Incorporating additional information for the estimation of interregional trade flows

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ABSTRACT:

This paper deals with the problem of estimating interregional trade flows. We apply entropy-based estimation procedure and compare its performance with double-constraint gravity model. We analyze two different entropy specifications - the data-constrained Generalized Cross Entropy (GCE) estimator and the moment-constrained GCE estimator. The performance of each estimator is evaluated by means of numerical simulations. We find that entropy-based estimators provide more accurate results than the gravity model once the possibility of potential errors in row or column totals is included. The bigger the potential error and the higher the number of regions the greater difference in accuracy between the gravity model and GCE estimators. Also, the moment-constrained GCE estimator tends to be more accurate than the data-constrained GCE estimator.

Keywords: entropy, input-output tables, CGE modelling, regionalization.

JEL codes: C67, O18, R15.

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1. Introduction

One frequent problem when studying interregional trade flows is the lack of sufficiently detailed data based on surveys. Interregional trade flows are not usually registered in surveys by statistical agencies and some indirect estimation method is applied. There are several alternatives when dealing with this problem in the input-output context. One of the most frequently applied solutions is to use a gravity approach (e.g. Miller and Blair, 2009), first suggested by Leontief and Strout (1963). Gravity models explain variation in the level of flows between origin and destination locations basing on three types of factors to explain spatial interactions: origin-specific variables; destination-specific variables and a separation function that reflects the effect of spatial distances between origins and destinations preventing or diminishing the interactions. If information on the totals of regional exports and imports is available, the predictions of the gravity equation are adjusted to make them consistent with these aggregates. Actually, several papers show that the gravity based estimates[1] of interregional trade are more accurate as compared to other existing methods (e.g. Fournier Gabela, 2020; Riddington *et al.,* 2006).

Alternatively, an estimation procedure based on entropy maximization -or information minimization - can be applied with the same purpose. Traditionally, the entropy-based approach minimizes a Kullback-Leibler divergence (e.g. Kullback and Leibler, 1951) with respect to an initial matrix while being consistent with the aggregate information of the target matrix. In this paper we explore the performance of alternative entropy formulations proposed by Golan et al. (1994), that rely not on these aggregates but in the cross-moments between the interregional trade flows and some additional covariates that are not considered in simple gravity-type equations. Furthermore, we compare the estimation accuracy of entropy estimators with double-constrained gravity model.

The paper is structured as follows. Section 2 explains the basics of the estimation problem and the solution traditionally applied. Section 3 present the two possible formulations of a Cross-Entropy (CE) estimator. A simple numerical simulation compares the performance of these estimator in section 4. Finally, section 5 concludes.

[1] Usually applied jointly with RAS adjusting procedure.

2. The problem: perfect v. noisy aggregate information

The lack of commonly available survey based data has forced researchers to put their attention on the development of non-survey regionalization methods. Four main strands can be distinguished in the literature concerning the non-survey,[2] trade-adjust technology based input-output tables (IOT) regionalization methods:

- Methods based on location quotients (LQ) such as the simple location quotient (SLQ), purchases-only location quotient (PLQ), cross-industry location quotient (CILQ), Flegg location quotient (FLQ) or industry-specific Flegg location quotient (SFLQ);
- Commodity balance approach and its extensions such as the cross-hauling adjusted regionalization method (CHARM) or modified CHARM (MCHARM);
- Regional purchase coefficient (RPC) approach;
- Bi-proportional methods such as RAS, entropy or mathematical programming models.

In recent years, there has been a growing body of methodological analyses devoted to the application of LQ type approaches in the regionalization of national IOT. For instance, Bonfiglio and Chelli (2008) show that FLQ[3] and its augmented version perform better than their predecessors such as SLQ. Additionally, they find that both methods tend to over/underestimate the impact with respect to the chosen values of parameter δ that influences the relative size of the regions within the formulae. The correct value of parameter δ has been a subject of other papers by Tohmo (2004), Bonfiglio (2009), Flegg and Tohmo (2013, 2016), Kowalewski (2015), Zhao and Choi (2015) or Flegg *et al*. (2016). In general, most of the authors claim the LQ approach to be one of the best non-survey alternatives to estimate regional IOT. Still, there is no agreement whether the FLQ or SFLQ is the better in terms of accuracy.

The commodity balance approach is an older methodology[4] that has been recently modified and extended within the CHARM framework. Following Kronenberg (2009), CHARM provides an improvement to the performance of the LQ approach in terms of estimating regional trade. By

[2] Actually, the term "non-survey" can be misleading since some of the above approaches actually require the use of some survey-based data (e.g. Lahr, 1993).
[3] See Flegg *et al*. (1995) for more details.
[4] Its roots can be found in the paper by Isard (1953).

accounting for cross-hauling, it deals with the problem of underestimation of regional trade flows that leads, in turn, to overestimation of regional multipliers. The initial CHARM approach was refined by Többen and Kronenberg (2015) in order to account for interregional trade.[5] Fujimoto (2019) claims that his version of the approach (called MCHARM) performs better in estimating regional trade flows in the case of Japan, as compared to LQ, FLQ and the standard CHARM.

The regional purchase coefficient (RPC) approach was first proposed by Stevens *et al*. (1983). It estimates the proportion of regional demand fulfilled from regional production using regional economic and interregional transportation data. Note, that in some studies gravity equation was used to estimate RPCs (e.g. Lindall et al., 2006). As shown by Brucker *et al*. (1990), RPC has been implemented in a number of ready-made models for the United States such as IMPLAN. Yet, several authors claim that RPC, is in fact, one of the weakest points of such models (e.g. Lazarus *et al*., 2002)[6].

The existing literature on RAS is wide and also discusses different variants of this regionalization method (e.g. Miller and Blair, 2009; Riddington *et al*., 2006; Oosterhaven and Escobedo, 2011; Wiebe and Lenzen, 2016). Actually, RAS can be applied alone, but most hybrid methods use RAS as an intermediate step in the procedure. For instance, Flegg and Tohmo (2013) suggest the combined use of FLQ and RAS to improve the accuracy of estimation. Yet, probably the most common is the combination of gravity model and RAS, where the former is used as a prior for further RAS adjustments (e.g. Cai, 2020; Fournier Gabela, 2020; Johansen *et al.,* 2018; Riddington *et al.,* 2006; Sargento *et al.,* 2012; Yamada, 2015).

Other bi-proportional methods are less often discussed. Lahr and de Mesnard (2004) provide a review of different bi-proportional methods that in general share many RAS features. Jackson and Murray (2004) show that the methods alternative to RAS can outperform the latter in certain situations. Canning and Wang (2005) implement a mathematical programming model to estimate multiregional IO tables based on an interregional accounting framework and initial information of interregional shipments. Finally, there exist several papers applying entropy based estimators to

[5] Kronenberg (2009) defined cross-hauling as a proportion to the sum of demand and output.
[6] Actually, for at least several years already, instead of RPC, IMPLAN has been applying a gravity based approach as a default trade regionalization method (e.g. Lindall *et al*., 2006).

the estimation of IOT (e.g. Fernandez Vazquez, 2015; Fernandez Vazquez *et al*., 2015; Johansen et al., 2018).

There exists a large number of studies comparing the outcomes of different non-survey methods, beginning with the early works by Czamanski and Malizia (1969) or Hewings (1971) and ending with the more recent papers by Oosterhaven *et al*. (2003), Tohmo (2004), Riddington *et al*. (2006), Miller and Blair (2009), Jiang *et al*. (2012), Fujimoto (2019) and Lahr *et al.* (2020). In most of the cases, regional IOT based on different LQ approaches and different versions of RAS are being compared, applying different statistical criteria. Due to the problems related to possible noisy aggregate information approaches applying more information are expected to provide better results (e.g. Szyrmer, 1989; Lahr *et al.,* 2020). Actually, as shown in several papers, the combination of gravity model and RAS usually provides more accurate results as compared to other regionalization approaches (e.g. Fournier Gabela, 2020; Riddington *et al.,* 2006; Sargento *et al.,* 2012). However, as shown in the present paper, if a noisy information is present it is possible to further improve the accuracy of regionalization procedure by applying a generalized CE estimator.

2.1. Predicting trade flows with a double-constrained gravity model

We will base our explanations on the matrix-balancing problem depicted in Figure 1 as in Golan (2006, page 105), where the goal is to predict the (unknown) cells of a matrix using the information that is contained in the aggregate data of the row and column sums.

<<Insert Figure 1 about here>>

Under this general framework, the problem under study is the estimation of the inter-regional trade flows for a system of region $i = 1,..N$ locations. The t_{ij} cells of the matrix are the unknown trade flows we would like to estimate (shaded in grey), where the row and column aggregates ($y_{i\cdot}$ and $x_{\cdot j}$ respectively) are known and where the sums $\sum_{j=1}^{N} t_{ij} = y_{i\cdot}$ and $\sum_{i=1}^{N} t_{ij} = x_{\cdot j}$ contain the total regional exports and imports respectively.

Assuming that we have information on variables that can be assumed to represent the mass of each region $i = 1,..N$ in our system (m_i) and a function that captures the impact of geographical separation between the regions $f(d_{ij})$, where d_{ij} is a variable that indicates distance, the double-constrained gravity model predicts the trade flows as in Griffith and Fischer (2013):

$$t_{ij}^{G} = A_i B_j m_i^{\alpha} f(d_{ij}) m_j^{\beta}; \text{ where}$$

$$A_i = y_{i\cdot} \left[\sum_{j=1}^{N} B_j m_i^{\alpha} f(d_{ij}) m_j^{\beta} \right]^{-1} \tag{1}$$

$$B_j = x_{\cdot j} \left[\sum_{j=1}^{N} A_i m_i^{\alpha} f(d_{ij}) m_j^{\beta} \right]^{-1}$$

The parameters α and β are assumed positive, reflecting that larger masses either at the origin or destination are expected to increase the interregional trade. In empirical implementations of these models to interregional trade flows, masses are usually set as regional GDPs or populations. The specific form of the function $f(d_{ij})$ depends on the choice of the researcher, but is should be defined assuring that $\partial t_{ij}^{G} / \partial\, d_{ij}$ is negative. The scaling factors A_i and B_j make the gravity predictions consistent with the total regional exports and imports respectively.

2.2. Issues: not-included factors and noisy totals

Predictions based on a double-constrained gravity model will be accurate if the mechanism that generates the trade flows actually follows this gravitational mechanism and if the scaling factors that act as constraints effectively adjust the predictions to be consistent with the true totals of regional exports and imports. But it is not infrequent that real datasets do not comply one or both conditions.

Starting with the first potential issue, it is possible that the data generating process of interregional flows is affected by other factors not considered in a gravity model. This situation can be sketched by assuming a double nature in the interregional flows, where some part of these flows are produced by a gravity mechanism, while other part is given by the effect of other variables not directly considered there. This can be easily depicted in the following expression:

$$t_{ij} = t_{ij}^{G} \times I_{ij}^{Z} \tag{2}$$

In equation (2) we assume that the true flows t_{ij} are produced by a gravity mechanism (t_{ij}^{G}) but also as the result of a series of H additional factors that are collapsed in the form of a correction index I_{ij}^{Z}. Note that the scaling factors A_i and B_j indirectly allow for controlling the effect of other drivers of the interregional trade that a simple gravity model does not account: the impact of any characteristic other than the origin and destination masses and the separation between regions could be affecting the flows, and the corrections made by A_i and B_j make the purely gravity-based predictions to be consistent with the observed totals.

Another issue arises if the totals $y_{i\cdot}$ and $x_{\cdot j}$ are not directly observable for the researcher. In many empirical applications, these vectors that contain the regional exports and imports are not perfectly known -or not at all- and only approximations can be used instead. This situation can be depicted by including observation error terms as in $\tilde{y}_{i\cdot} = y_{i\cdot} + \varepsilon_i^{y}$ and $\tilde{x}_{\cdot j} = x_{\cdot j} + \varepsilon_j^{x}$, where the terms ε_i^{y} and ε_j^{x} stand for the error in the measurement of the total of regional exports and imports respectively. In operational terms, this not a problem for the double constrained gravity model, since it will simply include the estimates $\tilde{y}_{i\cdot}$ and $\tilde{x}_{\cdot j}$ in the definition of the scaling factors A_i and B_j in (1).

3. Possible solution: GCE data-constrained and GCE moment-constrained formulations.

3.1. The data-constrained GCE estimator

We propose an estimation technique that exploits all the available information, we mainly base on the paper by Golan et al. (1994). This paper presented a Cross Entropy (CE) procedure to estimate intersectoral flows from incomplete data or, more generally speaking, for a problem of matrix balancing with partial information. This technique bases on transforming the initial trade flows t_{ij} by scaling them as $p_{ij} = t_{ij}/x_{\cdot j}$. This scaling allows for considering each column of a target matrix **P** as a set of N column probability distributions – note that their cells are positive and they sum up to one- to be estimated. Similar to RAS, the CE technique minimizes the Kullback-Leibler divergence between the target **P** and an initial matrix **Q**, provided that the solution is consistent with the observable information –vectors **x** and **y** in our problem-. A constrained minimization problem is applied and it can be posed as a program like:

$$\underset{\boldsymbol{P}}{Min}\, D(\mathbf{P},\mathbf{Q}) = \sum_{i=1}^{N}\sum_{j\neq i}^{N} p_{ij} ln\left(\frac{p_{ij}}{q_{ij}}\right) \tag{3}$$

Subject to:

$$\sum_{j=1}^{N} p_{ij}x_{\cdot j} = y_{i\cdot};\ i = 1, \dots, n \tag{4}$$

$$\sum_{i=1}^{N} p_{ij} = 1;\ j = 1, \dots, N \tag{5}$$

The CE program depicted above is known to produce the same solution as a RAS adjustment of the initial **Q** matrix given vectors **x** and **y**.

The previous CE program can be applied to estimate **P** in situations where vectors **x** and **y** are perfectly observable. However, as discussed in previous section, it is relatively frequent to deal with situations on which we do not have perfect observations on the elements of **x** and **y**, but instead we only have approximate figures on $\tilde{y}_{i\cdot} = y_{i\cdot} + \varepsilon_i^y$ and $\tilde{x}_{\cdot j} = x_{\cdot j} + \varepsilon_j^x$. In such cases, the CE program can be transformed into a Generalized Cross Entropy (GCE) problem in order to accommodate these errors terms.

In order to do that, the GCE estimator re-parametrizes the errors as (discrete) random variables that can take L different values. These realizations reported on a so-called supporting vector $\mathbf{v}'_{\mathbf{k}}$, symmetric and centred around zero as $\mathbf{v}'_{\mathbf{i}} = [v_{1i}, \dots, 0, \dots, v_{Li}]$, where L is an odd number and $v_{1i} = -v_{Li}$. The probabilities corresponding to the v_l elements are defined as $\mathbf{w'}_{\mathbf{i}} = [w_{1i}, \dots, w_{Li}\ ]$. While the elements on vector $\mathbf{v}_{\mathbf{i}}$ are imposed by the researcher, the corresponding probabilities $\mathbf{w}_{\mathbf{i}}$ are unknown and should be estimated**.**

Applying this GCE estimator implies modifying the CE program in equations (3) to (5) for accommodating the estimation of $\mathbf{w}_{\mathbf{i}}$. The strategy bases again in minimization of the Kullback-Leibler divergence between the target $\mathbf{w}_{\mathbf{i}}$ and an initial $\mathbf{w}_{\mathbf{i}}^{\mathbf{0}}$ that sets our prior belief about the errors. In absence of any information about the size and/or sign of these potential errors, a natural strategy is to set common supporting vectors for all them and set them as $\mathbf{v_i'} = \left[\frac{-1}{\sqrt{N}}, \dots, 0, \dots, \frac{1}{\sqrt{N}}\right]$. Regarding the specification of a prior probability distribution $\mathbf{w_i^0}$, a natural solution would be to set them as uniform probabilities. Note that this implies that our prior belief is that the errors are zero, and the GCE solution would deviate from this initial belief only if the observable information included as constraints forces the estimator to do it.

More specifically, the GCE estimator is depicted in the following set of equations:

$$\underset{\boldsymbol{P},\boldsymbol{Q}}{Min}\, D(\mathbf{P}, \mathbf{W}, \mathbf{Q}, \boldsymbol{W^0}) = \sum_{i=1}^{N} \sum_{j \neq i}^{N} p_{ij} ln\left(\frac{p_{ij}}{q_{ij}}\right) + \sum_{i=1}^{N} \sum_{l=1}^{L} w_{li} ln\left(\frac{w_{li}}{w_{li}^0}\right) \tag{6}$$

Subject to:

$$\tilde{y}_{i\cdot} = \sum_{j=1}^{N} p_{ij} \tilde{x}_{\cdot j} + \sum_{l=1}^{L} w_{li} v_{li}\, ;\ i = 1, \dots, n; \tag{7}$$

$$\sum_{i=1}^{N} p_{ij} = 1;\ j = 1, \dots, N \tag{8}$$

$$\sum_{l=1}^{L} w_{li} = 1;\ i = 1, \dots, N \tag{9}$$

Note that the GCE formulation (and also the pure CE program) can use a gravity-approach to produce their estimates on a natural way. Simply by defining the prior matrix **Q** basing on the predictions of a gravity mechanism as $q_{ij} = t_{ij}^G / \left[\sum_{i=1}^{N} t_{ij}^G\right]$. By doing that, the GCE produces estimates of the interregional trade matrix that implicitly accounts for the effect of the origin and destination masses and the geographical separation between regions while simultaneously incorporates the possibility of having imperfect observations of the regional exports and imports in a natural way.

3.2. The moment-constrained GCE estimator

While addressing the possibility of having these noisy constraints, this GCE formulation does not effectively deal with the possibility of having other factors not considered in the gravity mechanism affecting the interregional trade flows. However, a reformulation of the GCE estimator based on cross-moments between the trade totals and the other potential factors would account for this alternative source of variation in the t_{ij} cells. Assume that there is a set of covariates in a $(N \times H)$ matrix $\boldsymbol{Z}$ that are expected to be correlated with the $p_{ij} = t_{ij}/x_{\cdot j}$ scaled trade flows. Examples of these covariates can be, following the literature that estimate the so-called augmented gravity equations, income differences, stocks of infrastructures or indicators of specialization, among others. This information can be incorporated to the GCE estimator by following variant of the estimator proposed in Golan and Vogel (2000) or Golan (2018), which make use of the cross-moments equations:[7]

$$\boldsymbol{Z^T y} = \boldsymbol{Z^T}[\boldsymbol{Px} + \boldsymbol{u}]=\boldsymbol{Z^T}[\boldsymbol{Px} + \mathbf{Wv}] \tag{10}$$

that serve as constraints in the following optimization program:

$$\underset{\boldsymbol{P},\boldsymbol{Q}}{Min}\, D(\mathbf{P},\mathbf{W},\mathbf{Q},\boldsymbol{W^0}) = \sum_{i=1}^{N}\sum_{j\neq i}^{N} p_{ij} ln\left(\frac{p_{ij}}{q_{ij}}\right) + \sum_{i=1}^{N}\sum_{l=1}^{L} w_{li} ln\left(\frac{w_{li}}{w_{li}^0}\right) \tag{11}$$

subject to:

$$\sum_{i=1}^{N} z_{ih}\, \tilde{y}_{i\cdot} = \sum_{i=1}^{N} z_{ih}\left[p_{ij}\tilde{x}_{\cdot j} + \sum_{l=1}^{L} w_{li}v_{li}\right]; h = 1, \dots, H \tag{12}$$

$$\sum_{j=1}^{J} p_{ij} = 1; i = 1, .., n$$

$$\sum_{l=1}^{L} w_{ijl} = 1; i = 1, .., n; j = 1, .., J \tag{13}$$

[7] Superscript T indicates transposition.

Note that the information available for the data-constrained GCE estimation is included in equation (11), while the moment-constrained GCE estimator includes the correlation between the target matrix and the additional covariates in equation (12), being the rest of equations, including the objective function and the normalizing constraints, identical to the data-constrained GCE version. If the GCE program above is solved, estimates of $\boldsymbol{P}(\widehat{\boldsymbol{P}})$ are recovered, being this solution the closest one to matrix $\boldsymbol{Q}$ while being consistent with cross moments between $\boldsymbol{y}$ and $\boldsymbol{Z}$ given in the left hand side of equation (12). Without this piece of information, the solution of the GCE estimator will be matrix $\boldsymbol{Q}$, but this equation "pushes" the solution to be consistent with these observed moments.

4. Illustration by a (simple) numerical simulation

4.1. Simulation design

In order to illustrate how the studied GCE formulations operate in comparative terms to the double-constrained gravity, we rely on numerical experiments where artificial flows between regions are generated based on the (i) masses of the origin and the destination, (ii) a distance decay function that indicates separation or transport cost between origin and destination regions, and (iii) some other possible factors not accounted on the masses or the distance decay function. Note that the predictions produced by a gravity model would usually base on the information contained on (i) and (ii) and on setting some parameters. If totals of imports and exports are observable, the double constrained version of the gravity will yield predicted flows consistent with these totals. These predicted flows will be our benchmark in the comparative analysis.

We assume that there is a system of N regions on which the simulated trades (t_{ij}^S) are generated as $t_{ij}^S = \frac{m_i^\alpha m_j^\beta}{d_{ij}^\delta} \times I_{ij}^Z$; where α, β and δ are the parameters that quantifies the effect of the origin and destination masses and the geographical distance, respectively. In the simulations the masses m_i are generated as uniform distributions $m_i \sim U(1,2)$ and the distances as $d_{ij} = d_{ji} \sim U(0,1) \forall i \neq j$. For the sake of simplicity and following the same approach as in Sargento, Ramos, and Hewings (2012), all the parameters α, β and δ are set to 1.

There are other factors that are assumed to affect the t_{ij}^{S} flows, apart from the typical elements of the gravity mechanism. These other factors are captured in the indicator I_{ij}^{Z}, which in our simulation is assumed to have the form $I_{ij}^{Z} = exp[\gamma \boldsymbol{Z}\boldsymbol{Z}^{\boldsymbol{T}}]$, where $\boldsymbol{Z}$ is a matrix with dimension $(N \times H)$. The elements of $\boldsymbol{Z}$ are generated as $z_{ih} \sim U(0,1)$ and γ is a scaling parameter that fluctuates on each simulation drawn as $\gamma \sim U(0,1)$.

Some potential observational errors in the totals of regional exports and imports is included in the simulation. The idea is to reflect situations on which we only have approximate figures, like $\tilde{y}_{i\cdot} = y_{i\cdot} + \varepsilon_{i}^{y}$ and $\tilde{x}_{\cdot j} = x_{\cdot j} + \varepsilon_{j}^{x}$. While $y_{i\cdot} = \sum_{j=1}^{N} t_{ij}^{S}$ and $x_{\cdot j} = \sum_{i=1}^{N} t_{ij}^{S}$, the observed totals $\tilde{y}_{i\cdot}$ and $\tilde{x}_{\cdot j}$ are generated respectively as $\tilde{y}_{i\cdot} = \sum_{j=1}^{N}\left[t_{ij}^{S} + N(0, \sigma t_{ij}^{S})\right]$ and $\tilde{x}_{\cdot j} = \sum_{i=1}^{N}\left[t_{ij}^{S} + N(0, \sigma t_{ij}^{S})\right]$. In other words, we include the possibility of errors on these totals by means of an error that distributes normally with zero mean and a variance that is proportional to each cell t_{ij}^{S}. The size of this error is controlled by the scalar σ.

Our experiments consider several scenarios with variability in the number of N regions and the size of the observational errors in the regional totals. More specifically, we simulate systems with $N = 10, 20, 50$ regions and where the potential errors in the total of regional exports and imports is given by setting the scaling parameter $\sigma = 0;\ 0.05; 0.10$.

The predictions produced by a double constrained version of the gravity model will base on the information contained on the masses m_i, m_j and the distances d_{ij}, together with the setting of the α, β and δ parameters to predict trade flows consistent with the observable totals of exports and imports. We will assume a very favorable scenario for the gravity-predictions where the parameters are perfectly known, so there is no source of error that could come from "bad" choices of these parameters, even when this is hardly realistic in real-world problems where they are unknow. These predicted flows will be our benchmark in the comparative analysis.

The GCE formulations will be used as alternative ways of predicting these flows. The data-constrained version as in equations (6) to (9) and the moment-constrained formulations as in equations (11) to (13). In both cases the supporting vectors are defined as $\mathbf{v}_{\mathbf{i}}' = \left[\frac{-1}{\sqrt{N}}, \ldots, 0, \ldots, \frac{1}{\sqrt{N}}\right]$ and the prior matrix $\boldsymbol{Q}$ is defined based on the predictions produced by a free -unconstrainedgravity mechanism, which is not consistent with the totals of exports and imports (i.e., simply as $m_i^\alpha m_j^\beta / d_{ij}^\delta$ with $\alpha = \beta = \delta = 1$). This implies that the initial GCE estimations include the gravity mechanism, which are later adjusted to make them consistent with some additional information. The data-constrained GCE adjust these estimates to the totals of regional exports and imports, while the moment-constrained GCE formulation makes them consistent with the cross-moments with other additional covariates captured on matrix $\boldsymbol{Z}$. We have generated the elements of $\boldsymbol{Z}$ assuming that there are a set of $H = 10$ variables that affect the term I_{ij}^Z , although the moment-constrained GCE estimator only makes use of a subset of these covariates and assume that only $\boldsymbol{Z}^*$ (with dimension $N \times 4$) and not the full $\boldsymbol{Z}$ (with dimension $N \times 4$) is observable for the researcher.

The comparison between the estimates produced by these three techniques are evaluated by calculating the Weighted Absolute Percentage Error between the simulated trade flows and their corresponding predictions, being defined as:

$$WAPE = 100 \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} \left| t_{ij}^S - t_{ij}^P \right|}{\sum_{i=1}^{N} \sum_{j=1}^{N} \left| t_{ij}^S \right|} \tag{14}$$

where t_{ij}^P stands for the predictions.

4.2. Simulation results

The results, under different scenarios of potential error in the totals are reported in Tables 1, 2 and 3.

<<Insert Tables 1, 2 and 3 about here>>

The figures reported in these tables show how the gravity approach works well if reliable information about the margins is available: if this is the case, none of the GCE formulations seems to beat it. However, if the totals of regional exports and imports are not perfectly known, the GCE formulations produce lower deviation figures than the gravity predictions, being this difference larger if the size of the potential noise in these totals grows. Even when a relative small value of scalar $\sigma = 0.05$, which produces errors of around $\pm 5\%$ in the margins, the moment-constrained GCE estimator seem to perform better than gravity and the data-constrained GCE version, because it is not so much dependent on having accurate information on these margins that act as constraints. When this scalar is set to $\sigma = 0.10$, which produces errors of around $\pm 10\%$ in the margins, the difference in favor of the moment-constrained GCE estimator is even bigger, although in this case the data-constrained GCE formulation also does better than the gravity model, give its larger flexibility to accommodate measurement errors in its constraints.

5. Concluding remarks

This paper applies the moment-constrained GCE estimator developed in Golan and Vogel (2000) or Golan (2018) as a flexible way to incorporate additional data together with other structural variables into the estimation of interregional trade flow matrices. The results obtained in a simple numerical simulation illustrate how this approach can be considered as an alternative to other more traditional estimation procedures. Our results suggest that this technique could be particularly useful if (i) some additional information correlated with the network of interregional flows is available; and/or, (ii) we could have imperfect information in the totals that act as constraints in other estimation procedures.

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Table 1: Simulation results. WAPE between prediction methods and simulated flows. Perfect observation ($\sigma = 0$) of row and column totals

	Prediction method		
Number of regions	**Gravity**	**GCE-data constrained**	**GCE-moment constrained**
N=10	0.120	0.148	0.253
N=20	0.367	0.559	0.724
N=50	0.112	0.214	0.298

Table 2: Simulation results. WAPE between prediction methods and simulated flows. Noisy observations ($\sigma = 0.05$) of row and column totals

	Prediction method		
Number of regions	**Gravity**	**GCE-data constrained**	**GCE-moment constrained**
N=10	2.713	2.736	2.335
N=20	2.951	3.120	2.159
N=50	5.596	6.034	5.372

Table 3: Simulation results. WAPE between prediction methods and simulated flows. Noisy observations ($\sigma = 0.10$) of row and column totals

	Prediction method		
Number of regions	**Gravity**	**GCE-data constrained**	**GCE-moment constrained**
N=10	10.529	10.050	9.137
N=20	8.177	7.980	6.333
N=50	11.163	10.484	8.685

Figure 1: Matrix of inter-regional trade flows.

t_{11}	$\cdots$	t_{1j}	$\cdots$	t_{1N}	$y_{1\cdot}$
$\cdots$		$\cdots$		$\cdots$	$\cdots$
t_{i1}	$\cdots$	t_{ij}	$\cdots$	t_{iN}	$y_{i\cdot}$
$\cdots$		$\cdots$		$\cdots$	$\cdots$
t_{N1}	$\cdots$	t_{Nj}	$\cdots$	t_{NN}	$y_{N\cdot}$
$x_{\cdot 1}$	$\cdots$	$x_{\cdot j}$	$\cdots$	$x_{\cdot N}$	