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Aggregation Bias in CGE models

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Motivation of the Study

- Exact aggregation formula for elasticities for nested CES functions are not developed
  - Approximation by output-weighted averages of disaggregated elasticities has been applied
- General equilibrium elasticity of substitution differ from output-weighted averages
  - No closed-form or algorithm exists for the general equilibrium elasticities
- Recent progress on the general equilibrium elasticies need to be utilized for a better approximation of elasticity aggregations

Literature Review

- Jones (1965)
  - Formula for the general equilibrium elasticity for 2-sector 2-factor economy
  - GE elasticity is a weighted average of disaggregated elasticities and demand elasticity
- Overfield and Raval (2014)
  - Formula for the aggregated elasticities for a range of nested CES economy with intermediate demands
- Horridge (2018)
  - Propose 'K-approach' for the aggregated elasticities
- Baqaee and Farhi (2018)
  - Theoretical derivation of a formula of economy-wide general equilibrium substitution elasticities for general nested CES technologies

Formulas for Aggregated Elasticities

- Jones (1965)
  - $\sigma = Q_M\sigma_M + Q_F\sigma_F + Q_D\sigma_D$
    - ✓ $Q_M = a_{LM}\left(\frac{a_{TM}M}{T}\right) + a_{TM}\left(\frac{a_{LM}M}{L}\right), Q_F = a_{LF}\left(\frac{a_{TF}F}{T}\right) + a_{TF}\left(\frac{a_{LF}F}{L}\right), Q_D = \left(\frac{a_{LM}M}{L} - \frac{a_{TM}M}{T}\right)(a_{LM} - a_{LF})$
- Baqaee and Farhi (2018)
  - $1 - \frac{1}{\sigma_{fg}^F} = \left(I_{(g)} - I_{(f)}\right)'(I - \Gamma)^{-1}\delta_{(g)}$
    - ✓ $\Gamma_{hh'} = -\sum_{k\in 1+N}(\theta_k - 1)\lambda_k \mathrm{Cov}_{\Omega^{(k)}}\left(\Psi_{(h')}, \Psi_{(h)}/\Lambda_h\right)$
    - ✓ $\delta_{hh'} = \sum_{k\in 1+N}(\theta_k - 1)\lambda_k \mathrm{Cov}_{\Omega^{(k)}}\left(\Psi_{(h')}, \Psi_{(h)}/\Lambda_h\right)$
    - ✓ $\mathrm{Cov}_{\Omega^{(k)}}\left(\Psi_{(j)}, \Psi_{(i)}\right) = \sum_{l\in N+F}\Omega_{kl}\Psi_{lj}\Psi_{li} - \left(\sum_{l\in N+F}\Omega_{kl}\Psi_{lj}\right)\left(\sum_{l\in N+F}\Omega_{kl}\Psi_{li}\right)$
    - ✓ $\Psi \equiv (I - \Omega)^{-1}$, $\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i}$

Examples of 2-sector 2-factor economies

Scenario	A	B	C	D	E	F	G	H	I
a_{LM}	0.5	0.9	0.1	0.9	0.9	0.9	0.9	0.9	0.9
a_{LF}	0.5	0.1	0.9	0.1	0.1	0.1	0.1	0.1	0.1
M	5	5	5	9	5	9	1	5	5
F	5	5	5	1	5	1	9	5	5
L	5	5	5	8.2	5	8.2	1.8	5	5
T	5	5	5	1.8	5	1.8	8.2	5	5
σ_M	0.5	0.5	0.5	0.5	0	0.5	0.5	0.1	1
σ_F	0.1	0.1	0.1	0.5	0	0.1	0.1	0.5	5
σ_D	1	1	1	0.5	1	1	1	1	1
$\widetilde{Q_M}$	0.50	0.50	0.50	0.90	0.50	0.90	0.10	0.50	0.50
$\widetilde{Q_F}$	0.50	0.50	0.50	0.10	0.50	0.10	0.90	0.50	0.50
$\tilde{\sigma}$	0.30	0.30	0.30	0.50	0.00	0.46	0.14	0.30	3.00
Q_M	0.50	0.18	0.18	0.55	0.18	0.55	0.06	0.18	0.18
Q_F	0.50	0.18	0.18	0.06	0.18	0.06	0.55	0.18	0.18
Q_D	0.00	0.64	0.64	0.39	0.64	0.39	0.39	0.64	0.64
σ	0.30	0.75	0.75	0.50	0.64	0.67	0.48	0.75	1.72
%change	0.0%	149.3%	149.3%	0.0%	NA	45.8%	239.7%	149.3%	-42.7%

Design of Experiments

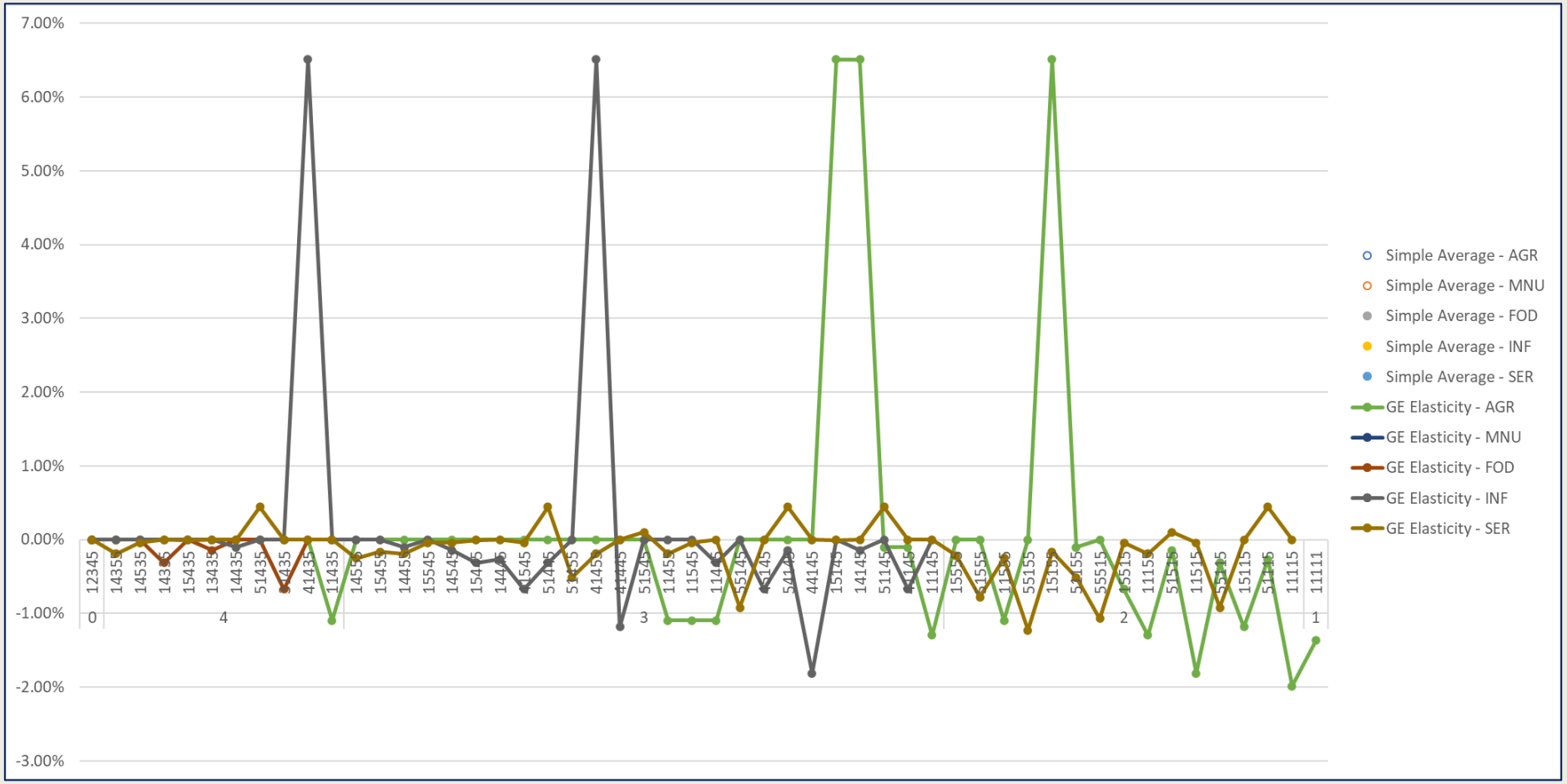
- Enumerate all the partitions of single-region 5-sector aggregation of GTAP 10 DB
  - Number of cases for n sectors: Sum of the Stirling numbers of the second kind for all $k \leq n$, $\sum_k S(n,k)$,
    - ✓ where $S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n$
- Compare the two kinds of aggregation formulas for substitution elasticities: 1) output-weighted average; 2) theoretically derived GE elasticities by Baqaee & Farhi (2018)
- Evaluate impacts from the increase of capital supply by 10% and the decrease of labor supply by 10%

1-region/5-sector Aggregation of GTAP 10 DB

	AGR	MNU	FOD	INF	SER	Consumption	SUM
AGR	669.9	4,546.4	1,791.4	387.3	115.2	1,734.5	9,244.6
MNU	1,107.7	22,586.2	660.8	7,228.6	4,252.5	14,185.0	50,020.9
FOD	352.6	203.9	1,513.8	665.9	410.0	4,383.7	7,530.0
INF	706.0	4,622.0	918.9	5,077.4	3,576.4	21,072.7	35,973.5
SER	528.6	3,407.8	469.2	4,856.9	10,599.3	32,886.0	52,747.7
Labor	1,953.1	8,665.7	1,154.0	10,895.2	19,685.5		42,353.7
Capital	3,926.8	5,988.7	1,021.9	6,862.2	14,108.8		31,908.3
SUM	9,244.6	50,020.9	7,530.0	35,973.5	52,747.7	74,261.9	
Elasticity	0.22	1.26	1.12	1.60	1.26	1.00	

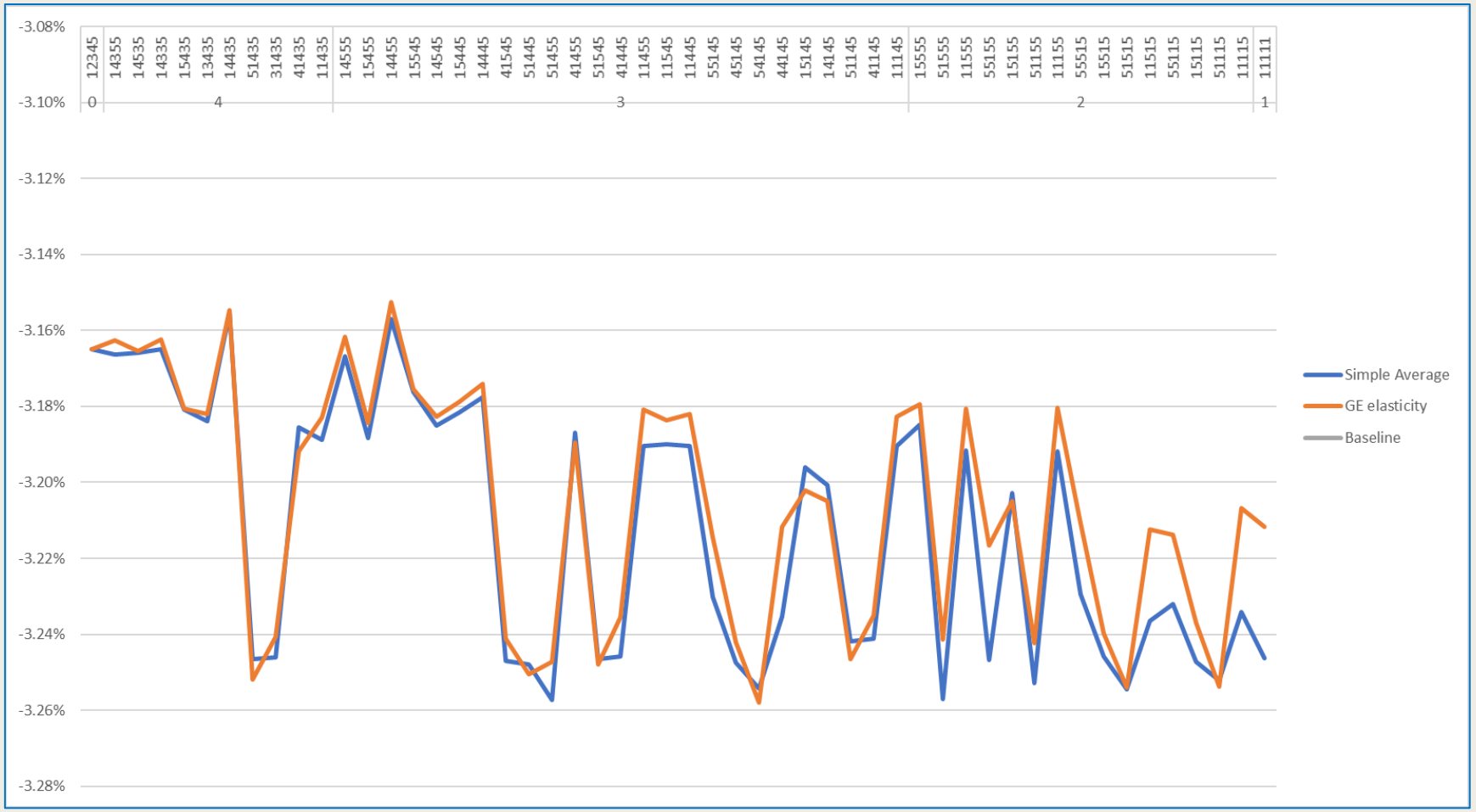
Aggregated elasticities by aggregation scenarios

- % difference from output-weighted average



Real GDP impacts by aggregation scenarios

- % change from the baseline (impacts from +10% of capital & -10% of labor)



Conclusion

- Theoretical GE elasticity formula tends to produce superior outcomes, particularly for higher levels of aggregation
- The gap between the theoretical GE elasticity formula and the output-weighted average is limited for a case of GTAP 10 DB
  - While the theoretical potential of the deviation from the exact aggregated elasticity is large
- Demand for future researches on the elasticity aggregation formula or algorithm for better representations of aggregated CGE models
  - With maintaining a balance of precision of anlysis and computational burden