

Modeling gender-based wage discrimination

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1. Introduction

A growing number of computable general equilibrium (CGE) models disaggregate labor by gender (along with other dimensions), typically if not always with higher wages for men than women for workers that otherwise belongs to the same category. To the best of our knowledge, by construction, to date all CGE models in the literature assume that these wage differences are not due to discrimination but to marginal productivity differences, something that, in the markets of the model, result in lower wages. This is a major shortcoming as it means that the models are unfit to analyze the consequences of removing or reducing the extent of wage discrimination. Accordingly, existing applications have, wisely, not addressed discrimination but instead focused on other issues, like analyses of the impacts of changes in policy or exogenous conditions (for example, an increase in world prices for sectors with a high employment share for women) on male vs. female labor.

The objective of this paper, which is inspired by the work of Gary Becker, is to extend the standard CGE framework so that it can analyze issues related to discrimination by splitting wage (or, more broadly, rent) differences into two categories: those related to marginal productivity and those related to other factors that are exogenous to the model which, for labor, include discrimination. While our focus is on discrimination based on gender, the approach is equally applicable to discrimination based on other characteristics like race, ethnicity, or religion. Given that the proposed extension only requires little additional information and minimal changes in model structure, it should be straightforward to apply it in different studies.

In outline, Section 2 presents the new approach and tests it in a model of a firm that practices wage discrimination against women; drawing on the rest results, we propose a measure of gendered wage discrimination. In Section 3, we embed the discriminating firm model in a simple static CGE model linked to a database for Mexico with labor disaggregated by gender and empirical data on wage discrimination. Using the CGE model, Section 4 presents and

analyzes a set of simulations in which wage discrimination is gradually reduced. The analysis reports on and explains the changes in sector production, trade, and employment; wages and incomes by labor type; and aggregate and disaggregated household welfare. The concluding Section 5 summarizes the findings and discusses potential applications and extensions.

2. Firm model

Our model of a profit-maximizing firm with wage discrimination deviates from the basic treatment of firms in CGE models and elsewhere in that it distinguishes between wage differences that are due to discrimination and those that are due to productivity differences.

As here defined, discrimination (positive or negative) is manifested when employers for some reason (for example prejudice) do not offer a category of workers, here women, wages that match their marginal value products (MVPs), i.e., the product of the output price and the marginal product, the latter defined by the production function.

Gaps between the MVP and the wage could also result from taxes or subsidies on wage payments specific to a subset of the worker categories; the difference is that payments due to taxes and subsidies are linked to the government budget, not recycled inside the firm, as is the case for our discrimination treatment when it is embedded in a CGE context.

2.1. Mathematical statement

The model describes a firm that maximizes profits in a short-run static setting. To reflect the short-run nature of the problem, capital employment is fixed and the capital rent flexible whereas, for labor, employment is flexible and wages fixed. Without wage discrimination, the basic optimization problem of the firm is

maximize

$$\pi = p \cdot Q - (WK \cdot qk + wm \cdot QM + wf \cdot QF)$$

subject to

$$Q = a \cdot qk^{shk} \cdot QM^{shm} \cdot QF^{shf}$$

where π = profits; p = output price; Q = output quantity; WK, wm, wf = wages (rents) for capital, male labor, female labor; qk, QM, QF = hiring levels for capital, male labor, female labor; shk, shm, shf = value shares (output elasticities) for capital, male labor, female labor; and a = efficiency parameter. The notation uses upper case for variables and lower case for parameters. In this exposition, we use a Cobb-Douglas production function for algebraic simplicity. In the CGE model, the embedded discriminating firm will use CES functions.

The Lagrangian function for the above problem may be stated as

$$Z = p \cdot Q - (WK \cdot qk + wm \cdot QM + wf \cdot QF) + \lambda(a \cdot qk^{shk} \cdot QM^{shm} \cdot QF^{shf} - Q)$$

where λ = the shadow price of output.

And the first-order conditions become

$$\frac{dZ}{dQ} = p - \lambda = 0$$

$$\frac{dZ}{dQK} = -WK + \lambda \cdot a \cdot shk \cdot qk^{shk-1} \cdot QM^{shm} \cdot QF^{shf} = 0$$

$$\frac{dZ}{dQM} = -wm + \lambda \cdot a \cdot shm \cdot qk^{shk} \cdot QM^{shm-1} \cdot QF^{shf} = 0$$

$$\frac{dZ}{dQF} = -wf + \lambda \cdot a \cdot shf \cdot qk^{shk} \cdot QM^{shm} \cdot QF^{shf-1} = 0$$

$$\frac{dZ}{d\lambda} = a \cdot qk^{shk} \cdot QM^{shm} \cdot QF^{shf} - Q = 0$$

The applied firm model, which is solved in GAMS, is derived from the first-order conditions and adjusted to account for potential discrimination between males and females (positive and/or negative). The equations are expressed as follows:

$$(1) \quad Q = a \cdot qk^{shk} \cdot QM^{shm} \cdot QF^{shf}$$

$$(2) \quad p \cdot a \cdot shk \cdot qk^{shk-1} \cdot QM^{shm} \cdot QF^{shf} = WK$$

$$(3) \quad p \cdot a \cdot shm \cdot qk^{shk} \cdot QM^{shm-1} \cdot QF^{shf} / (1 + dm) = wm$$

$$(4) \quad p \cdot a \cdot shf \cdot qk^{shk} \cdot QM^{shm} \cdot QF^{shf-1} / (1 + df) = wf$$

The only new notation is the two discrimination factors, dm and df , which show the degrees of male and female wage discrimination, respectively. For both, values above zero indicate negative discrimination and below zero the opposite. In addition, compared to the above FOCs, p has been substituted for λ .

The interpretations of the different equations, which together represent the profit-maximizing equilibrium, are straightforward. Equation 1 is the production function. Equation 2 states that, for capital, the MVP (output price times marginal production of capital) equals the rent.

Equations 3 and 4 show the same condition for male and female labor, respectively. Both equations impose equality between, on the right, the wage and, on the left, the MVP divided by 1 plus the rate of discrimination against the labor type in question. Thus, in equation 4, if the firm discriminates against female labor ($df > 0$), then, given a diminishing MP of female labor, it will hire fewer women and, at equilibrium require an MVP that exceeds the wage paid.

Equivalently, in equations 3 and 4, the denominators with the discrimination term could be moved to the right-hand side, respectively becoming.

$$(5) \quad p \cdot a \cdot shm \cdot qk^{shk} \cdot QM^{shm-1} \cdot QF^{shf} = wm(1 + dm)$$

$$(6) \quad p \cdot a \cdot shf \cdot qk^{shk} \cdot QM^{shm} \cdot QF^{shf-1} = wf(1 + df)$$

If so, discrimination against women could be interpreted as being generated by a perceived marginal cost of hiring women that adds to wage and thus leads to that, at equilibrium the MVP exceeds the wage. The latter formulation brings to the fore the fact that, in the model, discrimination has the same impact on hiring decisions as a tax.

2.2. A measure of wage discrimination

Using this model formulation, it is possible to define pure wage discrimination, i.e., firm-level wage gaps not matched by worker characteristics that influence productivity (like differences in education and experience). As indicated by the above model, at profit maximum

$$mvpm/(1 + dm) = wm; \text{ and}$$

$$mvpf/(1 + df) = wf$$

where $mvpm$ and $mvpf$ are the male and female MVPs (the numerators of the left-hand sides of equations 3 and 4, i.e., the products of output price and marginal products)

Given this, the degree of wage discrimination against females may be defined as follows, for illustration assuming that $df = 0.1$, $dm = -0.1$, $wm = 1.5$, and $wf = 1$:

$$wdiscr = \frac{mvpf/wf}{mvpm/wm} - 1 = \frac{1 + df}{1 + dm} - 1 = \frac{1.1}{0.9} - 1 = 0.22$$

The interpretation is that, relative to actual male and female MVPs, women are underpaid by 22%. This is here due to the combined effect of an underpayment to women and an

overpayment to men, but it could also stem from only one of these two sources. In effect, empirical data on wage discrimination would not be able to distinguish between the two sources of wage discrimination unless data on both wages and MVPs are available.

2.3. Simulation results

To test the formulation, the firm model was implemented in GAMS with the data shown in Table 2.3.1. The simulations are defined in Table 2.3.2. The main results, shown in Tables 2.3.3 and 2.3.4, may be summarized as follows:

- Discrimination is not consistent with profit maximization: after paying labor, the profit (remaining value-added) declines for all scenarios with discrimination.
- Whether discrimination is due to errors in the assessment of female productivity, male productivity, or of both has a strong impact on the results:
 - Underestimating female productivity (discr-f) reduces both male and, even more strongly, female employment.
 - Overestimating male productivity (discr-m) raises both female and, even more strongly, male employment.
 - The combination of over- and under-estimating (discr-mf) males and females, respectively, has a more mitigated aggregate impact since, at the aggregate level, the errors in estimation work in opposite direction. However, at the disaggregated male and female level, men gain and women lose in both employment and incomes.

These results are subject to the assumptions of Cobb-Douglas production functions and the partial-equilibrium setting. In a general-equilibrium context, the adjustments in output and employment would be more mitigated and the direction of change for sales revenue and labor incomes would depend on elasticities of demand and changes in aggregate incomes due to the removal of discrimination.

Table 2.3.1. Input data for firm model

output price	p	1.00
capital wage	wk0	1.00
male labor wage	wm	1.30
female labor wage	wf	1.00
quantity of output	q0	100.00
quantity of capital demand	qk	30.00
quantity of male labor demand	qm0	50.00
quantity of female labor demand	qf0	20.00
capital value share	shk	0.30
male labor value share	shm	0.45
female labor value share	shf	0.25

Table 2.3.2. Simulation definitions

base	given data, no wage discrimination
discr-f	same as base except for wage discrimination against female labor
discr-m	same as base except for wage discrimination in favor of male labor
discr-mf	same as base except for wage discrimination in favor of male labor and against female labor

Table 2.3.3. Simulation results (levels)

			base	discr-f	discr-m	discr-mf
output price	prices	p	1.0	1.0	1.0	1.0
capital wage	prices	wk	0.7	0.6	0.8	0.7
male labor wage	prices	wm	1.3	1.3	1.3	1.3
female labor wage	prices	wf	1.0	1.0	1.0	1.0
wage discrimination against female	prices	w-discr		0.2	0.1	0.2
quantity of output	quantities	q	69.4	59.5	83.0	69.8
quantity of capital demand	quantities	qk	30.0	30.0	30.0	30.0
quantity of male labor demand	quantities	qm	24.0	20.6	32.4	25.6
quantity of female labor demand	quantities	qf	17.3	12.4	20.7	15.9
$p \cdot q$	values	revenue	69.4	59.5	83.0	69.8
$wk \cdot qk + dm \cdot wm \cdot qm + df \cdot wf \cdot qf$	values	caprent-net	20.8	20.4	20.2	20.7
$wk \cdot qk$	values	caprent-gross	20.8	17.8	24.9	20.9
$dm \cdot wm \cdot qm + df \cdot wf \cdot qf$	values	distortion-adjust		2.5	-4.7	-0.3
$wm \cdot qm$	values	labinc-male	31.2	26.8	42.1	33.3
$wf \cdot qf$	values	labinc-female	17.3	12.4	20.7	15.9
= revenue minus total cost	values	gap-rev-cost	0.0	0.0	0.0	0.0

Table 2.3.4. Change from base by simulation (%)

			discr-f	discr-m	discr-mf
capital wage	prices	wk	-14.2	19.6	0.6
quantity of output	quantities q		-14.2	19.6	0.6
quantity of male labor demand	quantities qm		-14.2	34.8	6.6
quantity of female labor demand	quantities qf		-28.7	19.6	-8.6
$p \cdot q$	values	revenue	-14.2	19.6	0.6
$wk \cdot qk + dm \cdot wm \cdot qm + df \cdot wf \cdot qf$	values	caprent-net	-2.2	-3.1	-0.6
$wk \cdot qk$	values	caprent-gross	-14.2	19.6	0.6
$wm \cdot qm$	values	labinc-male	-14.2	34.8	6.6
$wf \cdot qf$	values	labinc-female	-28.7	19.6	-8.6

3. The CGE model: structure, assumptions, and database

3.1. Model structure and assumptions

Except for the fact that value-added is generated by a three-level nested CES structure, the CGE model incorporates the treatment of producer factor employment decisions of the firm model that was presented in Section 2, i.e., producers (represented by activities) are assumed to maximize profits, taking all prices, wages and rents as given, with potential discrimination of in their labor-employment decisions (depending on the choice of parameter values). More specifically, as indicated below (see Table 3.2.1), labor is disaggregated by skill (into skilled and unskilled) and gender (into male and female). In the CES nesting, the top level includes all non-labor factors and aggregate labor. At the second level, aggregate labor is defined as a function of aggregate skilled and unskilled labor and, at the third level, these two aggregates are generated from gender-disaggregated skilled and unskilled labor, respectively.

In other respects, the structure of the model is similar to that of many other CGE models. Nevertheless, here we summarize the assumptions as they matter to the interpretation of the

simulation results. (A full mathematical statement of this model is provided as Supplementary Material A.)

For the government the domestic receipts are made up of taxes, capital rents, and transfers from the domestic household (an aggregate domestic non-government institution. Its receipts from indirect (activity and commodity) taxes are determined by fixed rates times endogenous bases, capital rents are a fixed share of total capital rents, while transfers received from the household are a fixed share of household income net of direct taxes. On the side of government domestic outlays, the real level of government consumption is fixed while transfers to households and government savings are CPI-indexed. The government account is cleared by changes in the direct tax rates for the household.

In addition to transfers from the government, the household has earnings from labor and capital (both fixed shares of the total earning of each factor). Household spending other than transfers and direct taxes is made up of savings (with a rate that adjusts to clear the savings-investment balance) and consumption, the allocation of which is driven by the maximization of a Cobb-Douglas utility function. By construction, the total values for the household's expenditures and receipts are equal.

The remaining domestic final demands, fixed investment and stock changes, are both exogenous in real terms. Changes in their total value lead to changes in the household savings rate which, as noted above, is flexible; i.e., savings are driven by investment.

As in most CGE models, domestic markets for products and factors are cleared by flexible prices, wages, and rents. The employment of capital is fixed at the sector level whereas labor is mobile across sectors with a fixed level of total employment.¹

¹ The sensitivity of the results to an alternative formulation with wage curves is also presented.

For the rest of the world (the current account of the balance of payments), the quantities of exports and imports are endogenous while world prices for exports and imports, and all other payments, including foreign savings (the current account deficit) are exogenous in foreign currency. The allocation of output between exports and domestic sales and the split of domestic demands between imports and domestic output are based on the standard constant-elasticity-of-transformation (CET) and Armington approaches, respectively. The rest-of-the-world account is cleared by a flexible real exchange rate.

3.2. Model database

The model database includes a 2018 SAM for Mexico, constructed for this paper on the basis of the following data: supply and use tables (SUTs) prepared by the National Institute of Statistics and Geography (INEGI) Experimental Statistics group (INEGI 2021a); integrated economic accounts, also prepared by INEGI; and information on government financing from the Secretariat of Finance and Public Credit. The payments to the labor factor were disaggregated by gender and skill level using the National Survey of Occupation and Employment (ENOE) (INEGI 2021b). The disaggregation of the SAM, which coincides with the model disaggregation, is presented in Table 3.2.1. As can be seen, the SAM has 11 sectors (each of which is represented by an activity and a commodity), 7 factors, and 3 institutions, including a domestic non-government institution, referred to as the household.

Table 3.2.1. Disaggregation of 2018 Model SAM for Mexico	
Sectors	Agriculture
(Activities and commodities)	Mining
	Manufacturing
	Construction
	Trade
	Hotels and restaurants
	Transport and communications
	Financial and business services
	Education and health
	Public administration
	Other services
Factors	Labor, male, unskilled (<secondary)
	Labor, male, skilled (>=secondary)
	Labor, female, unskilled (<secondary)
	Labor, female, skilled (>=secondary)
	Capital
	Land
	Extractive resource
Institutions	Household
	Government
	RoW
Distribution margins	Domestic
	Imports
	Exports
Taxes	Activities
	Commodities
	Imports
	Income

Table 3.2.2 shows an aggregated version of the model SAM with percent of GDP as the unit (to make it easier to interpret the data) – the disaggregated SAM is provided in electronic appendix A. From the cells of the aggregated SAM, it is straightforward to extract standard national account aggregates (private and government consumption and investment, exports, and imports) and the budgets of the government, the domestic non-government, and the rest of the world (the balance of payments). The simulations are focused on the accounts related to labor. As can be seen, virtually all labor incomes are part of value added and they are all

received by the household. As percent of value-added (GDP at market prices net of indirect taxes), labor accounts for around 35 percent, split into 23 percent for men and 12 percent for women.

Table 3.2.2. Macro SAM for Mexico, 2018 (% of GDP)

	act	com	f-labm-unsk	f-labm-sk	f-labf-unsk	f-labf-sk	f-cap	tax	hhd	gov	row	sav-inv	dstk	total
act	169.4													169.4
com	75.0								65.7	11.7	36.2	22.1	3.4	213.9
f-labm-unsk	4.3										0.0			4.4
f-labm-sk	17.4										0.1			17.5
f-labf-unsk	1.3										0.0			1.4
f-labf-sk	9.7										0.0			9.7
f-cap	61.1													61.1
tax	0.6	5.5							4.0					13.3
hhd			4.4	17.5	1.4	9.7	10.3			5.9	3.6			74.3
gov							0.0	13.3	1.7					15.1
row		39.0												41.8
sav-inv									0.0	-2.4	1.8			25.4
dstk												3.4		3.4
total	169.4	213.9	4.4	17.5	1.4	9.7	61.1	13.3	74.3	15.1	41.8	25.4	3.4	

Source: Authors' elaboration.

Notation:

act	activity	tax	taxes
com	commodity	hhd	household
f-labm-unsk	male unskilled labor	gov	government
f-labm-sk	male skilled labor	row	rest of world
f-labf-unsk	female unskilled labor	sav-inv	savings and investment
f-labf-sk	female skilled labor	dstk	stock changes
f-cap	capital		

Apart from the SAM, the database covers physical employment by labor type, parameters for wage discrimination, and elasticities for value-added and trade.

- The rest of this section is in progress. It will present a summary of the model database and, drawing on this information, the structure of the Mexican economy, including labor from a gender perspective.

4. Simulations with the CGE model – in progress

- The simulations that are being implemented stepwise reduce the discrimination against the two categories of female labor, separately and simultaneously. The analysis is focused on the impact of these reductions on male and female incomes and employment, by sector and in total , as well as broader economic indicators, both at the national and sectoral levels (including GDP, foreign trade, and household welfare).

5. Conclusions – in progress

- This section will summarize the findings and discuss potential applications and extensions.

References – in progress

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