ij} \tag{A.4}$$

Substituting equations (A.2) and (A.4) into equation (A.3) and imposing zero profit from free entry gives an expression for total physical output $\overline{o}_i$:

$$\overline{o}_i = \frac{a_i(\sigma-1)}{b_i} \tag{A.5}$$

As a next step, N_i is defined as the mass of varieties produced in country i. N_i is identical for all destinations by absence of destination specific fixed costs. It follows from the following labor market equilibrium:

$$(b_i \overline{o}_i + a_i)\,N_i = L_i \tag{A.6}$$

Solving for N_i, substituting the expression for $\overline{o}_i$ in equation (A.5), gives the following expression:

$$N_i = \frac{L_i}{\sigma a_i} \tag{A.7}$$

The price of the representative good p_{ij} is defined such that substituting p_{ij} in the expression for the price index defined over representative prices in equation (3) generates the expression for the price index in the Ethier/Krugman economy as defined in equation (9). This gives:

$$p_{ij} = \left(\int\limits_{\omega \in \Omega_{ij}} p^o \left(\omega\right)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{A.8}$$

Given that all firms are identical and all varieties N_i are exported to all destinations, equation (A.8) can be rewritten as:

$$p_{ij} = N_i^{\frac{1}{1-\sigma}} p_{ij}^o \tag{A.9}$$

Substituting equation (A.2) for p_{ij}^o and equation (A.7) for N_i leads to:

$$p_{ij} = \left(1 + ta_{ij}\right) \tau_{ij} \left(\frac{L_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} b_i w_i \tag{A.10}$$

$p_{ij} = t_{ij} c_i w_i$ implies that generalized iceberg trade costs are given by $t_{ij} = \left(1 + ta_{ij}\right) \tau_{ij}$ and the marginal cost of variety scaled output are given by:

$$c_i = \left(\frac{L_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} b_i \tag{A.11}$$

Variety scaled output x_i (not taking into account iceberg trade costs) can be written as in equation (10) in the main text:

$$x_i = \frac{1}{c_i} L_i = \left(\frac{L_i}{\sigma a_i}\right)^{\frac{1}{\sigma-1}} \frac{1}{\frac{\sigma}{\sigma-1} b_i} L_i \tag{A.12}$$

Appendix B Melitz Economy

Appendix B.1 Demand and Production

Like in the Ethier/Krugman economy the goal of this section is to derive the expressions for generalized marginal costs c_i , generalized iceberg trade costs t_{ij} and the demand externality e_j

in the Melitz economy in equations (18)-(20) and to derive the demand externality.

Consumers in country j have the same CES preferences over varieties ω from different countries as in the Ethier/Krugman economy. The quantity and price index are thus given by equations (8)-(9) and demand for physical quantities $o_{ij}(\omega)$ of a variety ω by equation (A.1). In contrast to the Ethier/Krugman economy goods are produced by firms with heterogeneous productivity. Firms can sell both in domestic and foreign markets and have to pay fixed costs f_{ij} to sell in each market. The fixed costs are paid in wages of both countries with according to a Cobb Douglas specification a fraction μ paid in domestic input bundles. Exporting firms also face iceberg trade costs τ_{ij} to export. The cost function of a firm producing in country i and selling o_{ij} physical units of output in country j having productivity φ is thus:

$$C(o_{ij},\varphi)=\frac{\tau_{ij}o_{ij}}{\varphi}w_i+f_{ij}w_i^{\mu}w_j^{1-\mu} \tag{B.1}$$

Each firm produces a unique variety, so we can identify demand for variety ω by the productivity φ of the firm producing this variety. Demand $q_{ij}(\varphi)$ and revenues $r_{ij}(\varphi)$ of a firm with productivity φ producing in i and selling in j are equal to:

$$o_{ij}(\varphi)=\alpha_{ij}^{\sigma}p_{ij}^{o}(\varphi)^{-\sigma}P_j^{\sigma-1}E_j \tag{B.2}$$

$$r_{ij}(\varphi)=\alpha_{ij}^{\sigma}p_{ij}^{o}(\varphi)^{1-\sigma}P_j^{\sigma-1}E_j \tag{B.3}$$

Maximizing profits implies the following markup pricing rule:

$$p_{ij}^{o}(\varphi)=\frac{\sigma}{\sigma-1}\frac{(1+ta_{ij})\,\tau_{ij}w_i}{\varphi} \tag{B.4}$$

With tariffs as revenues shifters, profits for sales to destination market j are equal to:

$$\pi_{ij}(\varphi)=\frac{r_{ij}(\varphi)}{(1+ta_{ij})\,\sigma}-f_{ij}w_i^{\mu}w_j^{1-\mu} \tag{B.5}$$

Appendix B.2 Entry and Exit

Entry and exit are like in Melitz (2003), i.e. firms can draw a productivity parameter φ from a distribution $G_i(\varphi)$ after paying a sunk entry cost en_i. The productivity of firms stays fixed and firms face a fixed death probability δ in each period. Firms either decide to start producing for at least one of the markets or leave the market immediately. In equilibrium there is a steady state of entry and exit with a steady number of entrants NE_i drawing a productivity parameter, implying that the productivity distribution of producing firms is constant. Denoting φ_{ij}^* as the cutoff productivity, only firms with a productivity $\varphi \geq \varphi_{ij}^*$ from country i sell in market j.

Appendix B.3 Free Entry and Zero Cutoff Profit Conditions

Equilibrium is defined with a zero cutoff profit condition (ZCP) and a free entry condition (FE). According to the zero cutoff profit condition firms from country i with cutoff productivity φ_{ij}^* can just make zero profit from sales in country i:

$$r_{ij}\left(\varphi_{ij}^*\right) = \sigma\left(1 + ta_{ij}\right) f_{ij} w_i^{\mu} w_j^{1-\mu} \tag{B.6}$$

Using equations (B.3)-(B.5) the ZCP can be written as follows:

$$\varphi_{ij}^* = \frac{\sigma}{\sigma - 1} \frac{\left(1 + ta_{ij}\right) \tau_{ij} w_i}{P_j^e} \left(\frac{\alpha_{ij}^{\sigma} E_j}{\sigma\left(1 + ta_{ij}\right) f_{ij} w_i^{\mu} w_j^{1-\mu}}\right)^{\frac{1}{1-\sigma}} \tag{B.7}$$

The free entry condition (FE) equalizes the expected profits before entry with the sunk entry costs:

$$\sum_{j=1}^{J} \left(1 - G_i\left(\varphi_{ij}^*\right)\right) \pi_{ij}\left(\widetilde{\varphi}_{ij}\right) = \delta e n_i w_i \tag{B.8}$$

$\widetilde{\varphi}_{ij}$ is a measure of average productivity and defined as:

$$\widetilde{\varphi}_{ij} = \left(\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{g_i\left(\varphi\right)}{1 - G_i\left(\varphi_{ij}^*\right)} d\varphi\right)^{\frac{1}{\sigma-1}} \tag{B.9}$$

Using $\frac{r_{ij}(\varphi_1)}{r_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ and the ZCP in equation (B.6), the FE in equation (B.8) can be written as:

$$\sum_{j=1}^{J} \left(1 - G_i\left(\varphi_{ij}^*\right)\right) w_i^{\mu} w_j^{1-\mu} f_{ij} \left(\left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^*}\right)^{\sigma-1} - 1\right) = \delta e n_i w_i \tag{B.10}$$

The distribution of initial productivities $G_i\left(\varphi\right)$ is Pareto:

$$G_i\left(\varphi\right) = 1 - \frac{\kappa_i^{\theta_i}}{\varphi^{\theta_i}} \tag{B.11}$$

with θ_i the shape parameter and κ_i the size parameter. We impose $\theta_i > \sigma - 1$ to guarantee that expected revenues are finite. With a Pareto distribution $\widetilde{\varphi}_{ij}$ is proportional to φ^*_{ij}:

$$\widetilde{\varphi}_{ij} = \left(\frac{\theta_i}{\theta_i - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \varphi^*_{ij} \tag{B.12}$$

Substituting equations (B.11)-(B.12) into the fe, equation (B.10), gives:

$$\sum_{j=1}^{J} \left(\frac{\kappa_i}{\varphi^*_{ij}}\right)^{\theta_i} w_i^{\mu} w_j^{1-\mu} f_{ij} \frac{\sigma - 1}{\theta_i - \sigma + 1} = \delta e n_i w_i \tag{B.13}$$

Appendix B.4 Deriving The Representative Price and Price Index

The price of the representative good p_{ij} is defined excluding the demand side externality. Substituting this demand side externality stripped expression for p_{ij} into the expression for the price index defined over representative prices in equation (3) should generate the theoretical expression for the price index in the Melitz economy as defined in equation (9). Therefore p_{ij} is given by:

$$p_{ij} = e_j \left(\int\limits_{\omega \in \Omega_{ij}} p^o(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{B.14}$$

The representative price p_{ij} in equation (B.14) can be redefined as an integral over productivities of the producing firms as follows:

$$\frac{p_{ij}}{e_j} = \left(\int\limits_{\varphi^*_{ij}}^{\infty} N_{ij} p^o_{ij}(\varphi)^{1-\sigma} \frac{g_i(\varphi)}{1 - G_i\left(\varphi^*_{ij}\right)} d\varphi \right)^{\frac{1}{1-\sigma}} \tag{B.15}$$

$\frac{p_{ij}}{e_j}$ is the represenative price including the demand externality. Using equations (B.4) and (B.9) the representative price in equation (B.15) can be rewritten as a function of average productivities:

$$\frac{p_{ij}}{e_j} = \frac{\sigma}{\sigma - 1} \left(N_{ij} \left(\left(1 + ta_{ij}\right) \tau_{ij} w_i \right)^{1-\sigma} \widetilde{\varphi}_{ij}^{\sigma-1} \right)^{\frac{1}{1-\sigma}} \tag{B.16}$$

The mass of varieties sold from country i to country j, N_{ij} is related to the mass of entrants NE_i by the following steady state condition:

$$N_{ij} = \frac{\left(1 - G_i\left(\varphi^*_{ij}\right)\right) NE_i}{\delta} = \left(\frac{\kappa_i}{\varphi^*_{ij}}\right)^{\theta_i} \frac{NE_i}{\delta} \tag{B.17}$$

The steady state of entry and exit implies that NE_i can be written as a function of the number of workers L_i and the cutoff productivity φ_{ij}^*:

$$NE_i = \frac{\sigma - 1}{\sigma\theta} \frac{L_i}{en_i} \tag{B.18}$$

To derive equation (B.18) an equal shape parameter θ across countries was imposed, implying that the country subscript on this parameter is dropped. Using equations (B.12), (B.17) and (B.18), the representative price in equation (B.16) can be written as:

$$\frac{p_{ij}}{e_j} = \frac{\sigma}{\sigma - 1} \left(\frac{\sigma - 1}{\sigma\left(\theta - \sigma + 1\right)} \frac{\kappa_i^\theta L_i}{\delta en_i} \frac{\left(\left(1 + ta_{ij}\right) \tau_{ij} w_i\right)^{1-\sigma}}{\left(\varphi_{ij}^*\right)^{\theta-\sigma+1}} \right)^{\frac{1}{1-\sigma}} \tag{B.19}$$

The final step is to substitute the ZCP solved for φ_{ij}^* in equation (B.7) into equation (B.19) to generate the following expression for the representative price p_{ij}:

$$\frac{p_{ij}}{e_j} = \left(\gamma_m \left(\frac{\kappa_i^\theta L_i}{\delta en_i} \right) w_i^{-\left(\theta + \mu \frac{\theta-\sigma+1}{\sigma-1}\right)} \alpha_{ij}^{\sigma \frac{\theta-\sigma+1}{\sigma-1}} \left(\left(1 + ta_{ij}\right)^{1 + \frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tau_{ij} f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \right)^{-\theta} w_j^{-(1-\mu)\frac{\theta-\sigma+1}{\sigma-1}} \left(P_j^e\right)^{\theta-\sigma+1} E_j^{\frac{\theta-\sigma+1}{\sigma-1}} \right)^{\frac{1}{\cdots}} \tag{B.20}$$

With γ defined as:

$$\gamma_m = \left(\frac{\sigma}{\sigma - 1} \right)^{-\theta+1} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta - \sigma + 1} \tag{B.21}$$

Equation (B.20) shows that the representative price can be decomposed into three components, a source specific component, a bilateral component and a destination specific component. The source specific component in equation (B.20) gives the price of variety scaled output p_i^x before it is exported:

$$p_i^x = \left(\gamma_m \left(\frac{\kappa_i^\theta L_i}{\delta en_i} \right) w_i^{-\left(\theta + \mu \frac{\theta-\sigma+1}{\sigma-1}\right)} \right)^{\frac{1}{1-\sigma}} \tag{B.22}$$

Rearranging equation (B.22) the effect of w_i can be split up into a proportional term representing the intensive margin effect and an externalities term representing the extensive and compositional margin:

$$p_i^x = \frac{w_i^{\frac{\theta-\sigma+1}{\sigma-1} + \mu \frac{\theta-\sigma+1}{(\sigma-1)^2}}}{\left(\gamma_m \frac{\kappa_i^\theta}{\delta en_i}\right)^{\frac{1}{\sigma-1}} L_i^{\frac{1}{\sigma-1}}} w_i \tag{B.23}$$

Using $p_i^x = c_i w_i$ implies the expression for c_i in equation (18) in the main text.

The pairwise terms in equation (B.20) divided by the ad valorem tariff represent the generalized iceberg trade costs t_{ij} times the ad valorem tariff:

$$t_{ij}\left(1+ta_{ij}\right)=\left(\alpha_{ij}^{\sigma\frac{\theta-\sigma+1}{\sigma-1}}\left(\left(1+ta_{ij}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}\right)^{\frac{1}{1-\sigma}} \tag{B.24}$$

Rearranging leads to the expression for t_{ij} in the main text, equation (19), where the effect of iceberg trade costs τ_{ij} are split into a proportional term representing the intensive margin effect and an externalities term representing the extensive and compositional margin:

$$t_{ij}=\alpha_{ij}^{-\frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}}\left(\left(1+ta_{ij}\right)^{\frac{\theta-\sigma+1}{\sigma-1}+\frac{\theta-\sigma+1}{(\sigma-1)^2}}\tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}}f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)\tau_{ij} \tag{B.25}$$

Finally, the destination specific terms in equation (B.20) represent the demand externality, giving:

$$e_j=\left(w_j^{-(1-\mu)\frac{\theta-\sigma+1}{\sigma-1}}\left(P_j^e\right)^{\theta-\sigma+1}E_j^{\frac{\theta-\sigma+1}{\sigma-1}}\right)^{\frac{1}{\sigma-1}} \tag{B.26}$$

Equation (B.26) can be rearranged to generate equation (20) employing the relations $P_j^e=P_j$ and $E_j=P_jq_j$. In equation (20) the externality e_j is expressed as a function of the quantity index q_j and price index P_j without the demand externality:

$$e_j=\left(\frac{q_jP_j^{\sigma}}{w_j^{1-\mu}}\right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tag{B.27}$$

Substituting equation (B.20) into the definition of the price index in equation (3) gives:

$$P_j^e=\left(\sum_{i=1}^{J}\alpha_{ij}^{\sigma}\gamma\frac{\kappa_i^{\theta}L_i}{\delta en_i}w_i^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)}\left(\left(1+ta_{ij}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}w_j^{-(1-\mu)\frac{\theta-\sigma+1}{\sigma-1}}\left(P_j^e\right)^{\theta-\sigma+1}\left(\alpha_{ij}^{\sigma}E_j\right)^{\frac{\theta-\sigma+1}{\sigma-1}}\right)^{\frac{1}{1-\sigma}} \tag{B.28}$$

Since P_j^e is on the RHS and LHS we can solve for P_j^e to arrive at the following expression for the price index:

$$P_j^e=\left(\frac{w_j^{1-\mu}}{E_j}\right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\left(\sum_{i=1}^{J}\gamma\frac{\kappa_i^{\theta}L_i}{\delta en_i}w_i^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)}\alpha_{ij}^{\frac{\sigma\theta}{\sigma-1}}\left(\left(1+ta_{ij}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}\right)^{-\frac{1}{\theta}} \tag{B.29}$$

Equation (B.29) shows that P_j is a function of market size E_j: a larger market size reduces the price index, as more firms can enter in a larger market. This extensive margin effect reduces the price index. P_j rises in w_j if part of the fixed costs are paid in bundles of the destination country.

Appendix B.5 Deriving Representative Quantity Directly

In this section it is shown that representative demand q_{ij} can be expressed conventionally as follows with p_{ij} the price of the representative good:[3]

$$q_{ij} = \alpha_{ij}^{\sigma} p_{ij}^{-\sigma} P_j^{\sigma-1} E_j \tag{B.30}$$

q_{ij} is representative quantity excluding the demand side externality. Substituting this q_{ij} into the expression for the quantity index in equation (1) should generate the theoretical expression for the quantity index in equation (8). So, q_{ij} is defined as:

$$q_{ij} = \frac{1}{e_j} \left(\int\limits_{\omega \in \Omega_{ij}} o(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{B.31}$$

To derive equation (B.30) we redefine the representative quantity q_{ij} inclusive of the demand externality from equation (B.31) as an integral over the productivity of producing firms:

$$q_{ij} e_j = \left(N_{ij} \int\limits_{\varphi_{ij}^*}^{\infty} q_{ij}(\varphi)^{\frac{\sigma-1}{\sigma}} \frac{g(\varphi)}{1 - G\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{\sigma}{\sigma-1}} \tag{B.32}$$

Substituting the expression for $q_{ij}(\varphi)$ in equation (B.2), representative quantity in equation (B.32) can be written as a function of average productivity:

$$q_{ij} e_j = N_{ij}^{\frac{\sigma}{\sigma-1}} q_{ij}(\widetilde{\varphi}_{ij}) \tag{B.33}$$

The next step is to use $\frac{q_{ij}(\varphi_1)}{q_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$ and equation (B.12) to write q_{ij} as a function of cutoff quantity $q_{ij}\left(\varphi_{ij}^*\right)$:

$$q_{ij} e_j = N_{ij}^{\frac{\sigma}{\sigma-1}} q_{ij}\left(\varphi_{ij}^*\right) \left(\frac{\theta}{\theta - \sigma + 1} \right)^{\frac{\sigma}{\sigma-1}} \tag{B.34}$$

The ZCP in equation (B.6) can be employed to express cutoff quantity $q_{ij}\left(\varphi_{ij}^*\right)$ as follows:

$$q_{ij}\left(\varphi_{ij}^*\right) = (\sigma - 1)\, \varphi_{ij}^* \frac{f_{ij}}{\tau_{ij}} \frac{w_i^{\mu} w_j^{1-\mu}}{w_i} \tag{B.35}$$

[3] Remember that both q_{ij} and p_{ij} are defined without taking into account the demand externality.

Substituting equation (B.35) and also the expressions for N_{ij} and NE_i in equations (B.17)-(B.18) into equation (B.34) leads to:

$$q_{ij}e_j = \left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\right)^{\frac{\sigma}{\sigma-1}}\left(\sigma-1\right)\left(\frac{\kappa_i^{\theta}L_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}}\frac{f_{ij}}{\tau_{ij}}\frac{1}{\left(\varphi_{ij}^{*}\right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}}\frac{w_i^{\mu}w_j^{1-\mu}}{w_i} \tag{B.36}$$

Finally, the ZCP solved for φ_{ij}^{*} in equation (B.7) can be substituted into equation (B.36) to give the following expression for representative quantity inclusive of the :

$$q_{ij}e_j = \left(\gamma\left(\frac{\kappa_i^{\theta}L_i}{\delta en_i}\right)\left(\left(1+ta_{ij}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}w_i^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)}w_j^{-(1-\mu)\frac{\theta-\sigma+1}{\sigma-1}}\left(P_j^{e}\right)^{\frac{\theta\sigma-\sigma+1}{\sigma}}\left(\alpha_{ij}^{\sigma}E_j\right)^{\frac{\theta\sigma-\sigma+1}{\sigma(\sigma-1)}}\right)^{\frac{\sigma}{\sigma-1}} \tag{B.37}$$

It can be verified that substituting the expression for the representative price p_{ij} in equation (B.20) into the conventional expression for representative demand in equation (B.30) leads to the expression for representative quantity in equation (B.37). So, the primal and dual approach generate the same expressions and we could easily verify that the c_i, t_{ij} and e_j derived with the dual approach can also be derived from equation (B.37) using $q_{ij}e_j = t_{ij}c_iw_i$.

Appendix B.6 Summary of Equilibrium Equations and Implementation in GAMS

As a check on the correctness of the expressions, we show in GAMS that a solution of the model in a setting with a large number of countries generates the same solution using the initial equilibrium conditions of the model as using the two equilibrium conditions and the single equilibrium conditions. In particular, we programmed the following set of equations.

Employing the initial equilibrium conditions, we get the following set of equilibrium conditions, respectively the expression for the price index following from equation (B.19), the expression for the number of varieties following from equations (B.17) and (B.18), a demand equation, an expression for cutoff revenues following from equation (B.3), a markup pricing expression in equation (B.4) and a zero cutoff profit condition in equation (B.6). The free entry condition is substituted in both the expression for the number of varieties and the demand equation:

$$
\begin{aligned}
\left(P_i^e\right)^{1-\sigma} &= \sum_{j=1}^{J} N_{ji} \frac{\theta}{\theta-\sigma+1} p_{ji}\left(\varphi_{ji}^*\right)^{1-\sigma} \\
N_{ij} &= \left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta_i} \frac{\sigma-1}{\sigma\theta_i} \frac{L_i}{\delta en_i} \\
w_i L_i + \sum_{k=1}^{J} N_{ki} \frac{ta_{ki}}{1+ta_{ki}} r_{ki}\left(\varphi_{ki}^*\right) \frac{\theta}{\theta-\sigma+1} &= \sum_{j=1}^{J} N_{ij} \frac{\theta}{\theta-\sigma+1} r_{ij}\left(\varphi_{ij}^*\right) \\
r_{ij}\left(\varphi_{ij}^*\right) &= p_{ij}\left(\varphi_{ij}^*\right)^{1-\sigma}\left(P_i^e\right)^{\sigma-1} E_j \\
p_{ij}\left(\varphi_{ij}^*\right) &= \frac{\sigma}{\sigma-1} \frac{\left(1+ta_{ij}\right)\tau_{ij} w_i}{\varphi_{ij}^*} \\
\frac{r_{ij}\left(\varphi_{ij}^*\right)}{\left(1+ta_{ij}\right)} &= \sigma f_{ij} w_i^{\mu} w_j^{1-\mu}
\end{aligned}
$$

And with two equations we can use equations (B.29) and (22) in slightly rewritten form for programming reasons:

$$
1 = \left(P_i^e\right)^{\theta} E_i^{\frac{\theta-\sigma+1}{\sigma-1}} w_i^{-(1-\mu)\frac{\theta+\sigma-1}{\sigma-1}} \sum_{j=1}^{J} \gamma \frac{\kappa_j^{\theta} L_j}{\delta en_j} \alpha_{ji}^{\frac{\sigma\theta}{\sigma-1}} \left(\left(1+ta_{ji}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tau_{ji} f_{ji}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta} w_j^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)}
$$

$$
w_i = \sum_{j=1}^{J} \gamma \frac{\kappa_i^{\theta} L_i}{\delta en_i} w_i^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)} \alpha_{ij}^{\frac{\sigma\theta}{\sigma-1}} \left(\left(1+ta_{ij}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tau_{ij} f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta} w_j^{-(1-\mu)\frac{\theta+\sigma-1}{\sigma-1}} \left(P_j^e\right)^{\theta} E_j^{\frac{\theta-\sigma+1}{\sigma-1}} E_j
$$

With one equation we use equation (23) in slightly rewritten form:

$$
w_i = \sum_{j=1}^{J} \frac{\frac{\kappa_i^{\theta}}{\delta en_i} w_i^{-\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}} \alpha_{ij}^{\frac{\sigma\theta}{\sigma-1}} \left(\left(1+ta_{ij}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tau_{ij} f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}}{\sum_{k=1}^{J} \frac{\kappa_k^{\theta} L_k}{\delta en_k} w_k^{-\left(\theta+\mu\frac{\theta-\sigma+1}{\sigma-1}\right)} \alpha_{kj}^{\frac{\sigma\theta}{\sigma-1}} \left(\left(1+ta_{kj}\right)^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tau_{kj} f_{kj}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}} E_j
$$