Structural Parameters of Trade Models with Firm Heterogeneity

by

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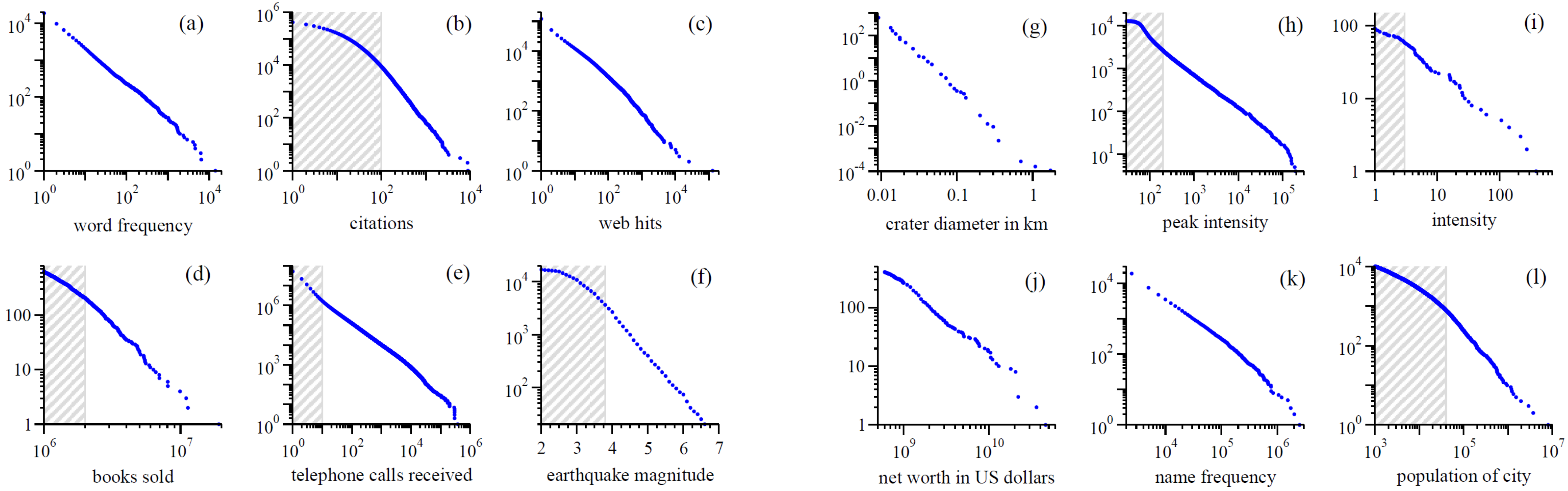
Overview

- When will the Melitz model be mainstream?
  - Current CGE models with Melitz mechanisms
  - Parameter needs
- Productivity follows Pareto distribution
- Pareto distribution and Power-law
- Estimation/calibration of key parameters
- Test the performance against alternatives distributions
- Sensitivity checks
- Conclusions

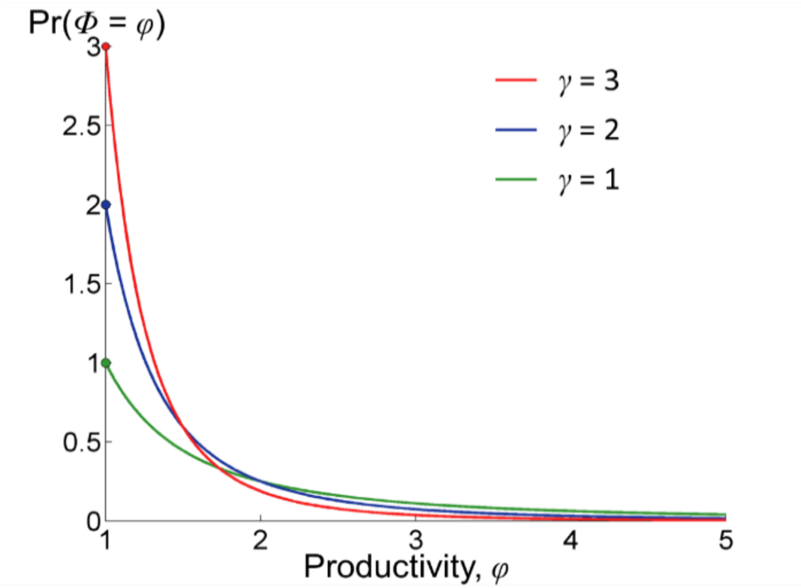
Pareto distribution and Power-law



- Cumulative distributions of 12 quantities reputed to follow Power-law (Newman, 2005)

Why Pareto distribution?

Probability density function of Pareto distribution



- Analytically tractable
  - Deriving aggregate properties of the analytical model is simplified (Head and Mayer, 2014)
- Stable to truncation from below
  - Right tail of the distribution (high-productivity firms) also follows Pareto (Chaney, 2008)
- Empirical relevance
  - Good fit for the observed distribution of firm sizes
  - American firms (Axtell, 2001)
  - French firms (Eaton, Kortum and Kramarz, 2011)
- Two key parameters in firm heterogeneity
  - shape parameter of Pareto distribution, γ
  - elasticity of substitution across varieties, σ
  - with a mathematical constraint, $\gamma > \sigma - 1$

Is Pareto the right choice?

- Omitting small firms due to poor performance in fitting the left-tail of export sales distribution (Head and Mayer, 2014; Sager and Timoshenko, 2016; Nigai, 2017).
- Empirical relevance may not apply to small firms since there may be a minimum size threshold for Power-laws to provide a good fit (di Giovanni et al., 2013).
- Log-normal distribution
  - Approximates a good fit for the complete distribution, not only the right tail (Head, Mayer and Thoenig, 2014)
  - Neither distribution alone provides a good fit for the entire support (Nigai, 2017; Sager and Timoshenko, 2016)
  - Analytical tractability?

Our approach

- **We use Pareto for firm size and productivity**
  - Analytical tractability in CGE
  - Good performance on the right tail above a lower-bound
- **We estimate shape and elasticity values for GTAP Version 10 manufacturing sectors**
  - improve the performance of the Power-law fit following Clauset, Shalizi and Newman (2009)
  - estimate the lower-bound
  - estimate the shape values based on the optimal lower-bound
  - compare performance of alternative distributions by Likelihood Ratio test
  - calculate elasticity values based on estimated shape parameters following Ahmad and Akgul (2018)

Empirical methodology

Step	Method	Result
1. Estimate lower-bound parameter	Kolmogorov-Smirnov (KS) Statistic	$\hat{x}_{min}$
2. Estimate power-law parameter	Maximum Likelihood	$\hat{\alpha}, \hat{\gamma}$
3. Impute elasticity of substitution	$\alpha = \dfrac{\gamma}{\sigma - 1}$	$\hat{\sigma}$

Pareto distribution is a Power-law

- PDF of a continuous Power-law model

$$p(x)dx = Pr[x \leq X < x + dx] = Cx^{-\alpha}dx$$

- The Power-law model only applies above a lower-bound

$$p(x) = \frac{\alpha - 1}{x_{min}}\left(\frac{x}{x_{min}}\right)^{-\alpha}$$

- CCDF = 1- CDF is

$$p(x)dx = Pr[X \geq x] = \int_x^{\infty} p(x)dx = \left(\frac{x}{x_{min}}\right)^{-\alpha+1}$$

Distribution

- **When firm productivity is Pareto, firm size is also Pareto**

- **Firm productivity**

$$\varphi \sim Pareto(x_{min}, \gamma)$$

- **Firm size**

$$S_i \sim Pareto(x_{min}^{1-\sigma}, \frac{\gamma}{\sigma - 1})$$

- **Step 1. Estimating the lower bound parameter (KS statistic)**
  - Minimize distance D:

$$D = \max_{x \geq x_{min}} | \underbrace{S(x)}_{\text{CDF of the data}} - \underbrace{P(x)}_{\text{CDF of the model}} |$$

Parameter estimation

- **Step 2. Estimating the Power-law exponent**
  - Maximum Likelihood Estimator

$$\hat{\alpha} = 1 + n\left[\sum_{i=1}^{n} ln\,\frac{x_i}{\hat{x}_{min}}\right]^{-1}$$

where x_i with i=1,2,...,n are the observed values of x such that $x \geq x_i$

- **Step 3. Imputed elasticity**

$$\hat{\alpha} = \frac{\hat{\gamma}}{\sigma - 1} \quad \Longrightarrow \quad \hat{\sigma} = \frac{\hat{\gamma}}{\hat{\alpha}} + 1$$

Data

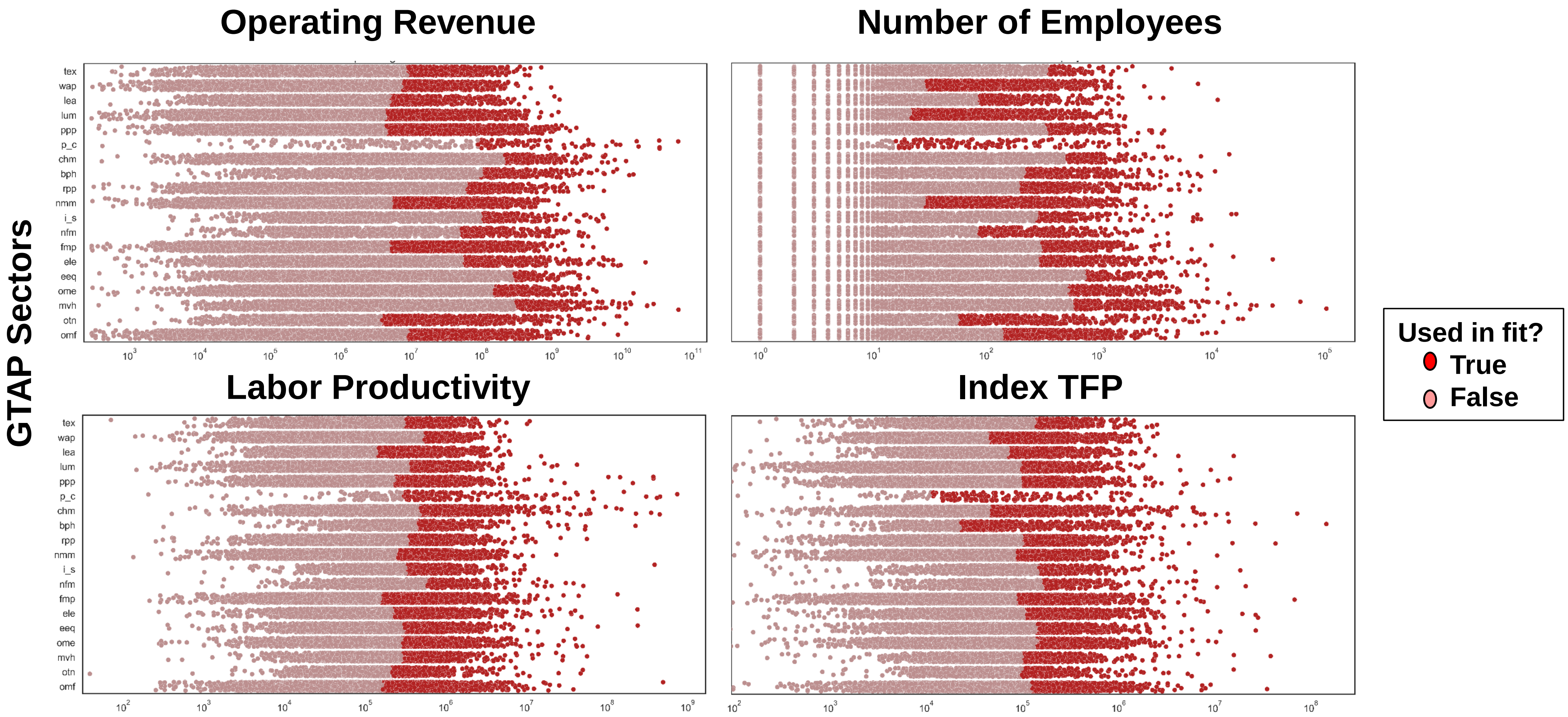
- **ORBIS**
  - Annual firm-level financial data on manufactures in the EU
  - Industry classification based on the Statistical Classification of Economic Activities in the European Community (NACE)
  - Level 4 of NACE Rev. 2 – 615 classes are identified by 4-digit codes
- **GTAP Version 10**
  - 65 sectors with 19 manufacturing
  - GSC3 Sectoral Identification
  - Provides mapping to ISIC Rev. 4
- **We map ORBIS firms to GTAP sectors via:**

NACE Rev. 2 → ISIC Rev. 4 → GTAP GSC3

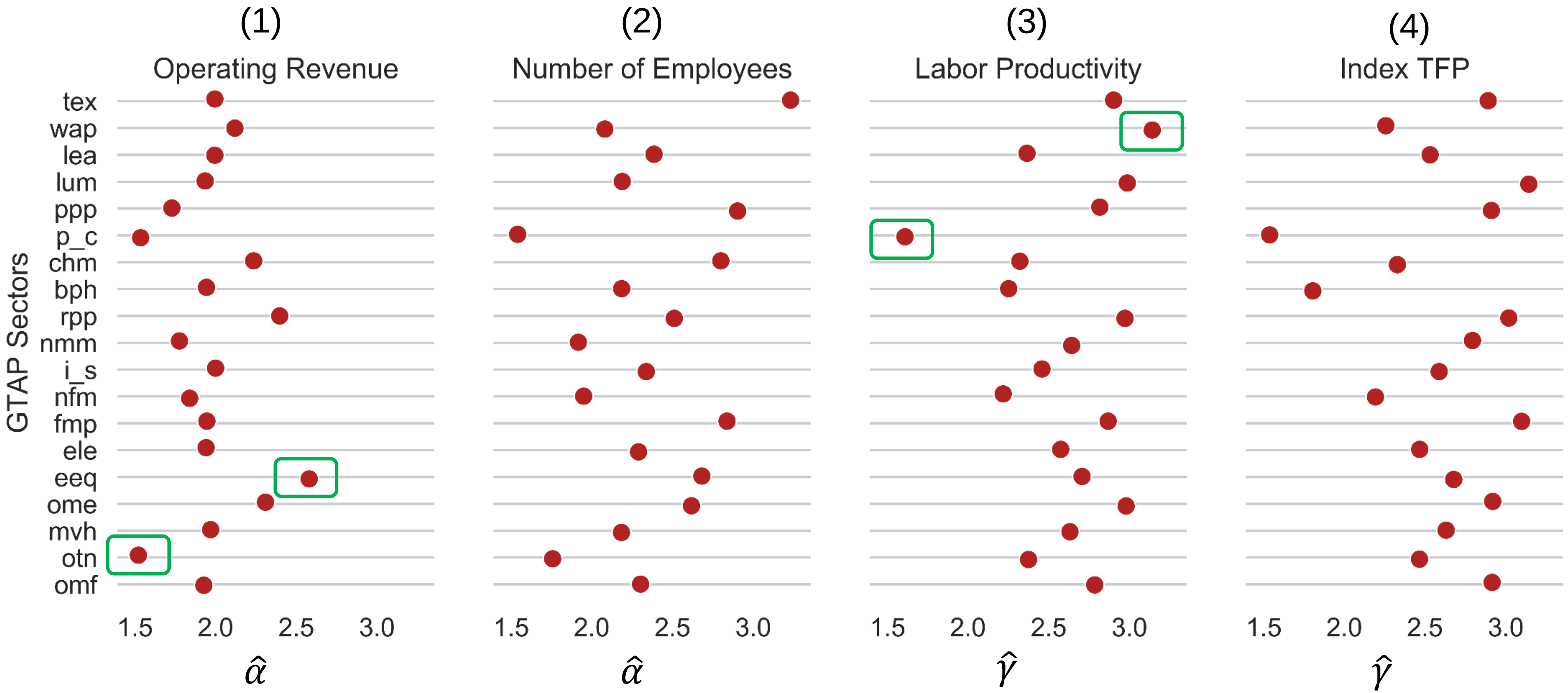
Measures

- **Firm Size**
  - Operating Revenue
  - Number of Employees
- **Firm Productivity**
  - Labor Productivity (Y/L)
  - Index TFP (Residual in a Cobb-Douglas production function)

Step1: Optimal lower-bound and data distribution in GTAP manufacturing

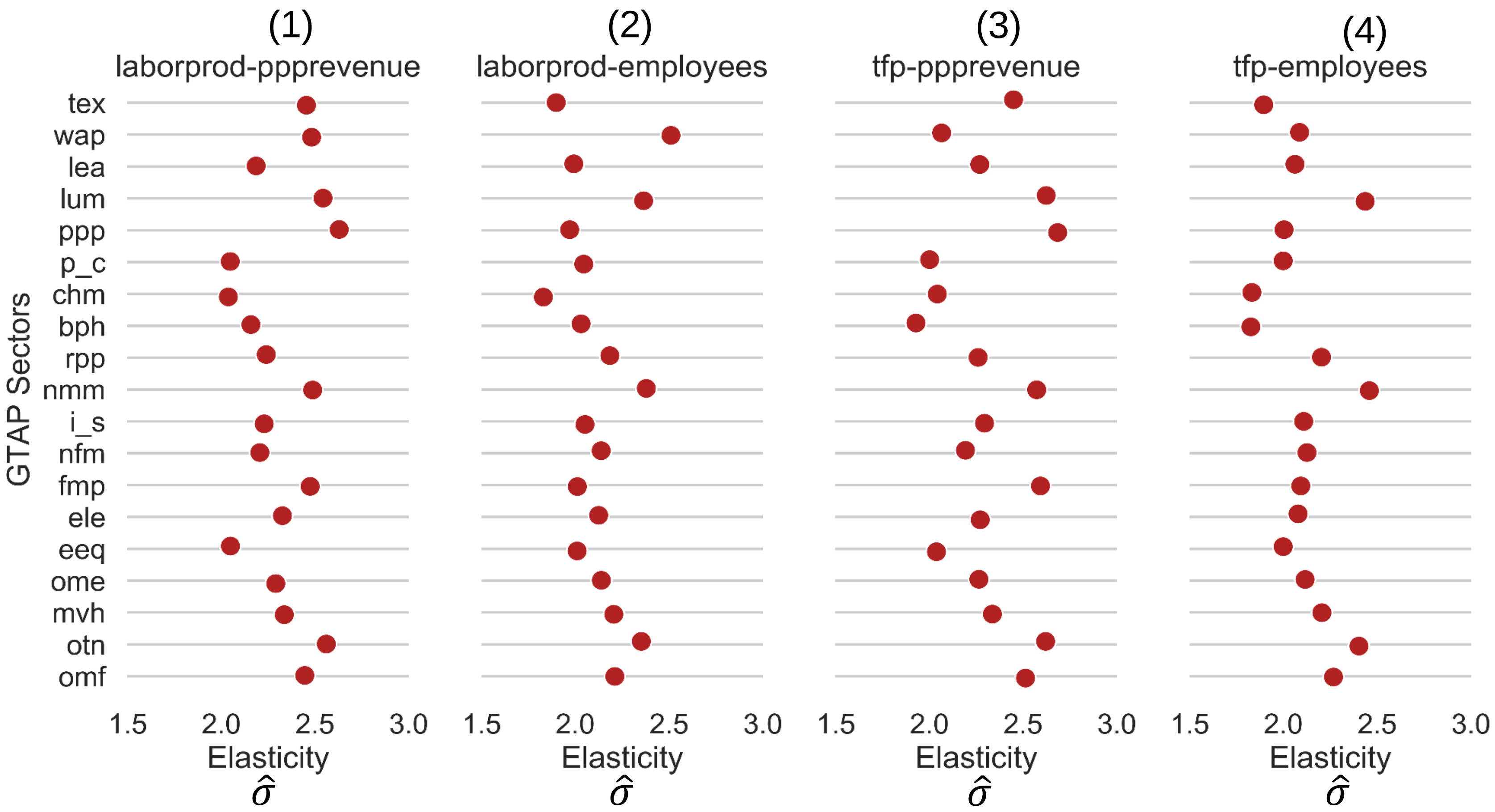


Step 2: Shape in GTAP manufacturing



- Estimates are based on optimal lower-bound
- Mathematical constraint is satisfied
- Productivity is less heterogenous than firm size within the GTAP manufacturing sectors

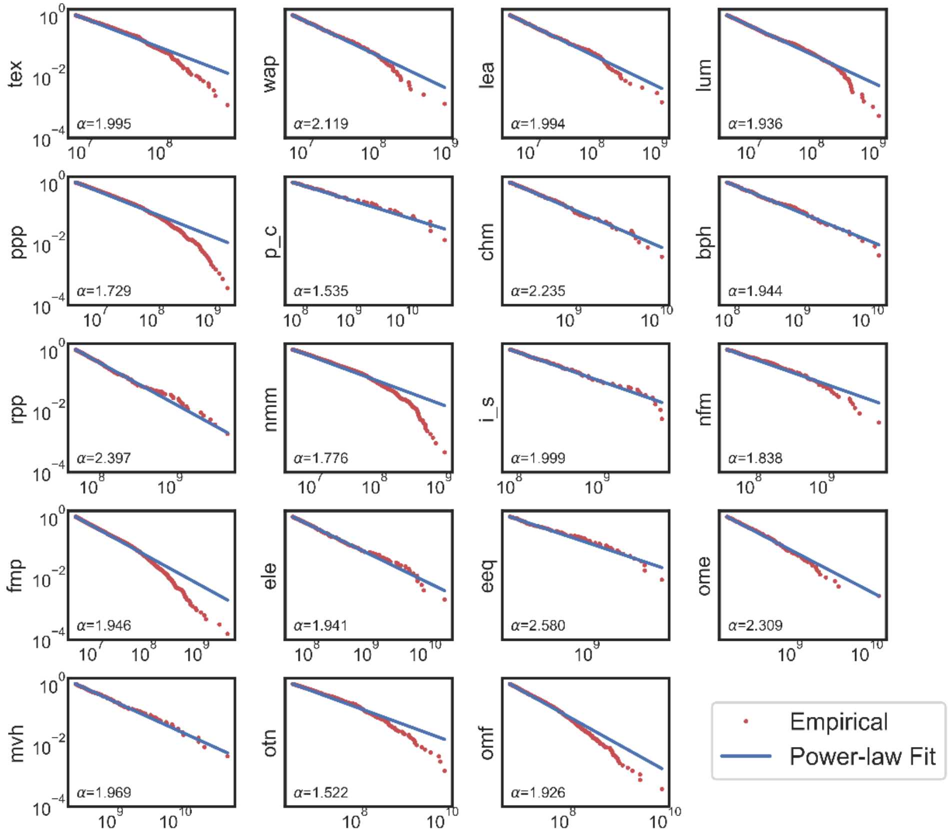
Step 3: Elasticity in GTAP manufacturing



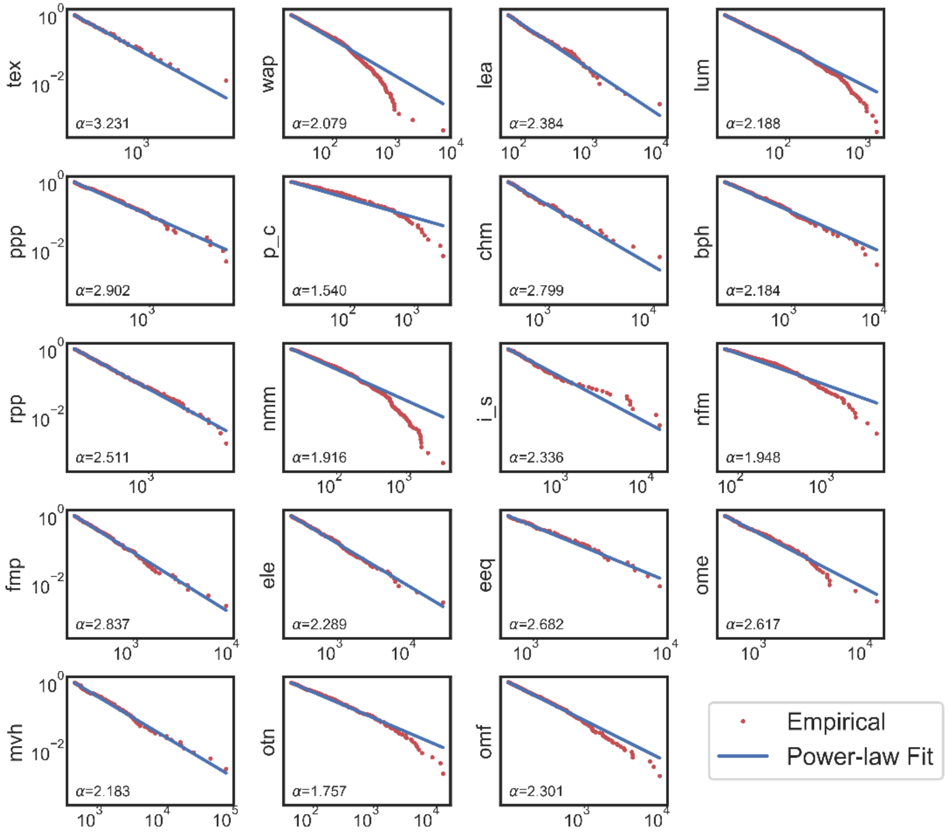
- Distribution across sectors is quite similar
- Firm size proxy drives the distribution

Empirical CCDF and Power-law fit: Firm size

Operating Revenue

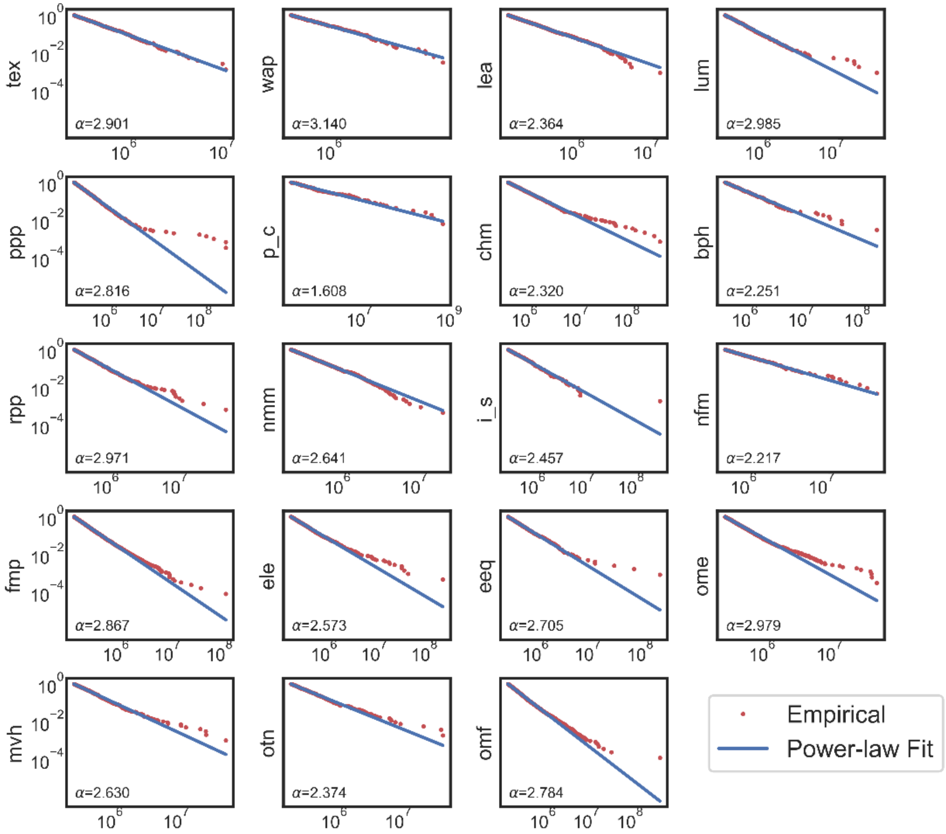


Number of Employees

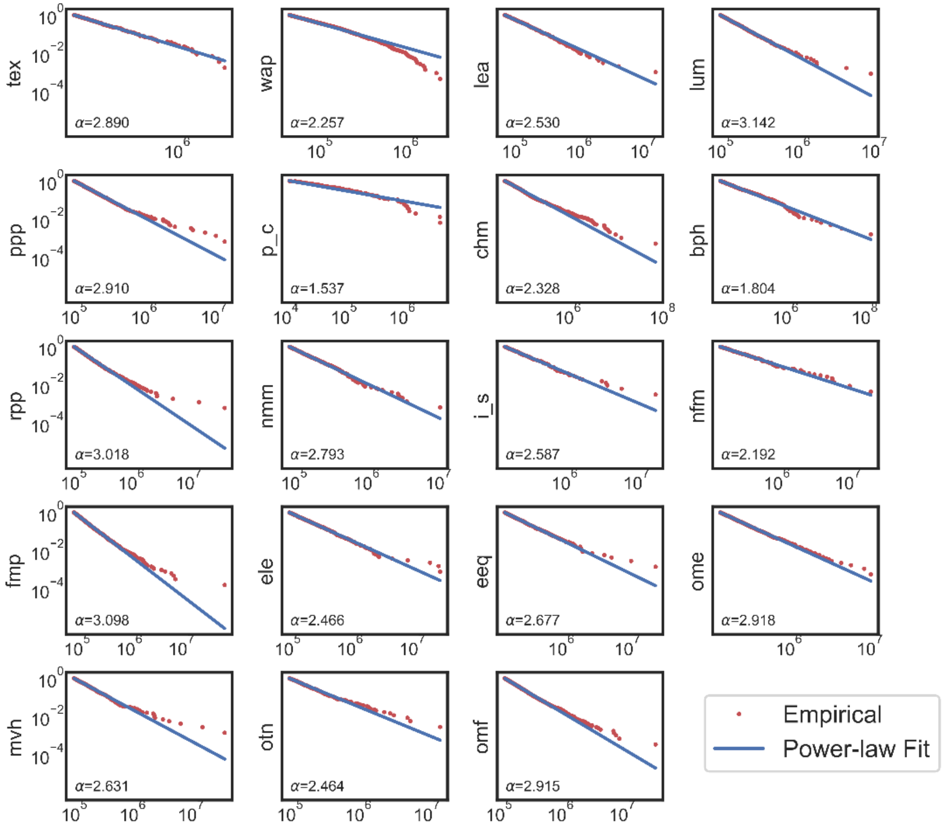


Empirical CCDF and Power-law fit: Productivity

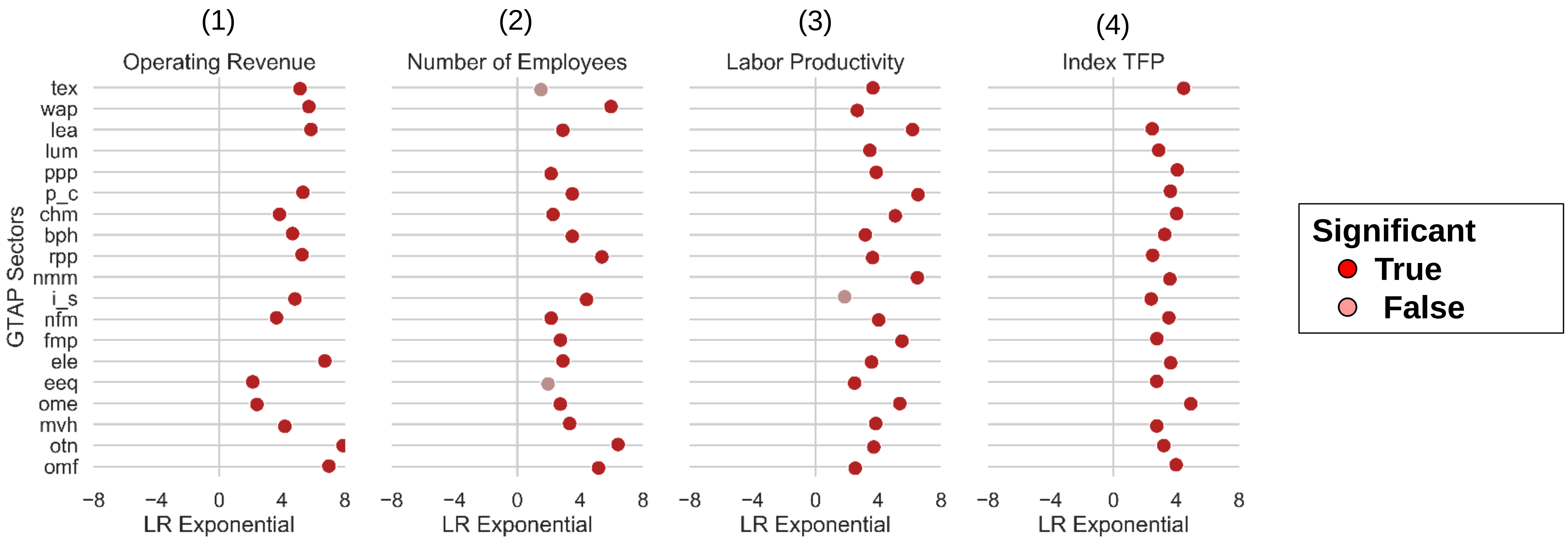
Labor Productivity



Index TFP

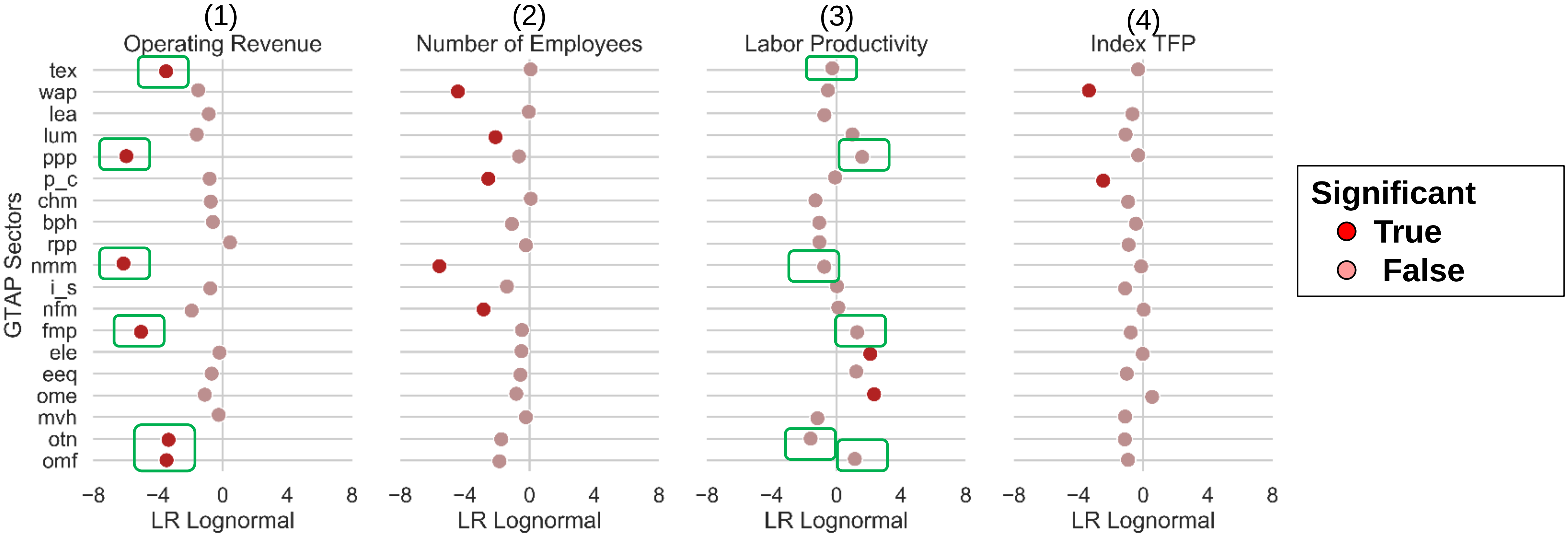


Likelihood Ratio Test: Exponential



- LR values are positive
- Power-law model is a better fit compared to exponential
- Test is inconclusive for a few sectors

Likelihood Ratio Test: Log-normal



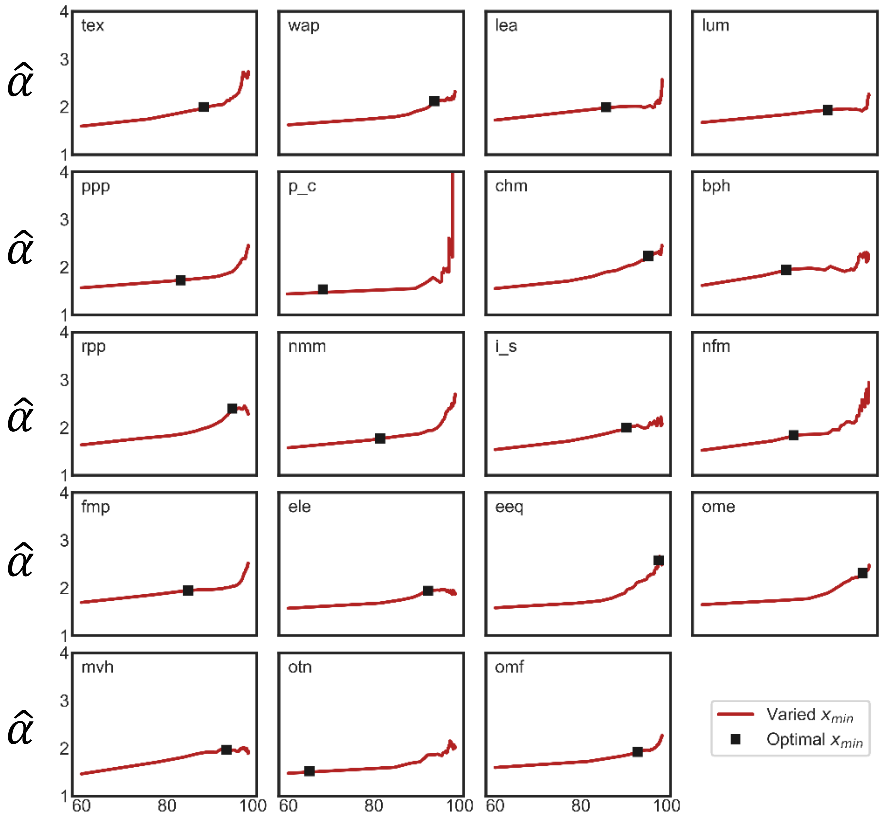
- LR values are mostly negative
- Test is inconclusive for most of the sectors
- Labor productivity may follow a power-law model, while operating revenue may not

Sensitivity analysis

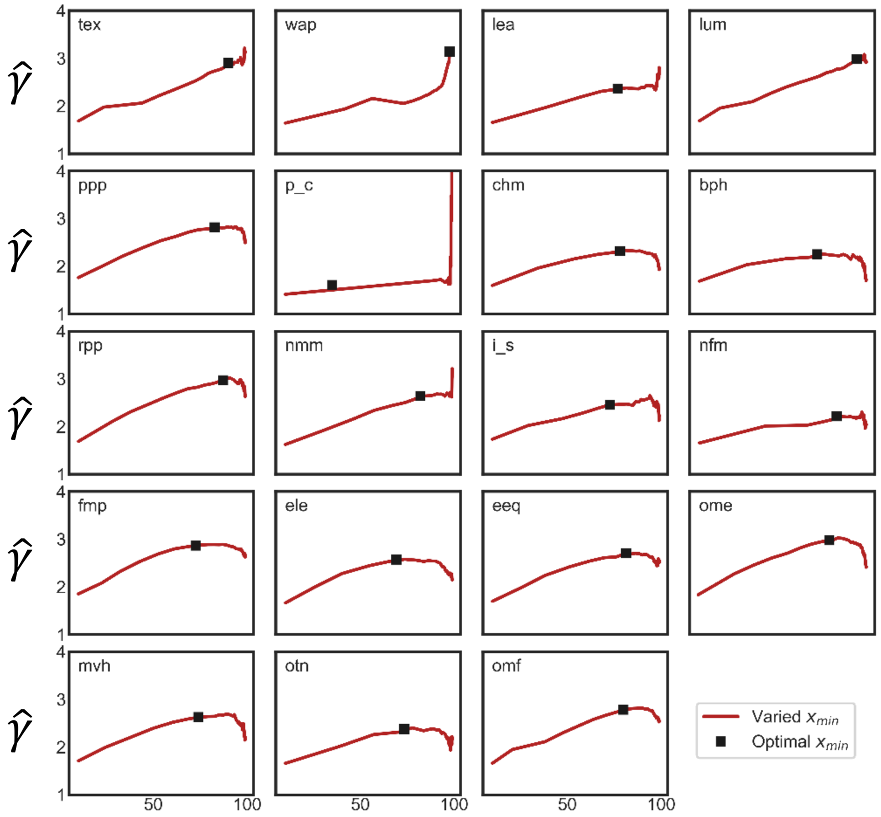
- Optimized x_{min} values reduce the sample size considerably in some sectors
- The size of the remaining sample differs across sectors
- Power-law fit is conducted with largest firms in some sectors and smaller firms in others
- Objective: Observe the effect of constraining sample size on the power-law exponent
- Method: Re-estimate the power-law in firm size and productivity for all firms in each sector while moving the lower-bound incrementally

Sensitivity analysis

Operating Revenue



Labor Productivity



Concluding remarks

- **Parameter values vary across GTAP sectors**
  - Power-law exponents for firm size
    - Operating revenue: 1.5 – 2.6
    - Number of employees: 1.5 – 3.4
  - Elasticity values: 1.9 - 2.8

- **Going forward**
  - Do firm size and productivity follow different distributions?
  - Alternative distributions for specific sectors may perform better
  - Policy simulations using GTAP-HET with new parameter estimates