

An Analysis for Carbon Constraint Modeling

Y.H. Henry Chen^φ

In computable general equilibrium modeling, a common strategy for simulating carbon constraint scenarios is to revise the fossil fuel input of the cost function so that the fossil fuel expenditure and carbon penalty are combined together with a new sub-nest of constant elasticity of substitution function, usually in a Leontief fashion. A critical drawback for this approach is that when the carbon price is low or equal to zero, using the aggregation for the fuel and carbon credit combo can indeed increase the simulated emissions relative to a no-policy case, although the issue is indiscernible when the carbon price gets higher. The study provides a theoretical framework explaining this peculiar observation. It then presents a new modeling approach that resolves the issue. The proposed strategy can be applied on any non-negative carbon price that is given, or on any emissions target level that is either binding or not, without erroneously increasing emissions relative to the no-policy level. A modeling exercise is offered as well for demonstration purposes.

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1. Introduction

In computable general equilibrium (CGE) modeling, a common strategy for simulating carbon constraint scenarios is to revise the fossil fuel input of the cost function (Figure 1a) so that the fossil fuel expenditure and carbon penalty are combined together with a new sub-nest of constant elasticity of substitution (CES) function, usually in a Leontief fashion (Figure 1b). The added CES nest is “scenario-specific,” i.e., it will be adopted only when carbon mitigation is in place. Otherwise the double-input nest is replaced by the original single-input setting for the fossil fuel. This method has been widely adopted by research focusing on energy and climate policy analyses (Bye et al., 2023; Montenegro et al., 2019; Naberneegg et al., 2017; Oh and Kim, 2020; Rajbhandari et al., 2019; and Rutherford, and Schreiber, 2019).

A critical drawback for this approach, which has been overlooked by current research, is that when the carbon price is low or equal to zero, using a CES aggregation for the fuel and carbon credit combo can indeed increase the simulated emissions relative to a no-policy case. For instance, applying the approach in a single country recursive-dynamic CGE for Taiwan’s economy, the carbon pricing simulation shows that fossil CO₂ emissions increase relative to the no-policy case, when the carbon price is lower than 3.1 USD/t-CO₂, and when the carbon price reduces to zero, projected emissions for 2025 will increase by 13.1% (Figure 2a). The issue is indiscernible, however, when the carbon price gets higher, and this is perhaps why the problem is left unnoticed by existing studies, usually focusing on more aggressive mitigation targets requiring much higher levels of carbon price.

The predicament motivates this study, which aims at achieving two goals. First, the study provides a theoretical framework explaining when the fossil fuel expense and carbon penalty are combined together in a Leontief fashion, why the simulated emissions can exceed the no-policy levels under a lower carbon price. Another intent of the study is to present a new carbon constraint modeling approach that resolves the aforementioned issue. Technically speaking, without resorting to the CES aggregation for fossil fuel and carbon credit, the proposed approach converts the carbon penalty into fossil fuel tax. The strategy can be applied on any non-negative carbon price that is given, or on any emissions target level that is either binding or not, without erroneously increasing emissions relative to the no-policy level under a low carbon price.

Applying the new methodology on the same carbon constraint scenario on Taiwan's economy, the study demonstrates that emissions decrease monotonically from the no-policy level as the carbon price is elevated (Figure 3). In particular, a carbon price of zero, whether it is given or is determined endogenously by a non-binding emissions level constraint, will now generate the same emissions level as that of the no-policy case.

The rest of the paper is organized as follows: Section 2 presents a theoretical framework explaining why the existing approach in modeling carbon constraint can increase emissions at lower carbon prices, Section 3 presents the new approach that resolves the issue and keeps the cost function consistent across scenarios, Section 4 analyzes the emissions and carbon prices results based on the proposed approach, and Section 5 concludes.

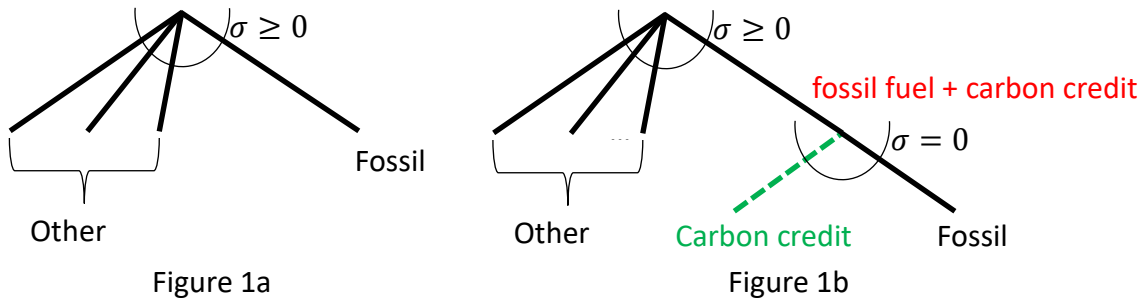


Figure 1. The cost function without carbon penalty (1a) and with carbon penalty (1b).

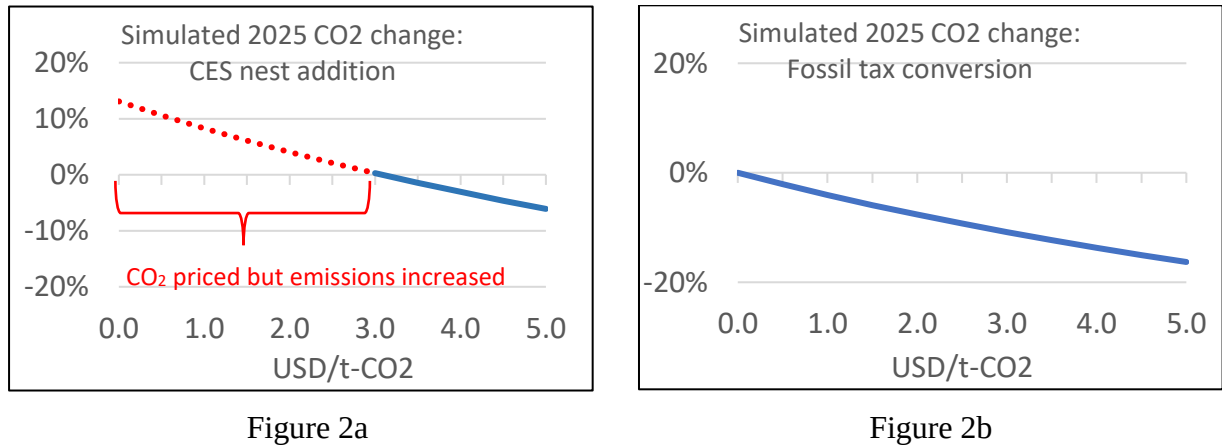


Figure 2. Projected CO₂ change for Taiwan: (a) existing approach; (b) proposed approach

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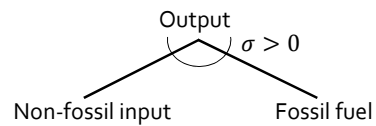
A Common Strategy

- A common strategy for carbon pricing modeling using CGE
 - Replace the fossil fuel input with a “fossil fuel & carbon permit” combo
 - The combo is a two-input CES aggregation (usually Leontief)

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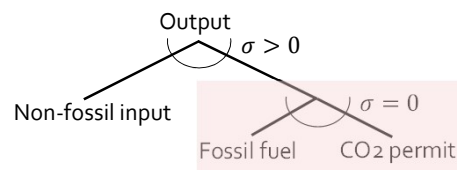
Without a carbon constraint



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Note: The CES functional form is changed.

A Common Strategy

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 - Replace the fossil fuel input with a “fossil fuel & carbon permit” combo
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- Examples

○ Antoszewski et al. (2015) ○ Bye et al. (2023) ○ Fæhn et al. (2005) ○ Montenegro et al. (2019) ○ Rajbhandari et al. (2019) ○ Rutherford, T. F. and A. Schreiber (2019)	○ Sue Wing. (2004) ○ Naberneegg et al. (2017) ○ Oh, I., W.-J. Yoo, and Y. Yoo (2019) ○ Oh, I., Y. Yeo, and J.-D. Lee. (2015) ○ Oh, I., W.-J. Yoo, and K. Kim. (2020)
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A Common Strategy

- A common strategy for carbon pricing modeling using CGE
 - Replace the fossil fuel input with a “fossil fuel & carbon permit” combo
 - The combo is a two-input CES aggregation (usually Leontief)
- A pitfall hiding in this approach! Solution?

An example

- Carbon pricing simulation using a single-country recursive dynamic CGE

A flexible framework for country-level analysis

Small open economy

As an example, using IO table for Taiwan with a base year of 2016

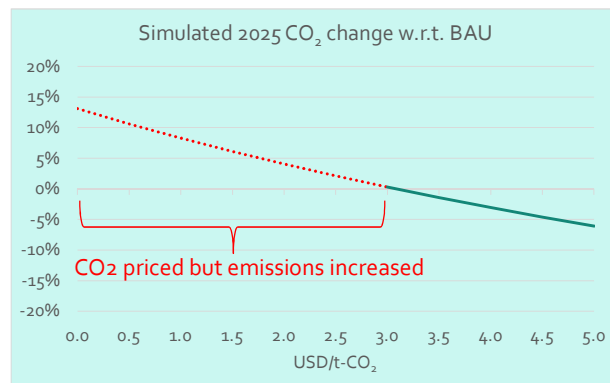
Using the "common strategy" listed above to look into changes in emissions

Time period: 2016-2025 or 2050 in this study.

Comparison in CO₂ emissions: BAU/no policy vs. CO₂ pricing in 2025

An example

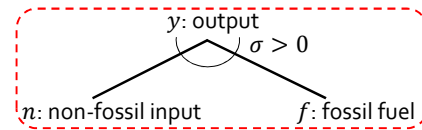
- Carbon pricing simulation using a single-country recursive dynamic CGE
 - Carbon pricing in 2025. When the CO₂ price < 3.1USD/t-CO₂, emissions > BAU level!



CES nest addition	
\$0.0	13.1%
\$0.5	10.6%
\$1.0	8.3%
\$1.5	6.1%
\$2.0	4.0%
\$2.5	2.1%
\$3.0	0.3%
\$3.5	-1.4%
\$4.0	-3.1%
\$4.5	-4.6%
\$5.0	-6.1%

Analysis for the common strategy

- **Without a carbon constraint**



- A representative firm uses fossil fuel f and other input n
- Two-input CES technology: $y = F(n, f)$; substitution elasticity $\sigma > 0$
- Competitive markets: p_y, p_n, p_f are exogenous

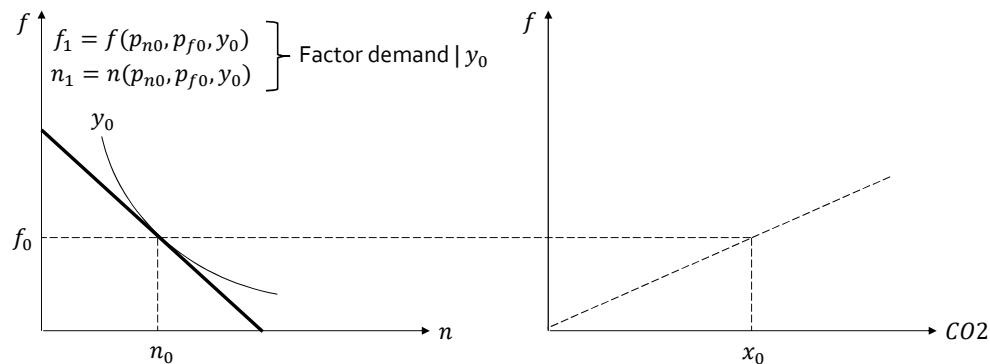
The technology is strictly convex $\Rightarrow$ demand for f is continuous $\forall p_f > 0$

Weak Axiom of Profit Maximization (WAPM) $\Rightarrow p_f \uparrow, q_f \downarrow$

Analysis for the common strategy

- **Without a carbon constraint**

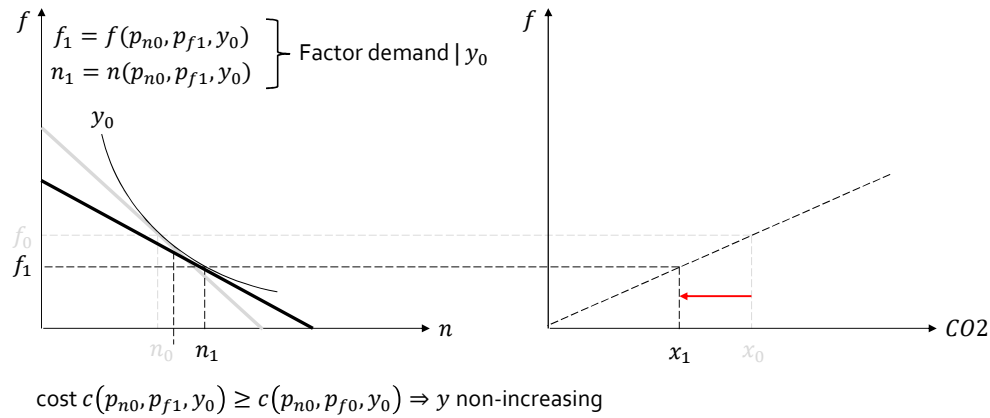
Fossil fuel price $p_f = p_{f0}$



Analysis for the common strategy

• Without a carbon constraint

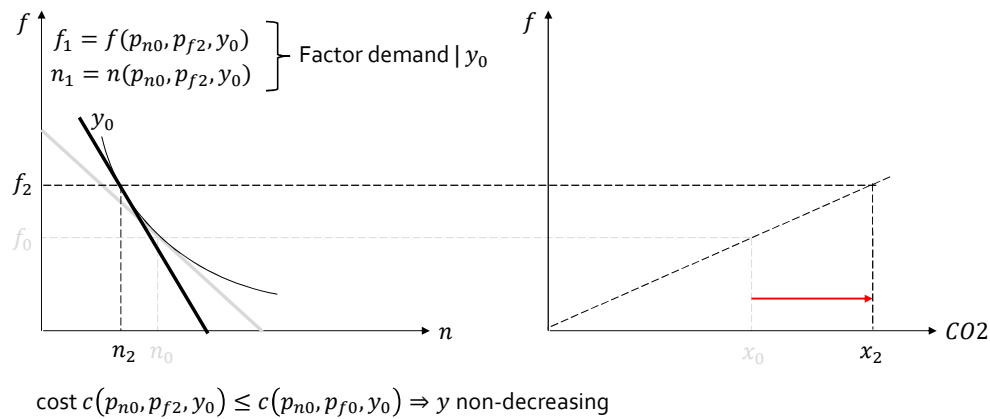
Fossil fuel price $p_f \uparrow (p_{f1} > p_{f0})$



Analysis for the common strategy

• Without a carbon constraint

Fossil fuel price $p_f \downarrow (p_{f2} < p_{f0})$



Analysis for the common strategy

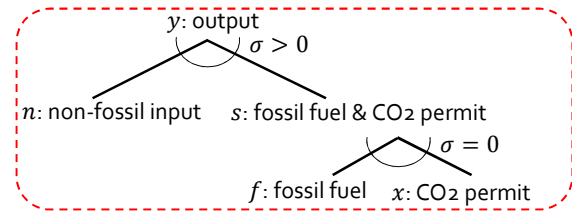
• With a carbon constraint

Consider the sub-nest "s" for fossil fuel f & CO₂ permit x

$s = s_0 \min\{f/f_0, x/x_0\}$ (production function for s)

$f = (f_0/s_0) \cdot s; x = (x_0/s_0) \cdot s$ (conditional factor demands for f and x)

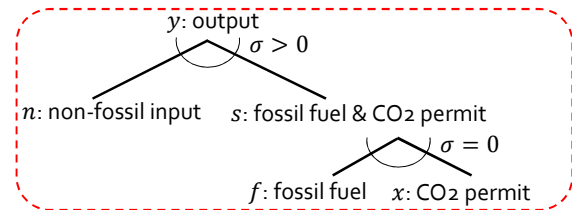
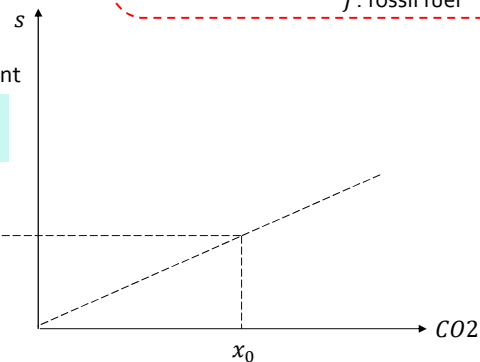
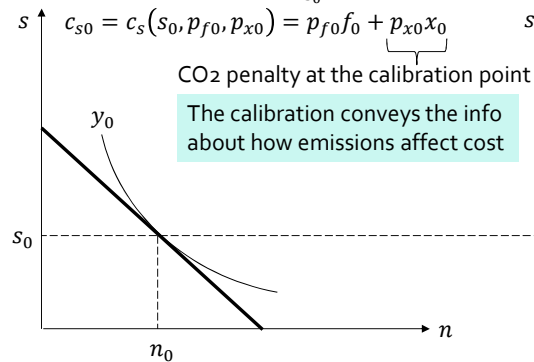
$c_s = c_s(s, p_f, p_x) = s/s_0 \cdot (p_f f_0 + p_x x_0)$ (cost function for s)



Analysis for the common strategy

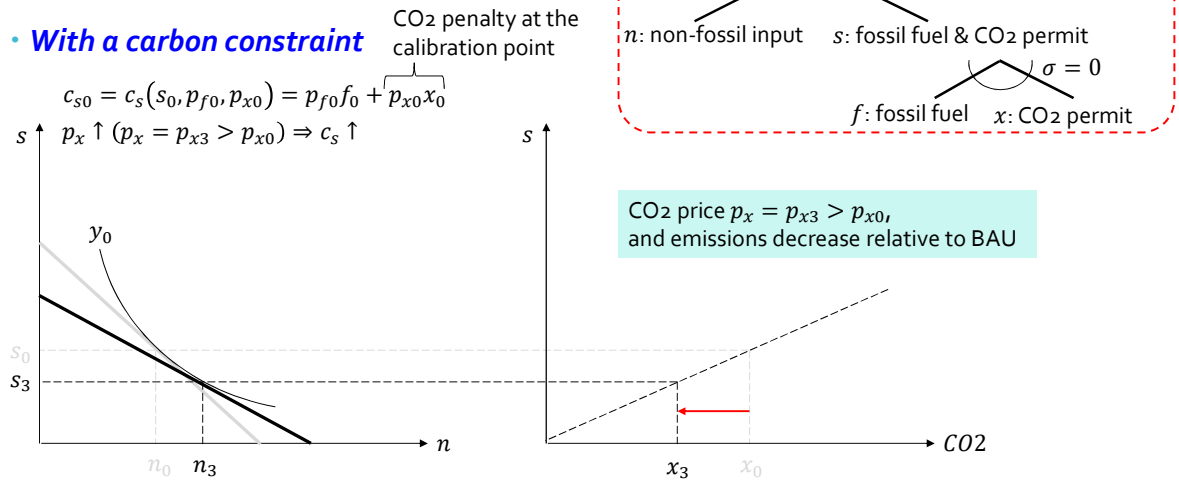
• With a carbon constraint

$c_s = c_s(s, p_f, p_x) = \frac{p_f f_0 + p_x x_0}{s_0} \cdot s$ (cost function for s)



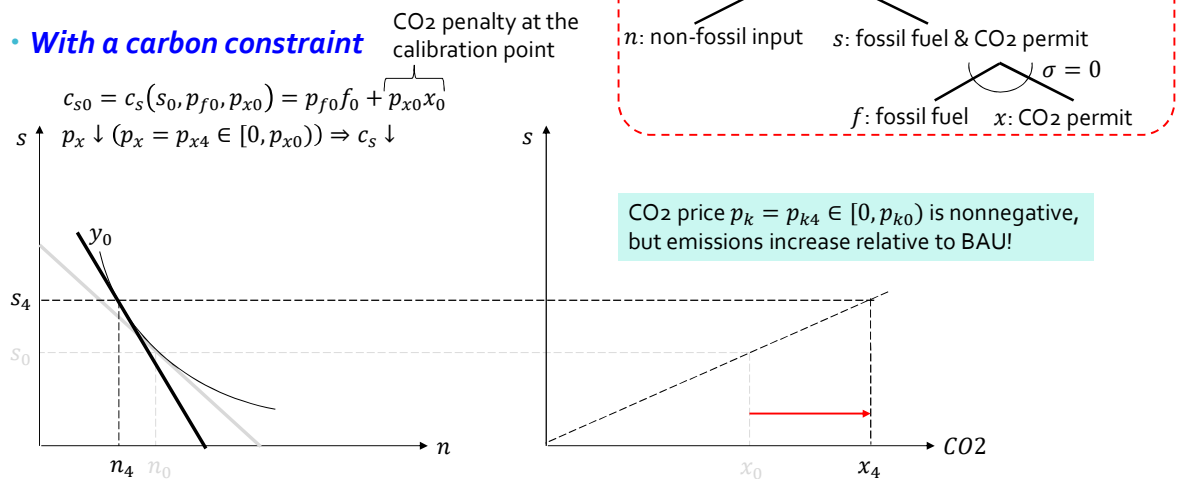
Analysis for the common strategy

• With a carbon constraint



Analysis for the common strategy

• With a carbon constraint



An alternative carbon pricing approach

- Step 1: Convert the CO₂ price into a fossil fuel tax rate
- Step 2: Impose the additional fossil fuel tax on fossil fuel use

The two steps are added and solved simultaneously with other components of the model.

Policy vs. no-policy are differentiated by the fossil fuel tax rate, rather than the technology's functional form.

An alternative carbon pricing approach

$$\overbrace{PA_j}^{\text{fossil fuel price index}} \cdot \overbrace{euse_{ji}}^{\text{fossil fuel use (\$)}} \cdot \overbrace{CTAXR_{ji}}^{\text{emissions}} = \overbrace{ejuse_{ji}}^{\text{emissions}} \cdot \overbrace{\epsilon_j}^{\text{carbon price (\$/t-CO}_2\text{)}} \cdot PC$$

PA_j = price index for fossil fuel j

$euse_{ji}$ = usage of j by sector i (monetary unit)

$CTAXR_{ji}$ = fossil fuel tax rate of using j imposed on i

$ejuse_{ji}$ = usage of j by sector i (physical unit)

ϵ_j = CO₂ emissions per physical unit of j use

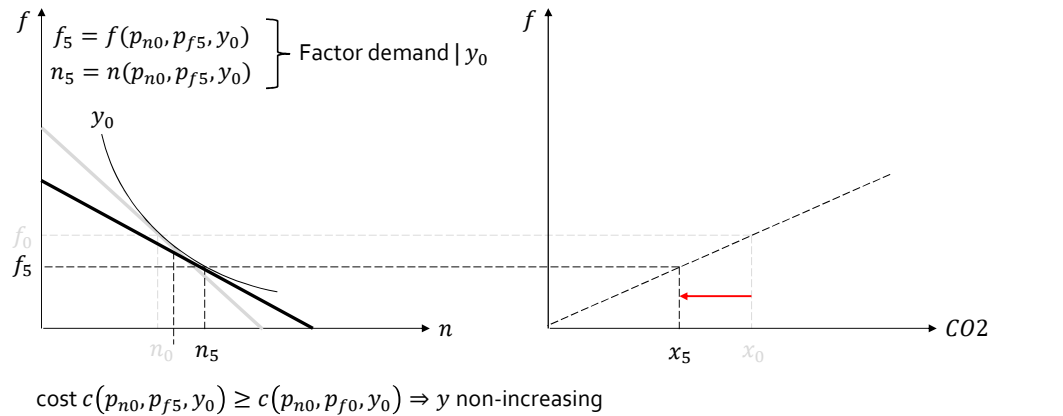
PC = CO₂ price per physical unit of emissions

The revision adds $i \times j$ variables $CTAXR_{ji}$ and $i \times j$ equations

An alternative carbon pricing approach

• With a carbon constraint

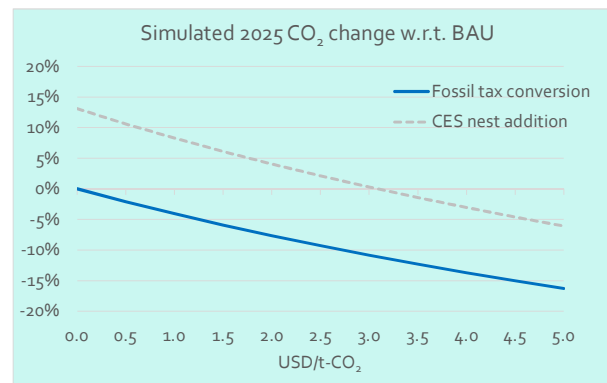
Fossil fuel price p_{f5} is tax-included ($p_{f5} > p_{f0}$)



An alternative carbon pricing approach

Using the single-country CGE framework with Taiwan's IO data

- Simulation with carbon pricing (0, 0.5, ..., 5.0 USD/t-CO₂ in 2025)
- Carbon pricing in 2025. Simulated emissions change relative to BAU

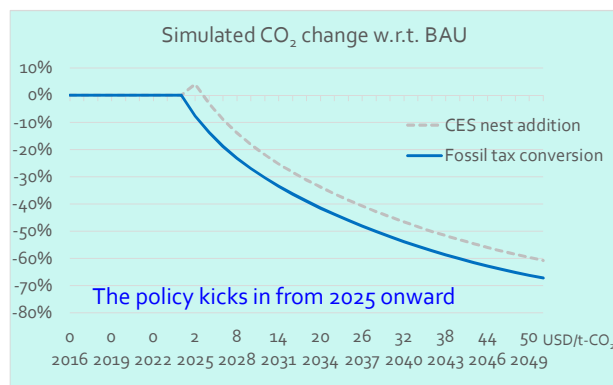


	CES nest addition	Fossil tax conversion
\$0.0	13.1%	0.0%
\$0.5	10.6%	-2.1%
\$1.0	8.3%	-4.1%
\$1.5	6.1%	-5.9%
\$2.0	4.0%	-7.6%
\$2.5	2.1%	-9.3%
\$3.0	0.3%	-10.8%
\$3.5	-1.4%	-12.3%
\$4.0	-3.1%	-13.7%
\$4.5	-4.6%	-15.0%
\$5.0	-6.1%	-16.3%

An alternative carbon pricing approach

Using the single-country CGE framework with Taiwan's IO data

- Simulation with carbon pricing (\$2/t-CO₂ in 2025, \$4 in 2026,..., \$52 in 2050)
- Simulated emissions change relative to BAU for 2016 to 2050

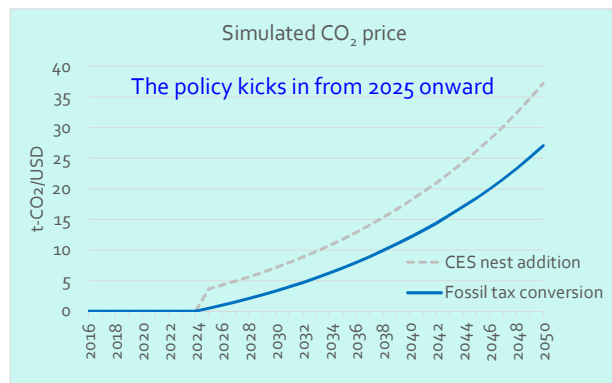


	CES nest addition	Fossil tax conversion
2025 (\$2)	4.0%	-7.6%
2026 (\$4)	-3.1%	-13.8%
2027 (\$6)	-9.0%	-18.8%
2028 (\$8)	-13.9%	-23.2%
2029 (\$10)	-18.2%	-27.0%
2030 (\$12)	-21.9%	-30.4%

An alternative carbon pricing approach

Using the single-country CGE framework with Taiwan's IO data

- Simulation with quantity constraint (-2% BAU emissions in 2025,..., -52% in 2050)
- Simulated CO₂ price for 2016 to 2050



	CES nest addition	Fossil tax conversion
2025 (-2%)	\$3.67	\$0.48
2026 (-4%)	\$4.28	\$0.98
2027 (-6%)	\$4.93	\$1.51
2028 (-8%)	\$5.62	\$2.08
2029 (-10%)	\$6.35	\$2.67
2030 (-12%)	\$7.12	\$3.31

Conclusions

- Analyze the pitfall of current carbon pricing approach
- Propose the well-behaved method for incorporating carbon penalty

Thank you! Comments or questions?

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Appendix

A1. The single-country CGE structure

