Endogenous Technological Change adapted to CGE framework

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Abstract

We introduce endogenous technological change in a multi-sector recursive dynamic Computable General Equilibrium (CGE) model. We consider the optimisation problem of technology firms that can choose the optimal level of R&D intensity given market conditions. The R&D intensity determines the number of innovations and speed of technological change in each sector. In addition, firms can choose the direction of technological change; for example, when designing, they could select less energy-intensive but more capital-intensive production when the price of energy increases. The incorporation of endogenous technological change has significant impacts on policy simulations. Once it is considered, the model predicts that the climate policy (in the form of a carbon tax) has a significantly larger impact on emissions in the long run than in the short run.

1 Introduction

The frequent function of recursive-dynamic Computable General Equilibrium (CGE) models is to simulate a path of economic variables after policy interventions. The major strength of this type of models in such simulations is that the predictions are derived from the changes in the optimal choices of economic actors; therefore, the predictions are always consistent with microeconomic theory. An important weakness is that, typically, the predictions are built on the questionable assumption that actors must make their choices taking technological variables as given; nor do they have an opportunity to influence the future characteristics of technologies. This paper is an attempt to provide a remedy for this weakness without compromising the fundamental strength of the CGE model, which is based on its microeconomic foundations.

We propose a framework that fits the typical structure of CGE models and allows us to take into account that changing market conditions will influence the speed of technological change and the factor intensity of technologies. At the same time, we preserve the microeconomics-based nature of CGE models by deriving all relations from the predictions of the optimisation of forward-looking profit-maximising R&D firms. We also consider that the blueprints that those firms produce are a non-rival good. This is an important consideration, because it implies that the accumulation and trade of knowledge (contained in blueprints) will be distinct from the accumulation and trade of capital goods.

From the methodological perspective, our study contributes to the literature by integrating insights from endogenous growth theory, evolutionary economics, inducted technological change and directed technological change, and by adapting them to the CGE framework. Following the endogenous growth literature (Romer 1989, 1990, Grossman and Helpman 1992, Aghion and Howitt 1992), we introduce monopolistic competition between owners of blueprints, quality ladder progress (every new blueprint allows an improvement in the productivity of production), and the business-stealing effect (the owner of a blueprint receives profit only until another firm comes up with a superior blueprint). Based on the literature on evolutionary economics (Aghion 2002, Goldfarb 2011, Young 1993), we allow for the possibility that innovation and progress in all sectors are bounded by the progress of general-purpose technology (GPT). Following the literature on induced technological change (Kennedy 1964, Samuelson 1965, Drandakis and Phelps 1966) and directed technological change (Acemoglu 1998, 2015, Acemoglue et al. 2012, Aghion et al. 2016), we consider that technology firms may choose not only the level of their R&D effort (which determines the number of innovations and the speed of technological change) but also the factor intensity of new technologies (the direction of technological change).

We also propose some simplifications and adjustments that allow us to adapt the approaches mentioned above to the modelling constraints of a standard country-level multi-sector recursive-dynamic CGE model. We assume that decisions regarding R&D activity and technological progress take place in each sector of the economy separately. We also assume that the progress of GPT – which, in our framework, determines a bound for the progress in individual sectors – is exogenous. This assumption is because the progress of the GPT is likely to be global and independent of country-level policy interventions. We also assume that firms are forward looking in the sense that they do consider that innovations will bring them future profits. However, we assume that they are naive in the sense that they believe that the critical parameters in their optimisation problem – interest rate, growth of sectoral revenue and innovation rate ¬– will stay constant in the future. We assume that R&D effort requires the use of investment goods, and that the productivity of R&D activities decreases over time, which is necessary to ensure the existence of a balanced growth path. We also propose and discuss approximations that significantly reduce the number of equations in the model, thereby improving the model's trackability and reducing the computational time required.

The key reason for incorporating an endogenous technological change framework in CGE analysis is the need to improve the dynamic predictions of CGE models. The immediate response of the economy to policy intervention will be different from its response in subsequent periods when actors can choose the characteristics of future technologies. For instance, after an increase in energy price (induced, for example, by a carbon tax), firms will initially have limited opportunities to replace energy with other factors of production, say capital; however, as time passes, they will conceive several ways to increase the energy efficiency of their production. Consequently, the drop in energy demand will be larger in the long run than in the short run, and the short-run price elasticity of demand will be different from the long-run elasticity. Another example is the impact of subsidies for renewable energy sources (RES) on the deployment of renewable energy. An initial increase in demand for RES will incentivise technology firms to invest more R&D resources in that sector, which will bring more future innovations, a gradual reduction in the cost of manufacturing, a gradual reduction in RES price, and thus, a further increase in demand. In this case, an initial increase in the deployment of renewables will be gradually amplified in subsequent periods.

In the paper, we demonstrate how the CGE model with endogenous technology change can be used to assess the macroeconomic impacts of climate policy (in the form of a hypothetical carbon tax) in India. The model possesses three features that, according to Grubb et al. 2020, are essential for the evaluation of the dynamic effects of a carbon tax: induced technological change, inertia and path dependency. Our simulations suggest that the response of emissions to the introduction of a constant carbon tax is significantly larger in the long run than in the short run. We also find that the deployment of RES will increase gradually. Moreover, both reduction in emissions and deployment of RES are larger when we take into account endogenous technological change, compared to the predictions of a standard CGE model with no endogenous technological change.

2 The microfounded model

2.1 Demand for varieties

We assume that output in sector a comprises a continuum of varieties that are imperfect substitutes:

$$Q_{a,t} = \left(\int_{\Omega_{a,t}} \left(\lambda_a^{top}(u)\, q_{a,t}(u) \right)^{\frac{\sigma_a^{var}}{\sigma_a^{var}-1}} du \right)^{\frac{\sigma_a^{var}-1}{\sigma_a^{var}}} \tag{1}$$

where $\Omega_{a,t}$ is a set of varieties available in sector a at time t, $q_{a,t}(u)$ is the quantity of variety u and $\lambda_a^{top}(u)$ is the productivity parameter for variety u.

Following Young (1998), we distinguish between vertical and horizontal technological progress. With vertical innovation, technology firms develop a new and superior blueprint for an existing variety; we refer to this using the label '*improvement*'. In horizontal innovation, technology firms develop a variety that did not previously exist; we label this *invention*.

Let $IMP_{a,t}$ denote the improvement rate, that is, the proportion of varieties existing at time $t-1$ that were improved at time t, and $INV_{a,t}$ denote the invention rate, that is, the number of newly invented varieties relative to the total number of varieties produced at time $t-1$. When $\sigma_a^{var} > 1$, there are new varieties added through invention and the total number of varieties increases. When $\sigma_a^{var} < 1$, every invention replaces two varieties, and the total number of varieties decreases.[1]. We will refer to the total number of new blueprints relative to the total number of varieties produced at time $t-1$ ($IMP_{a,t} + INV_{a,t}$) as the innovation rate.

Varieties improved or invented simultaneously are identical. Hence, in the discrete time setting, varieties can be grouped into cohorts indexed by τ, where τ is the year in which these were invented or last improved. The production function, 1, then becomes:

$$Q_{a,t} = \left(\sum_{\tau=0}^{t} N_{a,\tau,t} \left(\lambda_{a,\tau}^{top} q_{a,\tau,t}^{top} \right)^{\frac{\sigma_a^{var}}{\sigma_a^{var}-1}} \right)^{\frac{\sigma_a^{var}-1}{\sigma_a^{var}}} \tag{2}$$

where $q_{a,t,\tau}^{top}$ denotes the output of a variety representative for cohort τ, $N_{a,\tau,t}$ with $\tau > 0$ being the number of varieties in cohort τ and $N_{a,0,t}$ the number of varieties that were invented or improved before the first year of the simulation. We use $M_{a,t}$ to denote the total number of varieties at time t, $M_{a,t} = \sum_{\tau=0}^{t} N_{a,\tau,t}$.

As is clear below, technology firms that choose the variety they want to improve are indifferent between varieties. Hence, the probability that a given variety is improved is the same in every cohort and equal to $IMP_{a,t}$. Similarly, if $\sigma_a^{var} < 1$, the probability that a variety will be replaced by a new invention is $2INV_{a,t}$, because we assumed invention replaces two old varieties. Thus, over time, $N_{a,\tau,t}$ with $\tau > 0$ evolves according to the equation:

[1]For example, when $\sigma_a^{var} = 0$ all varieties are essential inputs in the production of the sector's output. Inventions reduce the number of essential inputs and thereby reduce the cost of sectoral production.

$$N_{a,\tau,t} = \begin{cases} (INV_{a,t} + IMP_{a,t})\, M_{a,t-1} & \text{for } \tau = t \\ (1 - IMP_{a,t} - 2\kappa INV_{a,t})\, N_{a,\tau,t-1} & \text{for } \tau < t \end{cases} \tag{3}$$

where $\kappa = 1$ if $\sigma_a^{var} < 1$, and $\kappa = 0$ otherwise. Similarly, $N_{a,0,t}$ evolves as follows: $N_{a,0,t} = (1 - IMP_{a,t} - 2\kappa INV_{a,t})\, N_{a,0,t-1}$ where $N_{a,0,0}$ is normalized to unity.

The output, $Q_{a,t}$, is assembled by a competitive firm. Its cost minimization implies that the demand for the representative variety in cohort τ is given by

$$q_{a,\tau,t}^{top} = \left(\lambda_{a,\tau}^{top}\right)^{\sigma_a^{var}-1} \left(\frac{P_{a,t}}{\hat{p}_{a,\tau,t}}\right)^{\sigma_a^{var}} Q_{a,t} \tag{4}$$

where $P_{a,t}$ is the price of output in sector a and $\hat{p}_{a,\tau,t}$ is the price of the variety invented at time τ.

The unit cost and price of output $Q_{a,t}$ are given by

$$P_{a,t} = \left(\sum_{\tau=0}^{t} N_{a,\tau,t} \left(\frac{\hat{p}_{a,\tau,t}}{\lambda_{a,\tau}^{top}}\right)^{1-\sigma_a^{var}}\right)^{\frac{1}{1-\sigma_a^{var}}} \tag{5}$$

2.2 Price of varieties

Varieties are produced by firms functioning under conditions of monopolistic competition. Firms can purchase a blueprint that allows them to manufacture the variety. Blueprints are excludable; that is, a firm with a blueprint has some monopoly power. In addition, we assume that the blueprint can be imitated costlessly, albeit imperfectly. If the blueprint is imitated, an imitating firm can produce the particular variety at a productivity lower than that allowed by the original blueprint by a factor of $\frac{1}{1+\gamma}$. This implies that the costs of the imitator are $(1+\gamma)$ times the marginal costs of a monopolist who owns the blueprint, and the markup charged by the monopolist cannot be higher than γ.

However, the strategic competition between the monopolist and potential imitators is not the only factor potentially affecting the monopolist's markup. The monopoly power may also depend on (i) the strategic interaction between the owner of the new blueprint (a newcomer) and the owner of the previously most-advanced blueprint (an incumbent) and (ii) on the competition between the monopolist and firms producing other varieties.

To simplify the model and rule out these two possibilities, we make the following assumptions. First, we assume that $\gamma < g$, where g is the smallest improvement of productivity between the two periods. In other words, we assume that γ is sufficiently small to ensure that the owner of the previous version of the blueprint has no incentive to continue producing under the old blueprint - they would rather imitate. Second, we assume that for all sectors a, the elasticity of substitution between varieties, σ_a^{var}, is sufficiently low to ensure that the monopolists have no incentive to set a markup lower than γ. The two assumptions imply that the markup chosen by the firm is equal to γ.

$$\hat{p}_{a,\tau,t} = (1+\gamma)\, p_{a,\tau,t}^{top} \tag{6}$$

where $p_{a,\tau,t}^{top}$ is the cost of manufacturing the representative variety in the cohort τ.

2.3 Manufacturing of varieties

The manufacture of each variety is described by a nested CES production function. For each nest, we label the nested output as a parent (par) and an input as a child (ch). We assume that each nest has only one output, so each nest corresponds to one parent and each parent is associated with a subset of children, $nest\,(par)$. The production function for the variety can then be written as a set of equations:

$$q_{a,\tau,t}^{par} = \left(\sum_{ch \in nest(par)} \left(\alpha_{a,\tau}^{ch}\right)^{\frac{1}{\sigma_a^{par}}} \left(\lambda_{a,\tau}^{ch} q_{a,\tau,t}^{ch}\right)^{\frac{\sigma_a^{par}-1}{\sigma_a^{par}}} \right)^{\frac{\sigma_a^{par}}{\sigma_a^{par}-1}}$$

,

which is defined for every par in the set of parents. The technological parameters $\alpha^{ch}_{a,\tau}$ and $\lambda^{ch}_{a,\tau}$ determine the input intensity ch in the nest par for variety τ. We use $\lambda^{ch}_{a,\tau}$ to control the productivity of the child (see the 'productivity path' section below), and $\alpha^{ch}_{a,\tau}$ to control the intensity of its use (see the 'direction of technological change' section below).

There is a single top nest, top, that belongs to the set of parents. The set of children, ch, consists of intermediate goods and factors of production. When a child is an intermediate good, it must be a parent in one of the lower nests. In our model, each element in set ch is assigned to exactly one element in set par; that is, a child cannot be used as input in more than one nest.

The demand for the child, $q^{ch}_{a,\tau,t}$, as a function of the demand for the parent, $q^{par}_{a,\tau,t}$, can be derived from the cost-minimization problem of a monopolist who manufactures a variety, as follows:

$$q^{ch}_{a,\tau,t} = \alpha^{ch}_{a,\tau} \left(\frac{p^{ch}_{a,\tau,t}}{p^{par}_{a,\tau,t}} \right)^{-\sigma^{par}} \left(\lambda^{ch}_{a,\tau} \right)^{\sigma^{par}-1} q^{par}_{a,\tau,t} \tag{7}$$

,

where p^{ch} denotes the cost the child, ch, and p^{par} denotes the cost of manufacturing the output of the parent, par.

The unit cost of production for the variety invented at τ can also be derived from the cost-minimization problem:

$$p^{par}_{a,\tau,t} = \left(\sum_{ch \in nest(par)} \alpha^{ch}_{a,\tau} \left(\frac{p^{ch}_{a,\tau,t}}{\lambda^{ch}_{a,\tau}} \right)^{1-\sigma^{par}} \right)^{\frac{1}{1-\sigma^{par}}} \tag{8}$$

2.4 Zero profit condition and R&D intensity

We assume free entry on the market of varieties, that is any firm can commission a new blueprint. Consequently, the price of blueprint will be equal to its value, which is equal to the discounted stream of cash flows. The cash flow in each period is the operating surplus; that is, the difference between the firm's revenue and the costs of manufacturing: $\left(\hat{p}_{a,\tau,t} - p^{top}_{a,\tau,t}\right) q^{top}_{a,\tau,t} = \gamma p^{top}_{a,\tau,t} q^{top}_{a,\tau,t}$, where γ is the markup rate (see equation 6). The discount factor is determined by the future interest rate, i_t^{RD}, and the future-improvement, $IMP_{a,t}$, and invention, $INV_{a,t}$, rates. The improvement rate is accounted for in the discount factor because of the business-stealing effect: a successful innovator will receive the cash flow from the blueprint only until another successful innovator produces a better blueprint that improves the technology of the variety; at time t, this happens with probability $IMP_{a,t}$. In addition, when $\sigma_a^{var} < 1$, a new invention can make the old variety obsolete. Since we assumed invention replaces two old varieties with one, the probability that an old variety becomes obsolete at time t is $2IMP_{a,t}$. Thus, the real value at time τ of the blueprint invented at time τ is equal to:

$$\sum_{T=t}^{\infty}\left(\prod_{s=t+1}^{T}\frac{(1-IMP_{a,s}-2\kappa INV_{a,s})}{(1+i_s^{RD})}\right)\gamma p_{a,t,T}^{top}q_{a,t,T}^{top}$$

We assume firms take into account what a blueprint will allow in terms of future production but that they are also myopic, in the sense that they assume that (i) revenue will grow at the exogenous rate, $g_{a,t}^{exp}$ ('expected growth'), (ii) the probability that their blueprint is replaced in the future is constant and given by the expected improvement rate next period, $IMP_{a,t}^{exp} \equiv IMP_{a,t+1}$, (iii) the probability that their blueprint becomes obsolete in the future is constant and given by $2\kappa INV_{a,t}^{exp} \equiv 2\kappa INV_{a,t+1}$, and (iv) the interest rate is exogenous and constant with $i_t^{RD} = i_0^{RD}$. We also assume that blueprints are created one period after they are commissioned by monopolists, which means that the cash flow at the time of commissioning is zero. In that case, the value of the blueprint commissioned at time t is given by:

$$\frac{\gamma p_{a,t+1,t+1}^{top}q_{a,t+1,t+1}^{top}}{1+i_0^{RD}}+\frac{\left(1-IMP_{a,t}^{exp}-2\kappa INV_{a,t}^{exp}\right)}{\left(1+i_0^{RD}\right)^2}\gamma p_{a,t+1,t+2}^{top}q_{a,t+1,t+2}^{top}+\ldots=$$

$$\left(\frac{1+g_{a,t}^{exp}}{1+i_0^{RD}}+\frac{\left(1-IMP_{a,t}^{exp}-2\kappa INV_{a,t}^{exp}\right)\left(1+g_{a,t}^{exp}\right)^2}{\left(1+i_0^{RD}\right)^2}+...\right)\gamma p_{a,t,t}^{top}q_{a,t,t}^{top}=$$

$$=\frac{\gamma p_{a,t,t}^{top}q_{a,t,t}^{top}}{\left(1+i^{RD}\right)/\left(1+g_{a,t}^{exp}\right)-\left(1-IMP_{a,t}^{exp}-2\kappa INV_{a,t}^{exp}\right)} \tag{9}$$

Because of the firms' myopia, expected profits might differ from actual profits. Therefore, we assume the existence of an 'insurance' actor that receives or pays the difference between the two.

New blueprints are generated by technology firms that conduct R&D. The Poisson arrival rate of original blueprints, that is, the expected number of new blueprints in one period, is given by

$$R_{a,t}^{1-\phi^{RDtoes}}\Gamma M_{a,t}$$

where $R_{a,t}$ is the total research intensity within activity a, and Γ represents the productivity of research, which we assume to be constant. The term $M_{a,t}$ captures the intertemporal spillover: the productivity of researchers increases with the number of past inventions. Parameter ϕ^{RDtoes} reflects the size of the stepping-on-toes effect (Jones 1995, Jones and Williams 2000): greater research effort is associated with lower research productivity, because of the higher risk of researchers duplicating ideas. Alternatively, the positive value of ϕ^{RDtoes} could reflect that the possibilities of developing improved or new varieties are heterogenous: some varieties are easier to develop than others. In this case, when there are a large number of researchers, some will work on research to develop a variety that has relatively low chances of success. The Poisson arrival rate per R&D effort would decrease with R&D effort. The value of parameter ϕ^{RDtoes} can, but does not have to be, zero. Implementation can be simplified by using ϕ^{RD}, which we define as

$$\phi^{RD}\equiv 1-\phi^{RDtoes}$$

Both, the spillover effect (captured by the term $M_{a,t}$) and the stepping-on-toes effect (captured by the term $R_{a,t}^{-\phi^{RDtoes}}$) are the externalities: an individual firm that considers investing an additional unit of R&D intensity does not take into account that it will increase future R&D productivity (through an increase in $M_{a,t}$) or that it decreases the productivity of current researchers (through a decrease in $R_{a,t}^{-\phi^{RDtoes}}$). In other words, from the firm's perspective one additional unit of R&D activity increases the Poisson arrival rate by $R_{a,t}^{-\phi^{RDtoes}}\Gamma M_{a,t}$ where $R_{a,t}$ and $M_{a,t}$ are taken as given. In this case the equilibrium level of R&D intensity in sector a is determined by the zero-profit condition for technology firms:

$$p_{RD,t}\Phi_{a,t} = R_{a,t}^{-\left(1-\phi^{RD}\right)}\Gamma\frac{\gamma p_{a,t,t}^{top}q_{a,t,t}^{top}}{\left(1+i^{RD}\right)/\left(1+g_{a,t}^{exp}\right)-\left(1-IMP_{a,t}^{exp}-2\kappa INV_{a,t}^{exp}\right)} \tag{10}$$

where $p_{RD,t}\Phi_{a,t}$ is the unit cost of R&D effort.

Some blueprints will be dedicated to improvements and some to the inventions of new varieties. Firms are indifferent between improvements and inventions because all improved and new varieties are characterized by exactly the same productivity and, therefore, offer the same revenue and profit streams. For the same reason, technology firms will be indifferent between the varieties they can choose to improve. We assume that the ratio of blueprints dedicated to improvements and to inventions is constant:

$$\frac{INV_{a,t}}{IMP_{a,t}} = \rho \tag{11}$$

Parameter ρ determines how much of technological progress can be attributed to horizontal technological change, which, in our setup, is bounded by the exogenous frontier, and how much can be attributed to vertical technological change, which, in our setup, is associated with path-dependency. We discuss the importance of this distinction in Section 4.

As noted above, new blueprints must be commissioned one period earlier. This means that their number at time t depends on the choice of R&D activity at time $t-1$. In combination with equation 4, this implies

$$M_{a,t-1}\left(IMP_{a,t}+INV_{a,t}\right)=R_{a,t-1}^{\phi^{RD}}M_{a,t-1}\Gamma \tag{12}$$

Given equation 11, we can then express the equilibrium improvement rate as:

$$IMP_{a,t}^{exp}=R_{a,t}^{\phi^{RD}}\frac{\Gamma}{1+\rho} \tag{13}$$

where

$$IMP_{a,t}^{exp}=IMP_{a,t+1} \tag{14}$$

2.5 Productivity path

We assume that in each period, the productivity of every variety in every sector is bounded by the state of the global technological platform, which we label as a frontier. The technological platform could be interpreted as a set of GPTs, such as ICT and automatization, and the frontier could be interpreted as the highest potential productivity attainable by a variety, given the current state of GPT.

We assume that the improvement of an existing variety allows firms to raise the variety's productivity to the level of the frontier and remain there until the next improvement. Similarly, when innovation results in the invention of a new variety, the productivity of that variety will be at the level of a frontier. Hence, the productivity of all varieties in the most advanced cohort at time t is:

$$\lambda_{a,t}^{top}=frontier_t \tag{15}$$

We assume that the frontier grows at the exogenous rate g:

$$frontier_t=(1+g)\,frontier_{t-1} \tag{16}$$

We set the value of $frontier_0$ at a level that ensures that, at $t = 1$, average productivity grows at the same rate as the frontier (see, Appendix A2).

The cost of production of sectoral output $P_{a,t}$ also depends on the number of varieties being produced (see equation 5). Inventions always result in that cost dropping. When $\sigma_a^{var} \geq 1$, this is achieved via an increase in the total number of varieties; when $\sigma_a^{var} < 1$, it is achieved via a decrease in the number of varieties (every invention replaces two existing varieties). In Section 4.2 we compare and further discuss improvement- and invention-driven growth.

2.6 Directed search

For projects commisioned at time $t-1$ that generate blueprints at time t, technology firms can adjust the design to the prices observed at time t. Specifically, technology firms choose parameters $\alpha_{a,\tau}^{ch}$ to minimize the expected costs of manufacturing varieties designed at time t:

$$\min_{\alpha_{a,t}^{ch}} \left(\sum_{ch \in nest(par)} \alpha_{a,t}^{ch} \left(\frac{p_{a,t,t}^{ch}}{\lambda_{a,t}^{ch}} \right)^{1-\sigma^{par}} \right)^{\frac{1}{1-\sigma^{par}}}$$

for every nest par, subject to the technology frontier (see, Caselli and Coleman 2007, Growiec 2008):

$$\sum_{ch \in nest(par)} \left(\frac{\alpha_{a,t}^{ch}}{\beta_{a,ch}} \right)^{\omega^{par}} = 1 \tag{17}$$

,

where $\beta_{a,ch}$ is a parameter calibrated to the Social Accounting Matrix. To ensure that the technology firm has an incentive to minimise units of production, we assume that the imitator can also costlessly choose parameters $\alpha_{a,t}^{ch}$.

The first order conditions for this problem determine optimal choices of $\alpha_{a,\tau}^{ch}$ for cohort τ:

$$\alpha^{ch}_{a,\tau} = \left(\frac{p^{ch}_{a,\tau,\tau}/\lambda^{ch}_{a,\tau}}{p^{par}_{a,\tau,\tau}} \right)^{\sigma^{par}-\tilde{\sigma}^{par}} \quad (\beta_{a,ch})^{\frac{1-\tilde{\sigma}^{par}}{1-\sigma^{par}}}$$

,

where $\tilde{\sigma}^{p} = \frac{\omega^{p}\sigma^{p}-1}{\omega^{p}-1}$ is the long-run elasticity of substitution. This also implies that, in the year of invention (i.e., for $t = \tau$),

$$q^{ch}_{a,t,t} = b_{a,ch} \left(\frac{p^{ch}_{a,t,t}}{p^{par}_{a,t,t}} \right)^{-\tilde{\sigma}^{par}} \quad \left(\lambda^{ch}_{a,t}\right)^{\tilde{\sigma}^{par}-1} q^{par}_{a,t,t} \tag{18}$$

and

$$p^{par}_{a,t,t} = \left(\sum_{ch} b_{a,ch} \left(\frac{P^{ch}_{a,t.t}}{\lambda^{ch}_{a,t}} \right)^{1-\tilde{\sigma}^{par}} \right)^{\frac{1}{1-\tilde{\sigma}^{par}}} \tag{19}$$

,

where $b_{a,ch} = (\beta_{a,ch})^{\frac{1-\tilde{\sigma}^{par}}{1-\sigma^{par}}}$. Hence, $\alpha^{c}_{a,t}h$ can be expressed as:

$$\alpha^{ch}_{a,t} \left(\lambda^{ch}_{a,t}\right)^{\sigma^{par}-1} = \left(\frac{p^{ch}_{a,t,t}}{p^{par}_{a,t,t}} \right)^{\sigma^{par}_{a}} \quad \frac{q^{ch}_{a,t,t}}{q^{par}_{a,t,t}} \tag{20}$$

The prices of intermediate products are determined by the optimal choice of $\alpha^{ch}_{a,t}$ in the lower nests (for example, if the intermediate good $ch1$ is produced with inputs $ch2 \in nest\,(ch1)$, then $p^{ch1}_{a,\tau,\tau}$ depends on the choice of $\alpha^{ch2}_{a,t}$).

The prices of factors of production (inputs at the bottom nests) are exogenous from the perspective of a firm. Again, in this formulation, we assume technology firms are myopic, believing that factor prices will remain constant.

3 Implementation in the CGE framework

In this section, we describe the set of equations that are implemented in the numerical CGE model. The equations are derived directly from the solution of the microfounded model presented in Section 2, with some exceptions involving the use of approximations. The approximations are optional, hence there are two versions of the model: one in which we apply the approximations and one in which we do not. We present and discuss these approximations in Section 3.2

The derivations are mostly straightforward and involve a rather tedious algebra, so we refrain from their extensive presentation here; instead, we describe these in Appendix A1.

3.1 Additional definitions

We introduce several additional variables to facilitate interpretation. First, we introduce the variable $Q_{a,t}^{top,new}$, which represents the output of all varieties improved at time t:

$$Q_{a,t}^{top,new} = N_{a,t,t} q_{a,t,t}^{top} \tag{21}$$

.

The variable $\lambda_{a,t}^{top,new}$, represents the productivity of varieties invented or improved at time t:

$$\lambda_{a,t}^{top,new} = \lambda_{a,t}^{top}$$

$P_{a,t}^{top,new}$ represents the unit cost of production (excluding markups) of $Q_{a,t}^{top,new}$:

$$P_{a,t}^{top,new} = p_{a,t,t}^{top} \tag{22}$$

.

For the lower nests, let $Q_{a,t}^{ch,new}$ be the total input ch demanded by the new varieties:

$$Q_{a,t}^{ch,new} = N_{a,t,t} q_{a,t,t}^{ch} \tag{23}$$

.

and let $\lambda_{a,t}^{ch,new}$ be the productivity of input ch for the new varieties

$$\lambda_{a,t}^{ch,new} = \lambda_{a,t}^{ch}$$

.

Similarly, let $P_{a,t}^{ch,new}$ be the unit cost of production of ch (when ch is an intermediate good) or a price of ch (when ch is a factor of production):

$$P_{a,t}^{ch,new} = p_{a,t,t}^{ch}$$

Next, we define a price index for old varieties, that is varieties that were invented or last improved at time $\tau \leq t-1$ as follows:

$$P_{a,t}^{top,old} = \left(\sum_{\tau}^{t-1} \frac{N_{a,\tau,t}}{\left(1 - IMP_{a,t} - 2\kappa INV_{a,t}\right) M_{a,t-1}} \left(p_{a,\tau,t}^{top}\right)^{1-\sigma_a^{top}}\right)^{\frac{1}{1-\sigma_a^{top}}} \tag{24}$$

We define the productivity index:

$$\lambda_{a,t}^{top,old} = \left(\sum_{\tau}^{t-1} \left[\left(\lambda_{a,\tau}^{top}\right)^{\sigma_a^{var}-1} \frac{N_{a,\tau,t}}{\left(1 - IMP_{a,t}\right) M_{a,t-1}}\right]\right)^{\frac{1}{\sigma_a^{var}-1}} \tag{25}$$

and the average demand for output of the old varieties as

$$Q_{a,\tau,t}^{top,old} = \sum_{\tau}^{t-1} N_{a,\tau,t} q_{a,\tau,t}^{top}$$

.

The average demand for inputs, ch, generated by old varieties is defined as:

$$Q_{a,\tau,t}^{ch,old} = \sum_{\tau}^{t-1} N_{a,\tau,t} q_{a,\tau,t}^{ch} \tag{26}$$

We define their productivity index as:

$$\lambda_{a,t}^{ch,old} = \left(\sum_{\tau}^{t-1} \left[\left(\lambda_{a,\tau}^{ch}\right)^{\sigma^{par}-1} \frac{N_{a,\tau,t}}{\left(1 - IMP_{a,t} - 2\kappa INV_{a,t}\right) M_{a,t-1}} \right] \right)^{\frac{1}{\sigma^{par}-1}} \tag{27}$$

The associated price index is expressed as follows:

$$P_{a,t}^{ch,old} = \left(\sum_{\tau}^{t-1} \frac{N_{a,\tau,t}}{\left(1 - IMP_{a,t} - 2\kappa INV_{a,t}\right) M_{a,t-1}} \left(p_{a,\tau,t}^{ch}\right)^{1-\sigma_a^{ch}} \right)^{\frac{1}{1-\sigma_a^{ch}}} \tag{28}$$

.

Further, let

$$\alpha_{a,t}^{ch,new} = b_{a,ch,t} = \left(\beta_{a,ch}\right)^{\frac{1-\tilde{\sigma}^{par}}{1-\sigma^{par}}} \tag{29}$$

$$\lambda_{a,t}^{ch,new} = \lambda_{a,t}^{ch}$$

$$\alpha_{a,t}^{ch,old} \left(\lambda_{a,t}^{ch,old}\right)^{\sigma^{par}-1} = \sum_{\tau}^{t-1} \frac{N_{a,\tau,t}}{\left(1 - IMP_{a,t} - 2\kappa INV_{a,t}\right) M_{a,t-1}} \alpha_{a,\tau}^{ch} \left(\lambda_{a,\tau}^{ch}\right)^{\sigma^{par}-1} \tag{30}$$

.

Finally, let

$$\sigma^{par,old} = \sigma^{par}$$

and

$$\sigma^{par,new} = \tilde{\sigma}^{par} = \frac{\omega^{par}\sigma^{par} - 1}{\omega^{par} - 1}$$

.

3.2 The short version of the model

In this section, we discuss the possibility of introducing several approximations related to the relative demand and manufacturing costs of old varieties, that is, varieties that were not improved or invented in the current period. The introduction of approximations is optional: in our numerical model the user can choose whether to include them or not. We label the model including approximations the short version and the one without these as the long version. The introduction of approximations has important advantages and equally important disadvantages, which we discuss towards the end of this subsection.

In both the long and the short version we assume that the firms' choices regarding varieties just invented (i.e., varieties represented by $\tau = t$) are consistent with firms' cost minimization and can be described by equations 4, 6, 7 and 8. In the long version we make the same assumption for all varieties. In the short version for all old varieties (i.e., those in cohorts $\tau \leq t-1$), we compute manufacturing costs by replacing the variety-specific cost of manufacturing in equation 8, $p^{ch}_{a,\tau,t}$, with the corresponding cost index, $P^{ch,old}_{a,t}$, which is constant across old varieties. In other words, we replace equation 8 with equation

$$p^{par}_{a,\tau,t} = \left(\sum_{ch \in nest(par)} \alpha^{ch}_{a,\tau} \left(\frac{P^{ch,old}_{a,t}}{\lambda^{ch,old}_{a,\tau}} \right)^{1-\sigma^{par}} \right)^{\frac{1}{1-\sigma^{par}}} \tag{31}$$

.

This, in combination with equation 28, results in a simple rule for computing the price index of manufacturing nests for old varieties:

$$\left(P^{par,old}_{a,t}\right)^{1-\sigma^{par}_{a}} = \sum_{ch \in nest(par)} \left(P^{ch,old}_{a,t}\right)^{1-\sigma^{par}} \alpha^{ch,old}_{a,t} \left(\lambda^{ch,old}_{a,t}\right)^{\sigma^{par}-1} \tag{32}$$

In addition, we compute the price of the final sector output by replacing

$p^{top}_{a,\tau,t}$ for $\tau \leq t-1$ with $P^{top,old}_{a,t}$:

$$P_{a,t} = \frac{N_{a,t,t} p^{top}_{a,t,t} q^{top}_{a,t,t} + \sum_{\tau}^{t-1} N_{a,\tau,t} P^{top,old}_{a,t} q^{top}_{a,\tau,t}}{Q_{a,t}} \tag{33}$$

Similarly, we compute demand by replacing the variety-specific cost of manufacturing, $p^{ch}_{a,\tau,t}$ and $p^{par}_{a,\tau,t}$, and outputs, $q^{par}_{a,\tau,t}$, in equation 7 with their corresponding indices, $P^{ch,old}_{a,t}$, $P^{par,old}_{a,t}$ and $Q^{par,old}_{a,t}$:

$$q^{ch}_{a,\tau,t} = \alpha^{ch}_{a,\tau} \left(\frac{P^{ch,old}_{a,t}}{P^{par,old}_{a,t}} \right)^{-\sigma^{par}} \left(\lambda^{ch}_{a,\tau} \right)^{\sigma^{par}-1} \frac{Q^{par,old}_{a,t}}{\left(1 - IMP_{a,t} - 2\kappa INV_{a,t}\right) M_{a,t-1}} \tag{34}$$

For the top nest, we compute the total demand for varieties $\tau \leq t-1$ (originally, equation 4) using the following:

$$q^{top}_{a,\tau,t} = \left(\lambda^{top,old}_{a,t} \right)^{\sigma^{var}_a - 1} \left(\frac{P_{a,t}}{(1+\gamma) P^{top,old}_{a,t}} \right)^{\sigma^{var}_a} Q_{a,t} \tag{35}$$

Discussion of approximations

The approximation is exact when all old varieties are identical. If we consider a scenario where, initially, all varieties are identical, but after an economic shock, they begin to vary, the predictions of the short version of the model will ignore several effects. The most important of these are: (i) changes in the shares of different old varieties after changes in factor prices and (ii) correlation between the intensity of use of one factor in nest *par* and the demand for this nest. We elaborate on these two effects below and, in Appendix A3, set out further detail of all effects that are ignored by making the approximation.

By introducing the approximations, we ignore that after a change in factor prices, the resulting change in manufacturing costs, price and demand will differ across old varieties. Consider, for example, a carbon tax introduced at time T_1. All pre-shock varieties, that is, all varieties invented at time $\tau < T_1$, will be characterized by high energy intensity (high α^{energy}) and post-shock varieties (with $\tau \geq T_1$) will be characterized by lower energy intensity (lower α^{energy}).

Now suppose that at time $T_2 > T_1$, there is another increase in energy prices. in the long version of the model, varieties with $\tau < T_1$ will immediately become less competitive, and their production will decrease relative to the production of varieties with $\tau \geq T_1$. In the short version we ignore this adjustment. By doing so we overestimate the production by relatively expensive varieties and, as a result, overestimate the increase in price $P_{a,t}^{top,old}$ after the second shock.

Notice that this is only a problem in the medium run. In the short run, almost all varieties are the same (pre-shock), and their behaviour is very close to the average. In the long run, all varieties are again the same (all are post-shock) and their behaviour is close to average.

Notice also that approximation does not mean that, following a price shock, firms make no adjustments in demand for particular varieties. First, even when we make the approximation, we distinguish between new and old varieties. Because new varieties are, in general, more productive and their intensity parameters ($\alpha_{a,t}^{ch}$'s) are better adjusted to current market prices, they will have lower manufacturing costs and higher demand than old varieties (see equations 21 and 4). Second, the demand for old varieties will also react to changes in factor prices; these will be transmitted through the change in the average price index $P_{a,t}^{top,old}$. Approximation does imply, however, that changes in all old varieties will be driven by the same price index $P_{a,t}^{top,old}$ (see equation 35).

The second important effect that we ignore by approximating is related to the computation of the average price index for old varieties in the nested production function. We compute the index by multiplying average intensity of use of input ch, $\alpha_{a,t}^{ch,old}\left(\lambda_{a,t}^{ch,old}\right)^{\sigma^{par}-1}$, by average price of input ch, $P_{a,t}^{ch,old}$ (see equation 32)[2]. This is correct only if the correlation between $\alpha_{a,\tau}^{ch}$ and $p_{a,\tau,t}^{ch}$ is zero. This assumption fails if varieties that are characterised by lower intensity use of input ch, also have a lower cost of manufacturing for that input. To compute

[2]Additionally, we ignore the role of the exponents $\left(1-\sigma_a^{par}\right)$. We discuss this in Appendix A3

the true average price we would need to weight each lower price $p^{ch}_{a,\tau,t}$ with a lower intensity $\alpha^{ch}_{a,\tau}$ before summing over all old varieties. When we ignore this weighting, we are miscomputing the average price of old varieties (in the case of a positive correlation between $p^{ch}_{a,\tau,t}$ and $\alpha^{ch}_{a,\tau}$, we are under-estimating this price).

The advantage of the short version is a significantly improved tractability with respect to the long version. All key productivity and intensity parameters can be expressed simply as a linear dynamic equation that resembles the AR1 process. The short version also reduces the number of effects that drive the results, which allows for a better exposition of effects driven purely by the choices of technology firms.

In addition, the short version allows us to significantly reduce the number of equations and ease the computational burden of the model. Note that, in the long version, all equations indexed with τ require a very large number of equations. For instance, assuming that the model is solved for the period 2020 to 2040, the simulation for 2040 requires finding a solution for variables indexed with τ for 20 periods (since τ can take any value from 2020 to 2039), times the number of sectors, times the number of nests.

In the next subsection, we describe the set of implementable equations for the short version of the model. The implementation of the long version is described in Appendix A1.

3.3 Implemented equations

We implement the module in seven blocks of equations, as described below. The detailed derivations are provided in Appendix A1. Here we limit our description to the outcome of these manipulations.

3.3.1 Demand and manufacturing costs of varieties

Equation 36 determines the manufacturing costs for each nest.

$$P_{a,t}^{par,v} = \left(\sum_{ch} \alpha_{a,t}^{ch,v} \left(P_{a,t}^{ch,new} / \lambda_{a,t}^{ch,new} \right)^{1-\sigma_a^{par,v}} \right)^{\frac{1}{1-\sigma_a^{par,v}}} \tag{36}$$

Equations 37 and 38 describe the demand for new and old varieties. The lower costs of production for new varieties will increase their contribution to the final output production of the sector. The demand also depends on the number of new and old varieties. An increase in innovation rate will again result in an increase in the contribution of new varieties. Finally, the contribution of new varieties is higher when they are more productive, that is, when they have a larger $\left(\lambda_{a,t}^{top,new}\right)^{\sigma_a^{xpvt}-1}$.

$$\begin{aligned} Q_{a,t}^{top,new} = \\ \left(IMP_{a,t} + INV_{a,t} \right) M_{a,t-1} \left(\lambda_{a,t}^{top,new} \right)^{\sigma_a^{var}-1} * \\ \left(\frac{P_{a,t}}{(1+\gamma) P_{a,t}^{top,new}} \right)^{\sigma_a^{var}} Q_{a,t} \end{aligned} \tag{37}$$

and

$$\begin{aligned} Q_{a,t}^{top,old} = \\ \left(1 - IMP_{a,t} - 2\kappa INV_{a,t} \right) M_{a,t-1} \left(\lambda_{a,t}^{top,old} \right)^{\sigma_a^{var}-1} * \\ \left(\frac{P_{a,t}}{(1+\gamma) P_{a,t}^{top,old}} \right)^{\sigma_a^{var}} Q_{a,t} \end{aligned} \tag{38}$$

The demand for inputs can be expressed using the standard rule for nested CES production functions:

$$Q_{a,t}^{ch,v} = \alpha_{a,t}^{ch,v} \left(\lambda_{a,t}^{ch,old} \right)^{\sigma^{par,v}-1} \left(\frac{P_{a,t}^{ch,v}}{P_{a,t}^{par,v}} \right)^{-\sigma^{par,v}} Q_{a,t}^{par,v} \tag{39}$$

.

3.3.2 Cost of sectoral output

The manufacturing costs of sector a can be calculated from the total cost of new and old varieties scaled by the markup:

$$P_{a,t}Q_{a,t} = (1+\gamma)\left(Q_{a,t}^{top,old}P_{a,t}^{top,old} + Q_{a,t}^{top,new}P_{a,t}^{top,new}\right) \tag{40}$$

We can then use equations 37, 38 and 40 to express the price as a function of M and productivity parameters, $\lambda_{a,t}^{top,new}$ and $\lambda_{a,t}^{top,old}$:

$$P_{a,t} = (1+\gamma)\, M_{a,t-1}^{\frac{1}{1-\sigma_a^{var}}} \left((1-(1-2\kappa\rho)\, IMP_{a,t}) \left(\frac{P_{a,t}^{top,old}}{\lambda_{a,t}^{top,old}}\right)^{1-\sigma_a^{var}} \right.$$

$$\left. + (1+\rho)\, IMP_{a,t} \left(\frac{P_{a,t}^{top,new}}{\lambda_{a,t}^{top,new}}\right)^{1-\sigma_a^{var}} \right)^{\frac{1}{1-\sigma_a^{var}}}$$

It is clear that a decline in costs at the sectoral level will be driven by (i) growth in productivity, $\lambda_{a,t}^{top,new}$ and $\lambda_{a,t}^{top,old}$ and (ii) growth in the total number of varieties, $M_{a,t-1}$, when $\sigma_a^{var} > 1$ or a decline in $M_{a,t-1}$, when $\sigma_a^{var} < 1$.

3.3.3 Productivity path

Given the settings specified with equations 15 and 16, all new varieties are simply assigned the same productivity as the frontier, which grows at an exogenous rate of g. Thus, the path of $\lambda_{a,t}^{top,new}$ can be summarised through equations 41 and 42:

$$\lambda_{a,t}^{top,new} = frontier_t \tag{41}$$

$$frontier_t = (1+g)\, frontier_{t-1} \tag{42}$$

Meanwhile, under the assumptions described in Section 3.2, the equation describing the path of $\lambda_{a,t}^{top,old}$, can be reduced to a simple relation that resembles the AR1 process:

$$
\begin{aligned}
(1+(1-2\kappa)\,INV_{a,t-1})\left(\lambda_{a,t}^{top,old}\right)^{\sigma^{par}-1} &= \\
(IMP_{a,t-1}+INV_{a,t-1})\left(\lambda_{a,t-1}^{top,new}\right)^{\sigma^{par}-1} & \\
+(1-IMP_{a,t-1}-2\kappa INV_{a,t-1})\left(\lambda_{a,t-1}^{top,old}\right)^{\sigma^{par}-1} &
\end{aligned} \tag{43}
$$

Evaluating this equation in reverse, the productivity measure, $\lambda_{a,t}^{top,old}$, can also be expressed as the weighted average of the productivity of individual variety cohorts weighted by cohort size.

Finally, using equations 3, we can express the growth (decline) of the total number of varieties, $M_{a,t}$, as a function of the invention rate:

$$
\frac{M_{a,t}-M_{a,t-1}}{M_{a,t-1}} = \begin{cases} INV_{a,t} & \text{if } \sigma_a^{var} \geq 1 \\ -INV_{a,t} & \text{if } \sigma_a^{var} < 1 \end{cases} \tag{44}
$$

.

3.3.4 Direction

As explained in Section 2.6, firms can choose the factor intensity of production for new varieties. Note that there are two equations that can be used to set firms' demand for inputs for new varieties: 7 and 18. In the model, we use the latter, and must thus set the factor intensity coefficient to $b_{a,ch,t}$:

$$
\alpha_{a,ch,t}^{new} = b_{a,ch,t} \tag{45}
$$

.

Given firm choices regarding inputs, $\frac{Q_{a,t}^{ch,new}}{Q_{a,t}^{par,new}}$, we can then recover the choices of $\alpha_{a,t}^{ch}$ (see equation 20)

$$
\alpha_{a,t}^{ch} = \left(\lambda_{a,t}^{ch}\right)^{1-\sigma^{par}} \left(\frac{P_{a,t}^{ch,new}}{P_{a,t}^{par,new}}\right)^{\sigma^{par}} \frac{Q_{a,t}^{ch,new}}{Q_{a,t}^{par,new}} \tag{46}
$$

.

Finally, we can use this information to update the factor intensity of old varieties:

$$
\begin{aligned}
(1+(1-2\kappa)\,INV_{a,t-1})\,\alpha^{ch,old}_{a,t} = \\
(IMP_{a,t-1}+INV_{a,t-1})\,\alpha^{ch}_{a,t-1} \\
+(1-IMP_{a,t-1}-2\kappa INV_{a,t-1})\,\alpha^{ch,old}_{a,t-1}
\end{aligned}
\tag{47}
$$

3.3.5 Speed of innovation

In this model, the speed of innovation is determined endogenously. The number of improvements and inventions is determined by the research production function (equation 13):

$$
IMP^{exp}_{a,t} = R^{\phi^{RD}}_{a,t}\left(\frac{IMP^{exp}_{a,0}}{R^{\phi^{RD}}_{a,0}}\right) \tag{48}
$$

.

where $IMP^{exp}_{a,0}$ and $R^{\phi^{RD}}_{a,0}$ are the parameters of the model.

The relation between $IMP^{exp}_{a,t}$ and $IMP_{a,t}$, and between $IMP_{a,t}$ and $INV_{a,t}$ is specified by equations 11 and 14:

$$
INV_{a,t} = \rho IMP_{a,t} \tag{49}
$$

$$
IMP_{a,t} = IMP^{exp}_{a,t-1} \tag{50}
$$

.

Following the logic in Section 2.4, firms invest in costly R&D because they expect a stream of profit that translates into a positive value for each blueprint. However, the value, in fact, decreases with an increase in research intensity for two reasons: reduction of the R&D success rate at the sectoral level (captured by the term $R^{1-\phi^{RD}}_{a,t}$ in equation 10) and the business-stealing effect (captured by the term $\left(1-IMP^{exp}_{a,t}-2\kappa INV^{exp}_{a,t}\right)$). The equilibrium level of research intensity is determined by the zero-profit condition (derived from equation 10)

$$R_{a,t}^{1-\phi^{RD}} p_{RD,t} \Phi_{a,t} = \frac{\Gamma \gamma p_{a,t,t}^{top} q_{a,t,t}^{top}}{(1+i^{RD}) / \left(1+g_{a,t}^{exp}\right) - \left(1 - IMP_{a,t}^{exp} - 2\kappa INV_{a,t}^{exp}\right)} \quad (51)$$

3.3.6 R&D finance equations

Equation 10) implies that the expected profit from all R&D projects in all sectors is given by

$$\sum_a R_{a,t}^{\phi^{RD}} \Gamma \frac{\gamma p_{a,t,t}^{top} q_{a,t,t}^{top}}{(1+i^{RD}) / \left(1+g_{a,t}^{exp}\right) - \left(1 - IMP_{a,t}^{exp} - 2\kappa INV_{a,t}^{exp}\right)}$$

As noted above, we assume there is no perfect foresight. Firms make their R&D decisions based on information on current profits in their sector, the expected growth rate (which is an exogenous parameter) and the obsolescence rate.

Actual profit from all past innovations is given by the following:

$$\sum_a \gamma \left(XPV_{a,t}^{old} PXV_{a,t}^{old} + XPV_{a,t}^{new} PXV_{a,t}^{new} \right)$$

.

Because firms are myopic, their expected profits could be different from the actual profits. The difference is covered by the sector of 'enterprises', which here act as an insuring actor (here we assume that no actor in the economy has perfect foresight, and the value of the blueprints reflects the best available knowledge regarding expected profits). The sector of enterprises in the model work as an intermediary that collect all capital rents from sectoral firms and transfer these to households. We assume that the difference between the actual and expected R&D profit (which could be positive or negative) is added to these capital transfers.

3.3.7 R&D demand for commodities

We assume that R&D activity requires the use of investment goods, which are purchased by technology firms: $P_{RD,t} = p_t^{investment}$, where $p_t^{investment}$ is the price of the investment good and

$$R_t^{INV} \equiv \sum_a p_t^{investment} \Phi_{a,t} R_{a,t}$$

The demand for investment goods dedicated to R&D projects is added to the demand for investment goods dedicated to capital accumulation, which is determined by the amount of available savings. The price of investment good, $p_t^{investment}$, is determined by the cost of manufacturing faced by a producer of investment goods, as in the standard CGE framework.

4 Discussion

4.1 Speed and direction of technological change

As outlined in the introduction, the primary purpose of our study is to improve the dynamic predictions of a CGE model by endogenizing technological parameters. We wish to describe an economic system where changes in market conditions that favour some technologies will result in their accelerated development and diffusion and the gradual restructuring of the economy. The model we propose allows the achievement of these goals in two ways: by allowing prices to influence (i) productivity growth and (ii) factor intensities.

The first of these is relatively straightforward. Any change in market conditions that increases the demand in a sector will result in an increase in its revenue. In our model, this will be associated with an expectation of higher profits and higher value of potential innovations. This will, in turn, lead to a larger investment in R&D activity in this sector and a gradual increase in the portion of varieties in the vicinity of the frontier. This, in turn, will lead to an acceleration of average productivity in the sector, a reduction in unit costs and a further increase in demand.

Perhaps the most obvious example of such a development is the impact of subsidies for RES on their diffusion: initially, the subsidy will lead to a single jump in the use of RES. In the long run, however, the initial hike in demand will induce efforts to develop more efficient value chains and adapt technologies to local physical conditions and institutional barriers. As a consequence, the unit cost of RES will gradually decrease, amplifying the initial impact of subsidies.

However, this feature of the model can only be exploited when the sectors included are sufficiently disaggregated. The endogenous development of a single technology can only be captured if this technology is explicitly named in the model as one of the sectors of the economy. RES technology, which we use in the example above, will typically satisfy this condition; most CGE models include the key electricity generation technologies as separate sectors. However, finding other examples is difficult. Most CGE models do not distinguish between alternative transport technologies or alternative production methods in heavy industry. At most, the models can provide information on the changes in factor intensities in these large sectors, which implicitly captures a change in their technology mix.

The second feature of our proposed model is that we allow technological change to be endogenous, even when alternative technologies are not explicitly named. After a change in prices of factors of production, firms will search for a way to deliver their output using less of the factor that has become relatively more expensive.

An example that illustrates this would be an evolution of energy demand by the transport sector after an increase in the price of oil. We expect that, as time passes, more and more firms will consider how to deliver their services with less reliance on oil and more on other inputs, such as electricity or capital. Implicitly, this can be interpreted as a gradual electrification of transport induced by a change in the price of fuels.

Firms' choices are constrained by the possibilities of the global technological frontier (see equation 17) and involve trade-offs: the choice of a lower intensity,

$\alpha^{ch}_{a,t}$, for some inputs, requires an increase in intensity for other inputs. In the context of an increase in the price of one input, the trade-off is that, for new varieties, the input will be used more efficiently, and other inputs will be used less efficiently, compared to the situation absent the price shock. Importantly, this does not necessarily mean that, in new varieties, other inputs will be used less efficiently than in old varieties.

To understand this, note that the efficiency in using other inputs is determined not only by $\alpha^{ch}_{a,t}$, which may decrease compared to $\alpha^{ch}_{a,t-1}$, but also by the productivity parameter, $\lambda^{top}_{a,t}$, which increases over time. As a result, unless the price change is very large, for all inputs, the total efficiency of use (measured, for instance, by $\frac{\lambda^{top}_{a,\tau} q^{top}_{a,\tau.t}}{q^{ch}_{a,\tau.t}}$), will be higher in new varieties than in old varieties. In other words, the efficiency of all inputs will benefit from technological progress, although the rate of improvement will differ between inputs.

Note also that at the sectoral level, the change in the demand for factors will be gradual; in our setup firms can adjust the intensities for new varieties only. If the price increase for input ch takes place at time T, only varieties invented or improved after that time will be characterized by the low use intensity in respect of that input; the pre-shock varieties will be characterized by high intensity of use. However, the share of pre-shock varieties will decrease over time as a result of improvements and inventions. Hence, the intensity of use of the input at the sectoral level will gradually decrease. The speed of the decay of pre-shock varieties depends on the arrival rate of the new blueprints.

Finally, note that the speed and direction of technological change interact. If a shock increases demand in the transport sector, that sector's revenue will increase, dragging up the innovation rate. A higher innovation rate will then increase the fraction of varieties with adjusted intensities. In this case, at the sectoral level, the adjustment in the demand for inputs will be faster than when the innovation rate is constant. Similarly, the adjustment of inputs will impact the total costs of production of the sector, its output, revenue and, through this channel, the innovation rate.

4.2 Vertical (bounded) and horizontal (unbounded) technological change

Another feature of the model is the existence of a frontier that bounds the productivity of each variety at every given moment. The frontier in the model is exogenous. It can be interpreted as the maximum productivity of varieties set by the state of GPT (such as ICT or automatization), which is independent of market conditions or policies. In our setup, every successful attempt to improve a variety brings a jump in its productivity $\lambda_{a,t}^{top}$ to the level of the frontier. As a result, all improved varieties exhibit the same productivity immediately after the improvement. In this respect, the model departs from the standard quality-ladder setup, where an innovation increases productivity by a fixed factor relative to the productivity of the variety preceding the innovation.

The introduction of a frontier is motivated by the local geographical scope of most CGE analyses. Typical CGE simulations involve policies or shocks that impact only a single country or a limited group of countries. The situations in which such policies could lead to, or prevent, major technological breakthroughs and have a significant impact on global GPT progress, are rare.

Policies or a change in local market conditions could influence secondary innovations that allow local firms to catch up with the global technological frontier. The role of secondary innovations corresponds to the role of improvements in our model. In the real world, these could take the form of adopting new ideas invented by the leading technology countries, learning and exploitation of new possibilities offered by GPT progress, or adaptation of new solutions to local conditions, for example, local consumer preferences or geographical constraints.

To understand the implications of the frontier assumption, we consider the evolution of average productivity across varieties: $\overline{\lambda}_{a,t}^{top} = \sum_{\tau=0}^{t} \frac{N_{a,\tau,t}}{M_{a,t}} \lambda_{a,\tau}^{top}$. To simplify our argument, suppose that the number of varieties is constant (i.e., there are no inventions, $\rho = 0$) and normalised to unity. In this case, given equation 3, average productivity would evolve as follows:

$$\overline{\lambda}_{a,t}^{top} = IMP_{a,t} frontier_{a,t} + (1 - IMP_{a,t}) \overline{\lambda}_{a,t-1}^{top} \tag{52}$$

.

The average productivity across varieties at every point in time will be a weighted sum of (i) the term that reflects the current state of the frontier and (ii) the average productivity from the previous period. The weights are determined by the sectoral innovation rate: a higher innovation rate results in more weight on the frontier, thereby increasing average productivity. Any shock that permanently increases the innovation rate will result in productivity growth that is faster than the growth of the frontier and a gradual reduction of the distance to the frontier.

In the long run, a permanent shock to the improvement rate will result in a permanently lower distance to the frontier. However, the growth rate will again equal the growth at the frontier. In fact, if improvement remains constant after the shock at the level IMP, the distance between the average productivity of the sector and the state of the frontier, $\frac{\overline{\lambda}_{a,t}^{top}}{frontier_{a,t}}$, will converge to a steady state given by $IMP\left(\frac{1+g}{g+IMP}\right)$. The setup prevents a situation where a policy leads to a permanent increase in the growth of a sector, such that there is explosive behaviour.

Because in some applications the framework with bounded technological change might be inappropriate, we allow for another type of technological change, which is unbounded. We do so by introducing the possibility that innovations result in 'inventions', that is, result in completely new varieties. In our model, invention is an outcome of R&D effort that is distinct from but perfectly correlated with improvement. The relation between the number of improvements and the number of inventions is governed by the parameter ρ. One possible interpretation of our setup is that when technology firms engage in R&D activity, there is some probability (given by $R_{a,t}^{-\phi^{RDtoes}} \Gamma M_{a,t}$) that their effort will lead to an improvement of existing varieties and some fixed probability (given by

$R_{a,t}^{-\phi^{RDtoes}}\Gamma M_{a,t})$ that it will result in a completely new variety.

Growth driven by inventions has very different properties than growth driven by improvements. Our approach to modelling invention is very close to the one in Romer (1992). We assume there are no bounds to the total number of inventions. Moreover, we assume intertemporal spillover effects in equation 12: the higher number of inventions in the past results in the higher productivity of researchers. In contrast to the case of improvements discussed above, the development of inventions will result in path dependency: a single temporary shock to the invention rate will result in permanent changes in sectoral costs.

The path dependency under alternative parameter values is illustrated in Figure 1, which shows the path of sectoral efficiency after a shock to the blueprint arrival rate. Here, we measure the efficiency of a sector as the inverse of the cost of manufacturing the sectoral output ($\frac{1}{P_{a,t}}$).

In the left panels, we illustrate the impacts under the assumption that the inventions are switched off ($\rho = 0$) and, therefore, the growth of sectoral efficiency is driven solely by improvements, that is, technological change bounded by the frontier.

In this case, the permanent shock to the blueprint arrival rate (the top panel) does not affect the long-run growth, which is always equal to the growth at the frontier. When the shock is temporary (the bottom panel), both the growth rate and the distance to the frontier return to their initial values.

In the right panels, we consider the full model with efficiency growth driven by both inventions and improvements ($\rho > 0$). In this case, the permanent shock to the innovation rate increases the long-run growth of sectoral efficiency. The temporary shock increases the growth only in the short run; in the long run, the growth returns to the initial level. However, the distance to the frontier also remains smaller in the long-run.

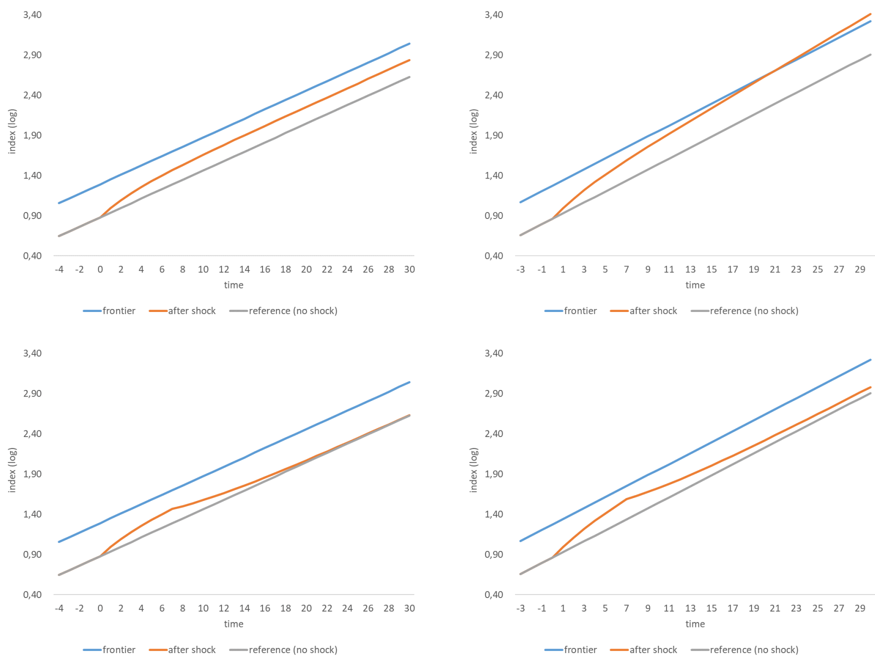

Figure 1: The impact of a change in the rate of arrival of blueprints on the path of efficiency of a sector. The top panels show the impact of a permanent increase in the arrival rate, and the bottom panels show a temporary increase (of 7 periods). The left panels show the impact when inventions are switched off ($\rho = 0$), and only improvements (bounded technological change) are activated. The right panels show the impact when both inventions and improvements are activated. The efficiency of a sector is measured as the inverse of the cost of manufacturing the sectoral output ($\frac{1}{P_{a,t}}$). In each graph, the orange line shows the path of efficiency affected by the shock. The blue line is the hypothetical path if all varieties are at the frontier and the number of varieties grow at the initial rate. The grey line is the efficiency path if there are no shocks to the arrival of blueprints.

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Appendix A1: Derivation of implementable equations

Additional definitions

In order to facilitate the exposition and implementation of the long version of the model, we introduce some additional definitions. First, we define variable $\hat{Q}^{old,top}_{a,t}$ as:

$$\hat{Q}^{old,top}_{a,t} = \frac{\sum_{\tau}^{t-1} p^{top}_{a,\tau,t} q^{top}_{a,\tau,t}}{P^{old,top}_{a,t}} \tag{53}$$

.

In addition, we define a variable, $\hat{q}^{top}_{a,\tau,t}$, which represents the output of varieties invented at time τ, relative to the value of $\hat{Q}^{old,top}_{a,t}$:

$$\hat{q}^{top}_{a,\tau,t} = N_{a,\tau,t} q^{top}_{a,\tau,t} / \hat{Q}^{old,top}_{a,t} \tag{54}$$

.

Further, $\hat{q}^{ch}_{a,\tau,t}$, represents the demand for the intermediate good or factor of production, ch, of varieties invented at time τ, relative to the demand generated by all varieties invented before t, $Q^{old,ch}_{a,t}$

$$\hat{q}^{ch}_{a,\tau,t} = N_{a,\tau,t} q^{ch}_{a,\tau,t} / Q^{old,ch}_{a,t} \tag{55}$$

.

Demand and manufacturing costs of variety outputs and inputs

Manufacturing cost

The long version For the new varieties, the relation can be established using equation 19:

$$P_{a,t}^{par,new} = \left(\sum_{ch} b_{a,ch,t} \left(P_{a,t}^{ch,new} / \lambda_{a,t}^{ch,new} \right)^{1-\tilde{\sigma}^{par}} \right)^{\frac{1}{1-\tilde{\sigma}^{par}}} \tag{56}$$

.

We can then use equation 29 to obtain the same expression as in 36.

For the old varieties, $P_{a,t}^{top,old}$ is defined in equation 24:

$$\frac{P_{a,t}^{top,old}}{\lambda_{a,t}^{top,old}} = \left(\sum_{\tau}^{t-1} \frac{N_{a,\tau,t}}{(1-x_{a,t}) M_{a,t-1}} \left(p_{a,\tau,t}^{top} / \lambda_{a,\tau,t}^{top} \right)^{1-\sigma_a^{xpvt}} \right)^{\frac{1}{1-\sigma_a^{xpvt}}} \tag{57}$$

.

In the long version, $p_{a,\tau,t}^{par}$ (including $p_{a,\tau,t}^{top}$), are determined using equation 8:

$$p_{a,\tau,t}^{par} = \left(\sum_{ch} \alpha_{a,ch,\tau} \left(\lambda_{a,\tau,t}^{ch} \right)^{\sigma^{par}-1} \left(p_{a,\tau,t}^{ch} \right)^{1-\sigma^{par}} \right)^{\frac{1}{1-\sigma^{par}}} \tag{58}$$

.

The short version In the short version, the derivation of $P_{a,t}^{par,new}$ follows exactly the same derivations as in the long version. That is, $P_{a,t}^{par,old}$ (including $P_{a,t}^{par,old}$), can be derived using equations 24 and31

$$P_{a,t}^{par,old} = \left(\sum_{ch} \alpha_{a,ch,\tau}^{old} \left(\frac{P_{a,t}^{ch,old}}{\lambda_{a,t}^{ch,old}} \right)^{1-\sigma^{par}} \right)^{\frac{1}{1-\sigma_a^{par}}}$$

As a result, we obtain equation 36 (restated below for convenience):

$$P_{a,t}^{par,v} = \left(\sum_{ch} \alpha_{a,ch,t}^{v} \left(P_{a,t}^{ch,new}/\lambda_{a,t}^{ch,new}\right)^{1-\sigma^{par,v}}\right)^{\frac{1}{1-\sigma^{par,v}}} \tag{59}$$

.

Demand for outputs

The long version $Q_{a,t}^{top,new}$ can be expressed using equations 4, 22, 6, 21 and 3. We obtain equation 37 as follows:

$$Q_{a,t}^{top,new} = (1+\rho)\, IMP_{a,t} M_{a,t-1} \left(\lambda_{a,t}^{top,new}\right)^{\sigma_a^{var}-1} \left(\frac{P_{a,t}}{(1+\gamma)\, P_{a,t}^{top,new}}\right)^{\sigma_a^{var}} Q_{a,t} \tag{60}$$

.

In order to express $\hat{Q}_{a,t}^{top,old}$, we first use equations 53 and 4, 6 to obtain:

$$P_{a,t}^{top,old} \hat{Q}_{a,t}^{top,old} = (1-IMP_{a,t})\, M_{a,t-1} \left(\frac{P_{a,t}}{1+\gamma}\right)^{\sigma_a^{var}} Q_{a,t} \sum_{\tau}^{t-1} \frac{N_{a,\tau,t}}{(1-x_{a,t})\, M_{a,t-1}} \left(p_{a,\tau,t}^{top}/\lambda_{a,\tau,t}^{top}\right)^{1-\sigma_a^{var}}$$

.

Then, we use equation 57 to obtain

$$\hat{Q}_{a,t}^{top,old} = (1-IMP_{a,t})\, M_{a,t-1} \left(\lambda_{a,t}^{top,old}\right)^{\sigma_a^{var}-1} \left(\frac{P_{a,t}}{(1+\gamma)\, P_{a,t}^{top,old}}\right)^{\sigma_a^{var}} Q_{a,t} \tag{61}$$

.

The Short version In the short version, the derivation of $Q_{a,t}^{top,new}$ follows exactly the same derivations as in the long version. Hence equation 60 is also implemented.

$Q_{a,t}^{top,old}$ can be derived using equations 26 and 35:

$$Q_{a,\tau,t}^{top,old} = (1 - IMP_{a,t})\, M_{a,t-1} \left(\frac{P_{a,t}}{(1+\gamma)\, P_{a,t}^{top,old}} \right)^{\sigma_a^{var}} \left(\lambda_{a,t}^{top,old} \right)^{\sigma_a^{var}-1} Q_{a,t}$$

.

Demand for intermediate goods

The long version

In the case of new varieties, the demand for intermediate goods for varieties is determined using equations 18, 23, 22 and 29

$$Q_{a,t}^{ch,new} = \alpha_{a,t}^{ch,new} \left(\lambda_{a,t}^{ch,new} \right)^{\tilde{\sigma}^{par}-1} \left(\frac{P_{a,t}^{ch,new}}{P_{a,t}^{par,new}} \right)^{-\tilde{\sigma}^{par}} Q_{a,t}^{par,new} \tag{62}$$

.

In the case of old varieties, for all $par \neq top$, the demand is determined using equations 55 and 26:

$$Q_{a,t}^{ch,old} = \alpha_{a,ch,t}^{ch,old} \left(\lambda_{a,t}^{ch,old} \right)^{\sigma^{par}-1} \left(\frac{P_{a,t}^{ch,old}}{P_{a,t}^{par,old}} \right)^{-\sigma^{par}} Q_{a,t}^{par,old} \tag{63}$$

,

where

$$\hat{\alpha}_{a,ch,t}^{old} = \sum_{\tau}^{t-1} \left[\alpha_{a,\tau}^{ch} \left(\frac{\lambda_{a,\tau,t}^{ch}}{\lambda_{a,t}^{ch,old}} \right)^{\sigma^{par}-1} \hat{q}_{a,\tau,t}^{par} \left(\frac{p_{a,\tau,t}^{ch}}{p_{a,\tau,t}^{par}} \right)^{-\sigma^{par}} \left(\frac{P_{a,t}^{ch,old}}{P_{a,t}^{par,old}} \right)^{\sigma^{par}} \right] \tag{64}$$

.

For $par = top$, we follow exactly the same steps, except that we replace

$Q_{a,t}^{par,old}$ with $\hat{Q}_{a,t}^{par,old}$:

$$Q_{a,t}^{ch,old} = \hat{\alpha}_{a,t}^{ch,old} \left(\lambda_{a,t}^{ch,old}\right)^{\sigma^{par}-1} \left(\frac{P_{a,t}^{ch,old}}{P_{a,t}^{par,old}}\right)^{-\sigma^{par}} \hat{Q}_{a,t}^{par,old} \tag{65}$$

Notice that we can express $\hat{q}_{a,\tau,t}^{par}$ using equations 55, 4 and 61:

$$\hat{q}_{a,\tau,t}^{top} = \frac{N_{a,\tau,t}}{(1-x_{a,t})\,M_{a,t-1}} \left(\frac{P_{a,t}^{top,old}}{p_{a,\tau,t}}\right)^{\sigma_a^{var}} \left(\frac{\lambda_{a,t}^{top,old}}{\lambda_{a,\tau,t}^{top}}\right)^{1-\sigma_a^{var}} \tag{66}$$

for the top nest and, using equation 55,

$$\hat{q}_{a,\tau,t}^{ch} = \frac{\alpha_{a,ch,\tau}\left(\lambda_{a,\tau,t}^{ch}\right)^{\sigma^{par}-1}}{\alpha_{a,ch,t}^{old}} \left(\frac{p_{a,\tau,t}^{ch}}{p_{a,\tau,t}^{par}}\right)^{-\sigma^{par}} \left(\frac{p_{a,t}^{ch,old}}{p_{a,t}^{par,old}}\right)^{\sigma^{par}} \hat{q}_{a,\tau,t}^{par} \tag{67}$$

for the nests with $par \neq top$.

The short version

In the short version, the intermediate demand for new varieties is derived in the same way as in the long version. In the case of old varieties, the demand is obtained by applying definitions 26 and 30 to equation 34:

$$Q_{a,t}^{ch,old} = \alpha_{a,t}^{ch,old} \left(\lambda_{a,t}^{ch,old}\right)^{\sigma^{par}-1} \left(\frac{P_{a,t}^{ch,old}}{P_{a,t}^{par,old}}\right)^{-\sigma^{par}} Q_{a,t}^{par,old}$$

In this equation, $\alpha_{a,t}^{ch,old}$ acts as an average intensity of use of factor ch across old varieties.

Price of sectoral output

The long version

Once the demand functions are established, we can determine the price of output, $P_{a,t}$, as follows:

$$P_{a,t}Q_{a,t} = (1+\gamma)\left(Q_{a,t}^{top,old}P_{a,t}^{top,old} + \hat{Q}_{a,t}^{top,new}P_{a,t}^{top,new}\right) \tag{68}$$

.

This will set exactly the same price as equation 5.

The Short version

In the short version, the output price is determined by equation 33. Combined with equations 21, 22 and 26, gives

$$P_{a,t} = \frac{P_{a,t}^{top,new}Q_{a,t}^{top,new} + \sum_{\tau}^{t-1} P_{a,t}^{top,old}Q_{a,t}^{top,old}}{Q_{a,t}}$$

.

Hence, the price can be again determined as follows:

$$P_{a,t}Q_{a,t} = (1+\gamma)\left(Q_{a,t}^{top,old}P_{a,t}^{top,old} + Q_{a,t}^{top,new}P_{a,t}^{top,new}\right).$$

Productivity path

Equations describing the path of $\lambda_{a,t}^{top,new}$ follow directly from equations 41 and 42:

$$\lambda_{a,t}^{top,new} = frontier_t \tag{69}$$

$$frontier_t = (1+g)\, frontier_{t-1} \tag{70}$$

The updating rule for $\lambda_{a,t}^{top,old}$ can be derived from from 25 and 3 as follows:

$$
\begin{aligned}
(1+INV_{a,t-1})\left(\lambda_{a,t}^{top,old}\right)^{\sigma^{var}-1} &= (INV_{a,t-1}+IMP_{a,t-1})\left(\lambda_{a,t-1}^{top,new}\right)^{\sigma^{var}-1} \\
&\quad +(1-IMP_{a,t-1})\left(\lambda_{a,t-1}^{top,old}\right)^{\sigma^{var}-1}
\end{aligned}
$$

Direction

As explained in Section 2, firms can choose the factor intensity of production for new varieties. Given firm choices regarding inputs $\frac{Q_{a,\tau}^{ch,new}}{Q_{a,\tau}^{par,new}}$, we can recover firms choices of $\alpha_{a,\tau}^{ch}$ (see equation 20); alphas chosen for a variety τ do not change over time. They are determined using equation 20:

$$
\alpha_{a,\tau}^{ch}\left(\lambda_{a,\tau}^{ch}\right)^{\sigma^{par}-1} = \left(\frac{PXFAC_{a,\tau}^{ch,new}}{PXFAC_{a,\tau}^{par,new}}\right)^{\sigma^{par}} \frac{XFAC_{a,\tau}^{ch,new}}{XFAC_{a,\tau}^{par,new}} \tag{71}
$$

.

$$
\alpha_{a,\tau}^{ch} = \left(\lambda_{a,\tau}^{ch}\right)^{1-\sigma^{par}} \left(\frac{P_{a,\tau}^{ch,new}}{P_{a,\tau}^{par,new}}\right)^{\sigma^{par}} \frac{Q_{a,\tau}^{ch,new}}{Q_{a,\tau}^{par,new}} \tag{72}
$$

Finally, we can use this information to update the factor intensity of the old varieties. The updating rule is different for the short and long versions. We describe them separately below:

The long version

The factor intensity parameter was derived above (see equation 64) and is restated here for convenience:

$$
\hat{\alpha}_{a,ch,t}^{old} = \sum_{\tau}^{t-1}\left[\alpha_{a,\tau}^{ch}\left(\frac{\lambda_{a,\tau,t}^{ch}}{\lambda_{a,t}^{ch,old}}\right)^{\sigma^{par}-1} \hat{q}_{a,\tau,t}^{par}\left(\frac{p_{a,\tau,t}^{ch}}{p_{a,\tau,t}^{par}}\right)^{-\sigma^{par}} \left(\frac{P_{a,t}^{ch,old}}{P_{a,t}^{par,old}}\right)^{\sigma^{par}}\right]
$$

.

The short version

In the short version, the updating rule can be derived directly from 30 and 3 as follows:

$$\alpha_{a,t}^{ch,old}\left(\lambda_{a,t}^{ch,old}\right)^{\sigma^{par}-1}=\frac{INV_{a,t-1}+IMP_{a,t-1}}{1+INV_{a,t-1}}\alpha_{a,t-1}^{ch}\left(\lambda_{a,t-1}^{ch}\right)^{\sigma^{par}-1}$$

$$+\frac{1-IMP_{a,t-1}}{1+INV_{a,t-1}}\alpha_{a,t-1}^{ch,old}\left(\lambda_{a,t-1}^{ch,old}\right)^{\sigma^{par}-1}$$

Appendix A2. Details of approximations for the short version

In this appendix, we describe the choice of the value of $frontier_0$ as follows. First, we define the average productivity in a sector as

$$\overline{\lambda}_{a,t}^{top} = \sum_{\tau=0}^{t} \frac{N_{a,\tau,t}}{M_{a,t}} \lambda_{a,\tau}^{top} = \frac{N_{a,t,t}}{M_{a,t}} \lambda_{a,t}^{top} + \frac{M_{a,t-1}}{M_{a,t}} \left(1 - IMP_{a,t}\right) \overline{\lambda}_{a,t-1}^{top}$$

.

Second, we set $\overline{\lambda}_{a,0} = 1$. We then set the value of $frontier_0$ at the level that ensures that, at $t = 1$, average productivity grows at the frontier growth rate:

$$\frac{\overline{\lambda}_{a,1}^{top} - \overline{\lambda}_{a,0}^{top}}{\overline{\lambda}_{a,0}^{top}} = \left(\frac{N_{a,t,t}}{M_{a,t}} \lambda_{a,t}^{top} + \frac{M_{a,t-1}}{M_{a,t}} \left(1 - x_{a,t}\right) \overline{\lambda}_{a,0}^{top} \right) - 1 = g$$

,

which requires

$$frontier_0 = \frac{\left(1+g\right)\left(1 + IMP_{a,0}\rho\right) - \left(1 - IMP_{a,0}\right)}{\left(1+\rho\right)\left(1+g\right) IMP_{a,0}}$$

Appendix A3

In this appendix we present in details what approximations are sufficient to obtain equations 31, 33, 34 and 35.

Suppose we consider a nested CES production function, where $ch0$ is an input for $ch1$, $ch1$ is an input for $ch2$, etc. Then, the demand for the bottom input, $ch0$ can be expressed as:

$$q_{a,\tau,t}^{ch0} = \alpha_{a,\tau}^{ch0} \left(p_{a,\tau,t}^{ch0}\right)^{-\sigma^{ch1}} \left(\lambda_{a,\tau,t}^{ch0}\right)^{\sigma^{ch1}-1} \left(p_{a,\tau,t}^{ch1}\right)^{\sigma^{ch1}-\sigma^{ch2}} \alpha_{a,\tau}^{ch1} \left(\lambda_{a,\tau,t}^{ch1}\right)^{\sigma^{ch2}-1} [...] *$$

$$* \left(p_{a,\tau,t}^{top}\right)^{\sigma^{top}-\sigma^{var}} \left(\lambda_{a,\tau}^{top}\right)^{\sigma_a-1} \left(P_{a,t}\right)^{\sigma_a^{var}} Q_{a,t}$$

Under the independence of $\alpha_{a,\tau}^{ch} \left(\lambda_{a,\tau,t}^{ch}\right)^{\sigma^{ch}-1}$ across nests:

$$E\left[q_{a,\tau,t}^{ch0}\right] = E\left[\alpha_{a,\tau}^{ch0} \left(p_{a,\tau,t}^{ch0}\right)^{-\sigma^{ch1}} \left(\lambda_{a,\tau,t}^{ch0}\right)^{\sigma^{ch1}-1}\right] *$$

$$E\left[\left(p_{a,\tau,t}^{ch1}\right)^{\sigma^{ch1}-\sigma^{ch2}} \alpha_{a,\tau}^{ch1} \left(\lambda_{a,\tau,t}^{ch1}\right)^{\sigma^{ch2}-1}\right] [...] *$$

$$*E\left[\left(p_{a,\tau,t}^{top}\right)^{\sigma_a^{top}-\sigma_a^{var}} \left(\lambda_{a,\tau}^{top}\right)^{\sigma_a^{var}-1}\right] \left(P_{a,t}\right)^{\sigma_a^{var}} Q_{a,t}$$

Ignoring the dependence between $\alpha_{a,ch,\tau} \left(\lambda_{a,\tau,t}^{ch}\right)^{\sigma^{par}-1}$ and $p_{a,\tau,t}^{par}$,

$$E\left[q_{a,\tau,t}^{ch0}\right] = E\left[\alpha_{a,\tau}^{ch0} \left(\lambda_{a,\tau,t}^{ch0}\right)^{\sigma^{ch1}-1}\right] E\left[\left(p_{a,\tau,t}^{ch0}\right)^{-\sigma^{ch1}}\right] *$$

$$*E\left[\alpha_{a,\tau}^{ch1} \left(\lambda_{a,\tau,t}^{ch1}\right)^{\sigma^{ch2}-1}\right] E\left[\left(p_{a,\tau,t}^{ch1}\right)^{\sigma^{ch1}-\sigma^{ch2}}\right] [...] *$$

$$*E\left[\left(p_{a,\tau,t}^{top}\right)^{\sigma_a^{top}-\sigma_a^{var}} \left(\lambda_{a,\tau}^{top}\right)^{\sigma_a^{var}-1}\right] \left(P_{a,t}\right)^{\sigma_a^{var}} Q_{a,t}$$

Ignoring that demand is a nonlinear function of prices: $E\left[\left(p^{ch1}_{a,\tau,t}\right)^{\sigma^{ch1}-\sigma^{ch2}}\right] \approx \left(E\left[p^{ch1}_{a,\tau,t}\right]\right)^{\sigma^{ch1}-\sigma^{ch2}}$ and $E\left[p^{ch2}_{a,\tau,t}\right] \approx \left(E\left[\left(p^{ch2}_{a,\tau,t}\right)^{1-\sigma^{ch2}}\right]\right)^{\frac{1}{1-\sigma^{ch2}}} = P^{old,ch2}_{a,t}$:

$$E\left[q^{ch0}_{a,\tau,t}\right] = E\left[\alpha_{a,ch0,\tau}\left(\lambda^{ch0}_{a,\tau,t}\right)^{\sigma^{ch1}-1}\right]\left(\frac{P^{ch0,old}_{a,t}}{P^{ch1,old}_{a,t}}\right)^{-\sigma^{ch1}} *$$

$$E\left[\alpha_{a,ch1,\tau}\left(\lambda^{ch1}_{a,\tau,t}\right)^{\sigma^{ch2}-1}\right]\left(\frac{P^{ch1,old}_{a,t}}{P^{ch2,old}_{a,t}}\right)^{-\sigma^{ch2}} [...] *$$

$$E\left[\left(\lambda^{top}_{a,\tau}\right)^{\sigma_a-1}\right]\left(\frac{P^{top,old}_{a,t}}{P_{a,t}}\right)^{\sigma^{top}-\sigma^{xpvt}} Q_{a,t}$$

Now, notice that

$$E\left[q^{top}_{a,\tau,t}\right] = E\left[\left(\lambda^{top}_{a,\tau}\right)^{\sigma^{var}_a-1}\right]\left(\frac{P^{top,old}_{a,t}}{P_{a,t}}\right)^{\sigma^{top}_a-\sigma^{var}_a} Q_{a,t}$$

$$E\left[q^{ch2}_{a,\tau,t}\right] = E\left[\alpha_{a,ch2,\tau}\left(\lambda^{ch2}_{a,\tau,t}\right)^{\sigma^{ch3}-1}\right]\left(\frac{P^{ch2,old}_{a,t}}{P^{ch3,old}_{a,t}}\right)^{-\sigma^{ch3}} E\left[q^{top}_{a,\tau,t}\right]$$

etc.

Hence, the expression above could be simplified to

$$E\left[q^{ch0}_{a,\tau,t}\right] = E\left[\alpha_{a,ch0,\tau}\left(\lambda^{ch0}_{a,\tau,t}\right)^{\sigma^{ch1}-1}\right]\left(\frac{P^{ch0,old}_{a,t}}{P^{ch1,old}_{a,t}}\right)^{-\sigma^{ch1}} E\left[q^{ch1}_{a,\tau,t}\right]$$

which can be written as:

$$Q^{ch0}_{a,t} = E\left[\alpha_{a,ch0,\tau}\left(\lambda^{ch0}_{a,\tau,t}\right)^{\sigma^{ch1}-1}\right]\left(\frac{P^{ch0,old}_{a,t}}{P^{ch1,old}_{a,t}}\right)^{-\sigma^{ch1}} Q^{ch1}_{a,t}$$

The same argument could be used for higher nests.

Regarding the prices: $p^{ch1}_{a,\tau,t}$ can be expressed using,

$$\left(p_{a,\tau,t}^{ch1}\right)^{1-\sigma^{ch1}} = \sum_{ch0} \left(p_{a,\tau,t}^{ch0}\right)^{1-\sigma^{ch1}} \alpha_{a,ch0,\tau} \left(\lambda_{a,\tau,t}^{ch0}\right)^{\sigma^{ch1}-1}$$

$p_{a,\tau,t}^{ch2}$ can be expressed using,

$$\left(p_{a,\tau,t}^{ch2}\right)^{1-\sigma^{ch2}} = \sum_{ch1} \alpha_{a,ch1,\tau} \left(\lambda_{a,\tau,t}^{ch1}\right)^{\sigma^{ch2}-1} \left(\sum_{ch0} \alpha_{a,ch0,\tau} \left(\frac{p_{a,\tau,t}^{ch0}}{\lambda_{a,\tau,t}^{ch0}}\right)^{1-\sigma^{ch1}}\right)^{\frac{1-\sigma^{ch2}}{1-\sigma^{ch1}}}$$

etc. Assuming independence of $\alpha_{a,\tau}^{ch} \left(\lambda_{a,\tau,t}^{ch}\right)^{\sigma^{ch}-1}$ across nests:

$$\left(P_{a,t}^{ch2,old}\right)^{1-\sigma^{ch2}} \equiv E\left[\left(p_{a,\tau,t}^{ch2}\right)^{1-\sigma^{ch2}}\right] = \sum_{ch1} E\left[\alpha_{a,ch1,\tau} \left(\lambda_{a,\tau,t}^{ch1}\right)^{\sigma^{ch2}-1}\right] E\left[\left(p_{a,\tau,t}^{ch1}\right)^{1-\sigma^{ch2}}\right]$$

Now, using again the approximation, $E\left[p_{a,\tau,t}^{ch1}\right] \approx \left(E\left[\left(p_{a,\tau,t}^{ch1}\right)^{1-\sigma^{ch1}}\right]\right)^{\frac{1}{1-\sigma^{ch1}}}$, we get:

$$\left(P_{a,t}^{ch2,old}\right)^{1-\sigma^{ch2}} = \sum_{ch1} E\left[\alpha_{a,ch1,\tau} \left(\lambda_{a,\tau,t}^{ch1}\right)^{\sigma^{ch2}-1}\right] P_{a,t}^{ch1}$$