Chapter 5
SIMPLE-G Model Specification: Mathematical Equations in a Multiscale Market Equilibrium Model

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Understanding the complex link between land, water, and environmental systems is required for addressing sustainability challenges. Earth observation provides valuable insights into these systems. Traditional earth analysis methods often rely on one-dimensional approaches, focusing on individual components like land cover, hydrology, or biogeochemical processes. However, real-world environmental challenges like climate change, water scarcity, and biodiversity loss necessitate a more holistic understanding that integrates these components with human system. Effective analyses require flexible and adaptable models that incorporate human decisions and responses. This chapter introduces the mathematical representation of SIMPLE-G framework, Simplified International Model of agricultural Prices, Land use, and the Environment- Gridded version. This framework can be applied on different grid cell sizes, from very fine resolution to coarse resolutions, and in multiscale analysis. SIMPLE-G addresses the need for a traceable and comprehensive framework by offering a modular and customizable framework that combines various geospatial data sources and modeling techniques. This allows researchers to tailor the model to specific research questions and environmental contexts, with a deeper understanding of the complex dynamics shaping our planet.

This chapter introduces five successively more complex model specifications of SIMPLE-G. Each model is designed for specific research purposes but can be applied to a wide range of questions. Each successive model has new features and new data requirements. Table 5.1 summarizes the diverse models discussed in this chapter. We start with the simplest model, then explain how to add new dimensions, such as new assumptions, new agricultural inputs, or new production technologies. However, new inputs may differ in the degree of spatial mobility or their implications for the spatial scope of their markets. Therefore, new market-clearing

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Table 5.1 SIMPLE-G models in Part III

	Agricultural inputs	New information	New features	Potential application
1	Land, other	Intensive and extensive margins	Regional and local markets	Deforestation, biodiversity
2	Land, water, other	Yield response to water	Subregional markets	Nonland changes
3	Land, water, fertilizer, other	Yield response to fertilizer	Market rigidity	Nitrogen policies
4	Land, water, fertilizer, other	Land conversion, yield response to fertilizer by technology	Multiple activities, land allocation	Water stress, yield, water quality, fertilizer policies
5	Land, surface water, groundwater, equipment, fertilizer, other	Water resources substitutions, irrigation technology	Product differentiation	Water scarcity, water management

conditions and new marketsheds must also be introduced. For example, fertilizer markets are typically spatially connected, while labor markets may be less well connected at the regional level. New parameters will also be added for each new feature, which may require new information to inform the model. For example, introducing fertilizer to a model may require information about yield response to fertilizer, which must be used to inform the new behavioral parameters.

1 Model 1: The Basic Two-Input Model

This section describes the simplest of the gridded models, SIMPLE-G1; each grid cell has one production practice, two agricultural inputs (*R* for natural resources and *H* for manufactured inputs and human services), and one composite crop commodity. Table 5.2 summarizes the equations of the production part of this model. For the demand side, we follow nongridded SIMPLE equations, which are common across all the models described in this chapter (see Table 5.2).

1.1 Production Structure

In SIMPLE-G, we adopt a nested constant elasticity of substitution (CES) structure, which can accommodate varied types of technological information about production processes. Figure 5.1 represents the production structure of the basic model, the local market for the natural resource input, and the regional market for the augmented human input. Here, *R* is a composite of agricultural inputs provided by natural and environmental system (e.g., land and water), and *H* is a composite of agricultural inputs provided by human systems (e.g., labor, capital, and fertilizer). The natural

Table 5.2 Summary of gridded equations for SIMPLE-G-1.2.1

Variable[a]	Equation	Description
Natural resources input market		
q_g^R	$q_g^R + a_g^R = q_g - a_g - \sigma_g\left(p_g^R - a_g^R - p_g - a_g\right)$	Agricultural demand for natural resources
p_g^R	$q_g^R = \eta_g^R p_g^R + s_g^R$	Economic supply of natural resources
Human services and manufactured input markets		
q_g^H	$q_g^H + a_g^H = q_g - a_g - \sigma_g\left(p_g^H - a_g^H - p_g - a_g\right)$	Demand for human and manufactured input
p_g^H	$p_g^H = p_r^H$	Price of human and manufactured input
p_r^H	$q_r^H = \eta_r^H p_r^H + s_r^H$	Supply of human and manufactured input
q_r^H	$Q_r^H = \sum_{g \in r} Q_g^H$ [b]	Total subregional/regional input supply
Crop markets		
q_g	$q_g - a_g = \theta_g^H\left(q_g^H + a_g^H\right) + \theta_g^R\left(q_g^R + a_g^R\right)$	Gridded zero profits
p_g	$p_g = p_r$	Gridded price of crops
$q_r = f(p_r)$	$Q_r = \sum_{g \in r} Q_g$ [b]	Total regional crop supply

[a]This column shows matching variables that are determined by each equation (usually price or quantity) with the matching dimension

[b]These equations are written in levels, not as percentage change

input markets are geographically separate, assuming that they are not directly linked; therefore, the natural input is not mobile. That is, land does not move from one grid cell to another grid cell. However, the human input market is aggregated, reflecting the fact that the grid cells compete for human input. Additional, human input can move among locations, which causes prices for the human input to move together across grid cells.

The single-nest Constant Elasticity of Substitution (CES) gridded production function in Fig. 5.1 comprises three key behavioral equations that are obtained from microeconomic theory based on assumptions of cost minimization, coupled with free entry and exit from these activities.

1.2 Input Demand Equations

In keeping with the analytical approach, model condensation and nonlinear solution strategy described in Chap. 8, we write these equations in linearized (percentage change) form (Dixon 1982). The cost shares are updated over the course of the nonlinear model solution. The following three equations pertain to the top-level nest,

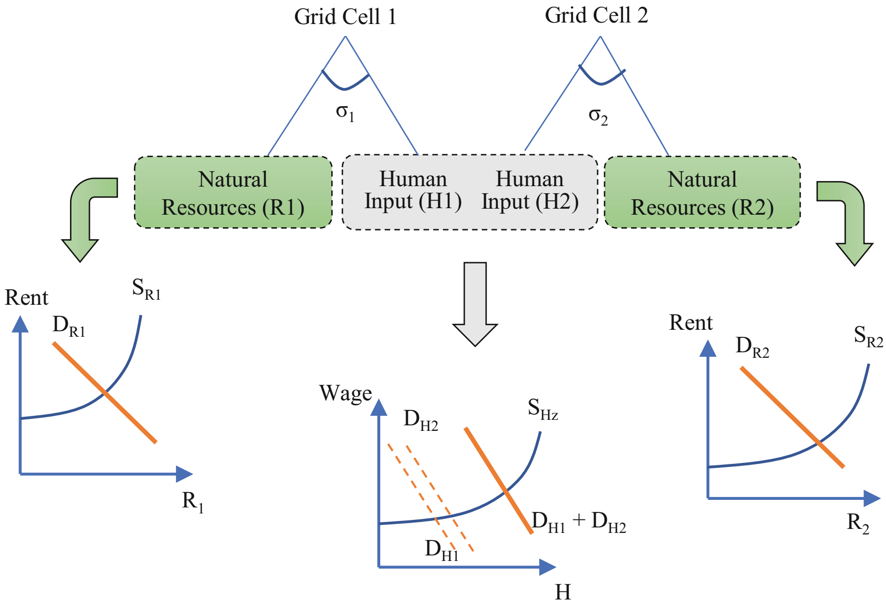

Fig. 5.1 Basic gridded production structure of a simple two-input model and associated markets

in which natural resources (R) and other inputs (H) are combined, in variable proportions, to produce aggregate crop output:

$$q_g^R + a_g^R = q_g - a_g - \sigma_g \left(p_g^R - a_g^R - p_g - a_g \right) \text{ : demand for natural input,} \quad (5.1)$$

$$q_g^H + a_g^H = q_g - a_g - \sigma_g \left(p_g^H - a_g^H - p_g - a_g \right) \text{ : demand for human input,} \quad (5.2)$$

$$p_g + a_g = \theta_g^H \left(p_g^H - a_g^H \right) + \theta_g^R \left(p_g^R - a_g^R \right) \text{ : zero profits,} \quad (5.3)$$

where lowercase variables denote percentage changes in the corresponding uppercase, "levels" variables (i.e., $p = 100(dP/P)$ is the percentage change in crop price and $a = 100(dA/A)$ is the percentage change in total factor productivity). The variables p_g, q_g, and a_g denote the percentage changes in gridded crop price, gridded crop production, and overall crop productivity, respectively. Similarly, p_g^j, q_g^j, and a_g^j denote the percentage changes in input price, input demand, and factor-augmenting productivity, respectively. Finally, θ_g is the share of each input in total crop production costs.

The zero-profit equation is the consequence of our assumption of unrestricted entry and exit from the crop sector. If the output price rises, given unchanged technology and input prices, then the sector will experience excess profits. This will attract new entrants or encourage the expansion of existing producers, which will drive up input prices and drive down output prices until zero pure economic profits are restored. Manipulation of these equations yields the following, equivalent, quantity-based expression of this condition, which we use in the model to facilitate our condensation strategy:

$$q_g - a_g = \theta_g^H \left(q_g^H + a_g^H \right) + \theta_g^R \left(q_g^R + a_g^R \right). \tag{5.4}$$

Thus, the percentage change in demand for natural resources depends on changes in technology (a, a^R), total crop output (q), and the price of natural resources (p^R) relative to an index of all input costs (p). Equation 5.1 shows that the demand for natural resources, q_g^R, will increase with increasing crop production, q_g. As the substitution elasticity is a positive parameter, the demand for natural resources will also increase with a reduction in input price (or cost) of natural resources, p_g^R. Finally, the demand for natural resources will increase with an increase in crop prices. The impact of productivity improvement on factor prices depends on the magnitude of the substitution elasticity and price responses. Holding the input prices constant, an increase in productivity will generally increase both output level and demand for both inputs.

In Chap. 7, we will discuss how the elasticity of substitution, σ, can be calibrated to reproduce grid-cell- and practice-specific agronomic characteristics of crop production. It is evident that a large substitution elasticity will result in a much greater response to, for example, a tax on natural resource use in crop production. Therefore, σ is a key parameter in sustainability analysis.

1.3 Natural Resource Input Supply Equations

Input supply functions are written as mathematical equations that show the relationship between the price of agricultural inputs and the quantity of the input that is supplied to the market by the seller. The supply function for the natural resource inputs is typically written for each grid cell in the following linearized form:

$$q_g^R = \eta_g^R p_g^R + s_g^R, \tag{5.5}$$

where q is the percentage change in quantity supplied, p is the percentage change in price, η is the supply elasticity in the linearized supply function, and s is the shifter of the supply curve. The supply elasticity is a fixed parameter and defined as the percentage change in the quantity supplied divided by the percentage change in the price, holding everything else constant. The parameter of supply elasticity is positive, which means that the quantity supplied increases as the price increases.

1.4 Human and Produced Input Supply Equations

The supply function for human-produced inputs (e.g., labor, capital, fertilizer) is typically written in the following linearized form:

$$q_r^H = \eta_r^H p_r^H + s_r^H, \tag{5.6}$$

where r is an index for regions. Equation 5.6 shows that the total supply of human input in each region depends on regional prices (e.g., wages) and regional supply elasticities. Additionally, a shift in supply at the regional level (s_r) can affect the input market. Note that Eq. 5.6 can be specified for subregions depending on the application, indicating less mobility among human inputs.

1.5 Market Equilibrium Equations

For a natural resource input, the market-clearing condition implies that the gridded economic supply of the natural resource input should be equal to the gridded agricultural economic demand for natural resource input at the equilibrium price. For a human input, the market-clearing condition holds at the regional level. Table 5.2 summarizes the production side equations for Model 1 (SIMPLE-G-1.2.1).

Box 5.1 describes an application of Model 1 in SIMPLE-G-Mini that is designed for understanding the determinants of local responses and decisions about resources.

Box 5.1 SIMPLE-G Mini and AnalyseGE: The Minimodel for Understanding Gridded Production and Gridded Markets

Before moving on to more complex models, it is useful to pause and reflect on what can be learned from working solely with the supply side of the model. (The theory behind this gridded analysis was introduced in Part II.) If the market prices of outputs and variable inputs are known, then the outcome in each grid cell can be treated as an independent solution to a problem in which the market prices and grid-cell-specific policies (e.g., groundwater sustainability policies) are treated as exogenous shocks and we can solve the model for the local prices and quantities. We dub this the "minimodel" since it can be readily solved and analyzed in depth on a grid cell by grid cell basis. Of course, to know what the market price changes are for any comprehensive policy, one must first solve the "maxi-model" (i.e., the full-blown SIMPLE-G model with consumption, trade, and production in the rest of the world). This is therefore a sequential exercise.

The great advantage of working with the minimodel is that we have access to theoretical and numerical results that can facilitate insights and help us

(continued)

Box 5.1 (continued)
identify key parameters in the model and behavioral assumptions governing the outcomes (recall Part II.2). Thus, for example, referring to Eq. 3.9 of Box 3.1 (reproduced in a simplified form that assumes $a = 0$), we can see that the change in production in a given grid depends on three components: the extensive margin, the intensive margin, and the conservation policy (see also Ray et al. 2023). Each of these components hinges on grid cell parameters and data. % change in production (Q): $q = p^* \frac{\nu_R}{\theta_R} \Gamma_R + p^* \frac{\sigma \theta_H}{\theta_R} \Gamma_\sigma - \phi_R \Gamma_R$.

Therefore, any surprising outcome at the grid-cell level can be traced back to the underlying parameter and data combinations, drawing our attention back to the foundational work of estimating these parameter values and data: Are these values sound? If so, we may have uncovered an important finding. If not, then we need to revisit our data and/or parameters, which, in turn, will alter the grid-cell outcome.

To facilitate analysis with the minimodel, we use the GEMPACK software "AnalyseGE," which allows the user to load the solution, database, and parameter values into the algebraic representation of the model in order to quickly evaluate every component of an equation, such as that shown above, so that the analyst can quickly identify which term and which parameter is driving the unusual outcome. The concept of a minimodel is original to SIMPLE-G modeling, and this novel invention is important step forward in global-to-local sustainability analysis. It is developed in more detail in the application in Chap. 11.

2 Model 2: Introducing Subregional Input Markets

Model 2 introduces subregional markets and multiple inputs, which requires slightly different production functions and a slightly different supply structure. The production function is written in a nested CES form, a CES function of multiple composite inputs. In a nested CES function, the inputs to a production process are grouped into different nests that reflect the multiple stages of a cost-minimizing producer's decision making. For example, in Fig. 5.2, the farmer first decides how much irrigation water to apply to a field, after which they then decide how much of the human inputs to apply. The land–water composite input will have a price index and a quantity index (usually weighted based on values). Then, the composite land–water input is included in the top nest, where it is combined with human inputs.

In a nested CES function, demand for each individual input is written as a function of the demand for the composite input, which is linked to the scale of production. More output requires additional composite input. Here, the demand for the composite land–water input can be expressed as

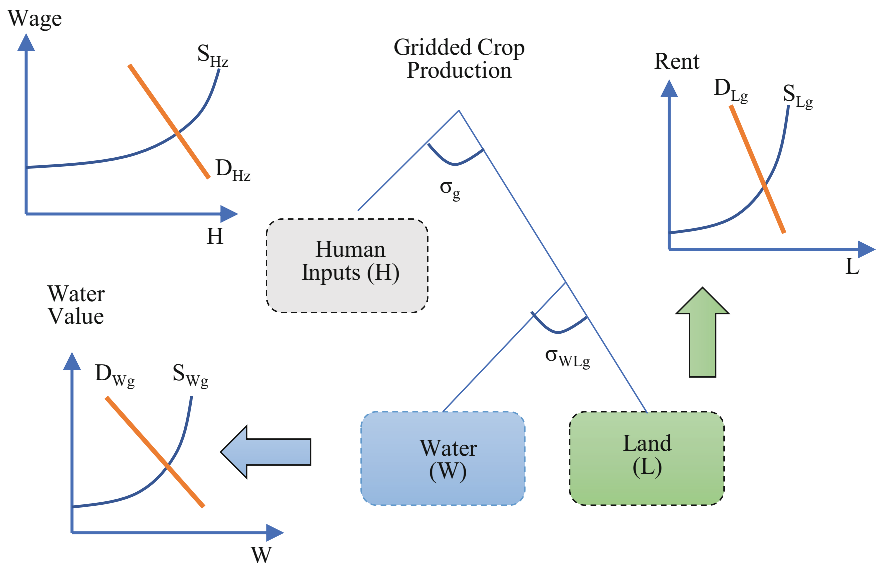

Fig. 5.2 Basic production structure of a SIMPLE-G model with three-input nested CES
The presence of two natural resource inputs, land and water, allows for different availability of these two critical inputs

$$q_g^{LW} = q_g - a_g - \sigma_g \left(p_g^{LW} - p_g - a_g \right), \tag{5.7}$$

where LW is the land–water composite, q is percentage change in quantity, and p is the percentage change in price. The structure of Eq. 5.7 is similar to the demand for natural resources in Model 1: The demand for land–water composite depends on the production scale effect, $q_g - a_g$, and the substitution effect, $-\sigma_g \left(p_g^{LW} - p_g - a_g \right)$.

The demand for individual land and water inputs is derived under the assumption of cost minimization, subject to the CES functional form:

$$q_g^L + a_g^L = q_g^{LW} - \sigma_g^{LW} \left(p_g^L - a_g^L - p_g^{LW} \right) \tag{5.8}$$

$$q_g^W + a_g^W = q_g^{LW} - \sigma_g^{LW} \left(p_g^W - a_g^W - p_g^{LW} \right) \tag{5.9}$$

$$p_g^{LW} = \beta_g^L p_g^L + \beta_g^W p_g^W \tag{5.10}$$

Equation 5.10 introduces a price index for the land–water composite, which is a weighted average of percentage changes in land rent and value of water. The cost shares sum to 1 and are shown by β, which represents the value shares in the land–water composite. This is different from θ, which was introduced in Model 1 to represent the input cost shares in total costs.

In a subregional market for inputs, the constant elasticity supply function for the human inputs (e.g., labor, capital, and fertilizer) is typically written in the following linearized form:

$$q_z^H = \eta_z p_z^H + s_z^H, \tag{5.11}$$

where z is an index that denotes the scope of the human-produced input markets, ranging from the grid-cell to the regional level. Depending on factor mobility, there are three potential relationships between gridded prices and regional prices for the human inputs:

$$\text{Equilibrium}: \begin{cases} p_g^H = p_z^H = p_r^H & \text{if } z = r, \text{and perfect mobility} \\ p_g^H = p_z^H \neq p_r^H & \text{if } z \subset r, \text{and perfect mobility} \\ p_g^H \neq p_z^H \neq p_r^H & \text{if } z \subset r, \text{and imperfect mobility} \end{cases}. \tag{5.12}$$

Depending on the size of grid cells and assumptions about mobility, index z can show one of the following:

- All the grid cells in a region are paid a uniform input price (or move at the same rate) as a result of perfect mobility of inputs. There is no mobility across regions.
- A subset of grid cells in a region faces a uniform input price for the subregion and perfect mobility of inputs within the subregion, but there is no mobility across subregions. Input prices are equal for all grid cells in a subregion but not for all grid cells in a region.
- An individual grid cell: Assuming no mobility and heterogeneous unrelated markets for inputs. There is no mobility across grid cells. Input prices can vary for grid cells in a subregion and in a region.

Table 5.3 summarizes the equations for Model 2 (SIMPLE-G-1.3.1).

2.1 Factor Mobility

The economic production factors (e.g., land, water, and human inputs) can be broadly categorized into mobile and immobile factors. In SIMPLE-G, this corresponds to spatial mobility (i.e., the ability of production factors to move freely among geographical locations). This mobility is essential for efficient resource allocation. For an efficient outcome, the marginal productivity of a given input must be equalized across locations and activities. Since cost-minimizing producers equate the value of marginal productivity for each input to its price, an efficient allocation of the input is achieved when the changes in input prices are equal in all grid cells. This also reduces spatial income inequality and promotes social mobility. Capital in the form of machinery can be mobile because it can be moved or used in

Table 5.3 Gridded equations for SIMPLE-G-1.3.1 with three inputs

Variable[a]	Equation	Description
p_z^H	$q_z^H = \eta_z p_z^H + s_z^H$	Subregional supply of human input
p_g^W	$q_g^W = \eta_g^W p_g^W + s_g^W$	Gridded supply of water input
p_g^L	$q_g^L = \eta_g^L p_g^L + s_g^L$	Gridded supply of land input
q_g^H	$q_g^H + a_g^H = q_g - a_g - \sigma_g \left(p_g^H - a_g^H - p_g - a_g\right)$	Gridded demand for human input
q_g^W	$q_g^W + a_g^W = q_g^{LW} - \sigma_g^{LW} \left(p_g^W - a_g^W - p_g^{LW}\right)$	Gridded demand for water input
q_g^L	$q_g^L + a_g^L = q_g^{LW} - \sigma_g^{LW} \left(p_g^L - a_g^L - p_g^{LW}\right)$	Gridded demand for land input
q_g^{LW}	$q_g^{LW} = q_g - a_g - \sigma_g \left(p_g^{LW} - p_g - a_g\right)$	Gridded demand for land–water composite (*LW*) input
p_g^{LW}	$p_g^{LW} = \beta_g^L p_g^L + \beta_g^W p_g^W$	Gridded price for *LW*
q_g	$q_g - a_g = \theta_g^L \left(q_g^L + a_g^L\right) + \theta_g^W \left(q_g^W + a_g^W\right) + \theta_g^H \left(q_g^H + a_g^H\right)$	Gridded zero profits
$q_r = f(p_r)$	$Q_r = \sum_{g \in r} Q_g$ [b]	Total regional crop supply
q_z^H	$Q_z^H = \sum_{g \in z} Q_g^H$ [b]	Total subregional input supply
p_g^H	$p_g^H = p_z^H$	Gridded price of mobile input
p_g	$p_g = p_r$	Gridded price of crops

[a]This column shows matching variables that is determined by each equation (usually price or quantity) with matching dimension
[b]These equations are written in levels not the percentage change

different locations. Technology is typically also mobile within a region because it can be transferred from one location to another. Examples of immobile factors of production include land, natural resources, and infrastructure. Land, natural resources, and infrastructure are considered immobile due to their nature and high cost of moving them. Of course, there can be interactions. For example, the construction of canals can facilitate the movement of water from one location to another.

For immobile factors, demand and supply must be in equilibrium in each individual grid cell. The prices of immobile factors can vary significantly by location, and they are not directly related. For mobile factors, farmers compete across a wide geographical range and synchronize mobile input prices across space. For example, fertilizer prices are usually very similar across locations; but land rents are usually different.

2.2 Marketsheds

A marketshed is a geographical area that represents the catchment area for a particular market. It is the area from which a market draws its supplies and demands. The size and shape of a marketshed can vary depending on the type of market, the market's location, and transportation infrastructure in the area.

For some research questions, it is necessary to determine the scope of factor mobility. For example, while the labor can typically move freely from one grid cell to neighbor grid cells or other locations within a certain distance, it is not easy to move across national borders. The concept of a labor-market commuting zone is related to the concept of a marketshed in that it defines a geographical area from which workers commute to a particular location. This information can be used to estimate the size of the potential labor force for a particular location. This topic comes up in the application documented in Chap. 13.

3 Model 3: Introducing Factor Market Rigidity

In Model 3, we introduce fertilizer as a distinct agricultural input for the purposes of analyzing sustainability related to water quality and food security. We also introduce factor market rigidity. Figure 5.3 illustrates the production tree structure and markets.

In this system, nitrogen (N) fertilizer use is determined endogenously in the model by considering relative prices, technology, substitution possibilities, and overall output level. The potential for nitrogen–land substitution is grid-cell- and activity-specific and is obtained from agronomic yield functions. The price of N fertilizer is determined at the regional level through a market-clearing condition wherein regional supply equals demand, which is in turn determined by aggregating N use across all grid cells and practices.

The four-input model has four input supply functions, each of which depends on the elasticities and shifter variables at the relevant scale. These functions can be expressed as

$$q_g^L = \eta_g^L p_g^L + s_g^L, \tag{5.13}$$

$$q_g^W = \eta_g^W p_g^W + s_g^W, \tag{5.14}$$

$$q_r^N = \eta_r^N p_r^N + s_r^N, \tag{5.15}$$

$$q_z^H = \eta_z p_z^H + s_z^H, \tag{5.16}$$

where L, W, N, H are land, water, N fertilizer, and other human-produced inputs, respectively; g, z, and r are indices for the grid cells, marketsheds, and regional scopes; q is the percentage in quantity supplied; η is the supply elasticity; p is the

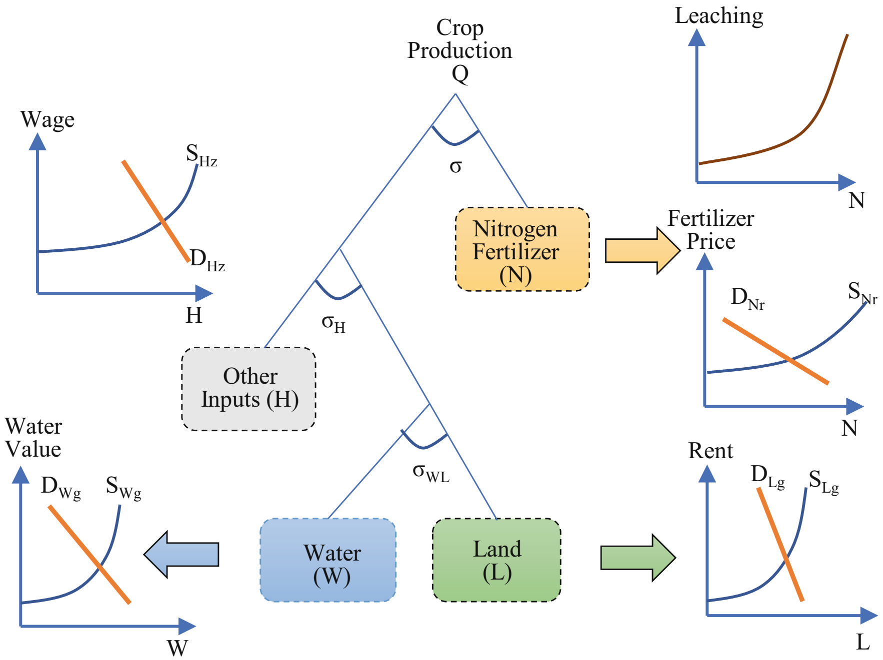

Fig. 5.3 Basic production structure with four inputs
The addition of fertilizer at the top of the production tree allows for the incorporation of information about the responsiveness of crop output to additional fertilizer applications. The leaching function varies by grid cell

percentage change in market price; and s is the percentage change due to exogenous shifters.

The demand for each composite and individual input is derived following the nested CES structure summarized in Table 5.4.

Explicitly modeling market rigidity in gridded economic models is important for several reasons. First, it allows researchers to better understand the geographical impact of conservation and sustainability policies, which often lead to leakage and other unanticipated effects. Second, it provides a framework for better understanding the market-mediated effects. Third, it allows policy makers to design more effective policies to address market rigidity and its implications for conservation policies.

To introduce rigidity in markets, we use a constant elasticity of transformation (CET) function, which is analogous to the CES function; rather than determining demands, it governs supply response to changing relative prices. In linearized form, it is expressed as

$$q_g^H = q_z^H - \tau_z^H \left(p_g^H - p_z^H\right), \tag{5.17}$$

Table 5.4 Gridded equations for SIMPLE-G-1.4.1 multiscale markets

Variable[a]	Equation	Description
p_g^L	$q_g^L = \eta_g^L p_g^L + s_g^L$	Supply of land input
p_g^W	$q_g^W = \eta_g^W p_g^W + s_g^W$	Supply of water input
p_r^N	$q_r^N = \eta_r^N p_r^N + s_r^N$	Supply of fertilizer input
p_z^H	$q_z^H = \eta_z p_z^H + s_z^H$	Supply of other inputs
q_g^L	$q_g^L + a_g^L = q_g^{LW} - \sigma_g^{LW}\left(p_g^L - a_g^L - p_g^{LW}\right)$	Demand for land input
q_g^W	$q_g^W + a_g^W = q_g^{LW} - \sigma_g^{LW}\left(p_g^W - a_g^W - p_g^{LW}\right)$	Demand for water input
q_g^N	$q_g^N + a_g^N = q_g - a_g - \sigma_g\left(p_g^N - a_g^N - p_g - a_g\right)$	Demand for fertilizer input
q_g^H	$q_g^H + a_g^H = q_g^{HLW} - \sigma_g^{HLW}\left(p_g^H - a_g^H - p_g^{HLW}\right)$	Demand for other inputs
q_g^{LW}	$q_g^{LW} + a_g^{LW} = q_g^{HLW} - \sigma_g^{HLW}\left(p_g^{LW} - a_g^{LW} - p_g^{HLW}\right)$	Demand for land–water composite (*LW*)
q_g^{HLW}	$q_g^{HLW} = q_g - a_g - \sigma_g\left(p_g^{HLW} - p_g - a_g\right)$	Demand for composite *HLW*
p_g^{LW}	$p_g^{LW} = \beta_g^L p_g^L + \beta_g^W p_g^W$	Price for *LW*
p_g^{HLW}	$p_g^{HLW} = \beta_g^H p_g^H + \beta_g^{LW} p_g^{LW}$	Price for composite *HLW*
q_g	$q_g - a_g = \theta_g^L\left(q_g^L + a_g^L\right) + \theta_g^W\left(q_g^W + a_g^W\right) + \theta_g^N\left(q_g^N + a_g^N\right) + \theta_g^H\left(q_g^H + a_g^H\right)$	Zero profits
q_r	$Q_r = \sum_{g \in r} Q_g$ [b]	Total crop supply
q_z^H	$Q_z^H = \sum_{g \in z} Q_g^H$ [b]	Total input supply
p_g^H	$q_g^H = q_z^H - \tau_z^H\left(p_g^H - p_z^H\right)$	Supply of other inputs
p_g^N	$p_g^N = p_z^N$	Price of nitrogen fertilizer
p_g	$p_g = p_r$	Price of crops

[a]This column shows matching variables that are determined by each equation (usually price or quantity) with matching dimension

[b]These equations are written in levels, not in percentage change

where q_g^H is the percentage change in supply of input to grid cell g; q_z^H is the percentage change in total supply in marketshed z; $\tau_z^H \leq 0$ is the transformation elasticity parameter determining the responsiveness of transformation in the supply function; p_g^H is the factor price (wage) in location g; and p_z^H is the marketshed average factor price. Equation 5.17 shows that the supply of an input to one location depends on the factor price (wage) in other locations in addition to the local price. However, as long as the transformation elasticity is not infinite, factor prices can differ across grid cells within the marketshed.

3.1 Market Rigidity

Market rigidity refers to the inability of market quantities to adjust to changes in spatially disaggregated supply and demand conditions. Spatial market rigidity is important in global-gridded economic modeling because it can have a significant impact on a variety of economic outcomes, including wages, employment, land use, and agricultural production patterns. In the case of farmworkers, labor market rigidity refers to the fact that workers are not perfectly mobile across jobs and locations. In the case of capital, market rigidity can arise from the inability to move agricultural capital to another location. This typically depends on the share of immobile installed capital and infrastructure. Farm capital in the form of machinery is easier to move to another location than buildings and irrigation canals are. Factor immobility gives rise to spatially heterogeneous prices for the same input, with prices being (weakly) related.

Labor market immobility within a marketshed is an important rigidity explored in Chap. 13. This immobility can stem from market forces, institutions, or individual preferences. For example, labor market rigidity can arise due to geographical barriers, employment contracts, local attachments, social/cultural differences, language barriers, or simply lack of information on job opportunities and wage differentials. This can make it difficult for employers and employees to adjust to changes in demand or supply and can lead to farmworker wage disparities across locations, as those in high-yield areas can ask for higher compensations.

The CET function outlined above is an adaptable tool used in SIMPLE-G to model a variety of types of factor market rigidity. By including the difference in degree of mobility between different locations, economists can gain insights into the impacts of global and local changes on the spatial reallocation of resources and agricultural production.

4 Model 4: Multiple Activities and Land Allocation

Thus far, we have defined only one production function for each grid cell. Here, we introduce two activities in each grid cell, with different production functions, cost structures, input usages, yields, and revenue streams in each. Most of the applications in Part IV distinguish between irrigated and rainfed production, although this approach could also be used to explore other differences (e.g., traditional and commercial production). Figure 5.4 divides cropland area into rainfed and irrigated practices, and the different land rents for these two activities are endogenously determined in the model. Land rents are also grid-cell-specific and depend on local biophysical characteristics, irrigation yield gaps, prices, and the technologies available to each production unit. The allocation of land to rainfed and irrigated production in response to a policy shock is determined by their relative returns (i.e., land rental rates), which are determined endogenously for each grid cell on the basis of

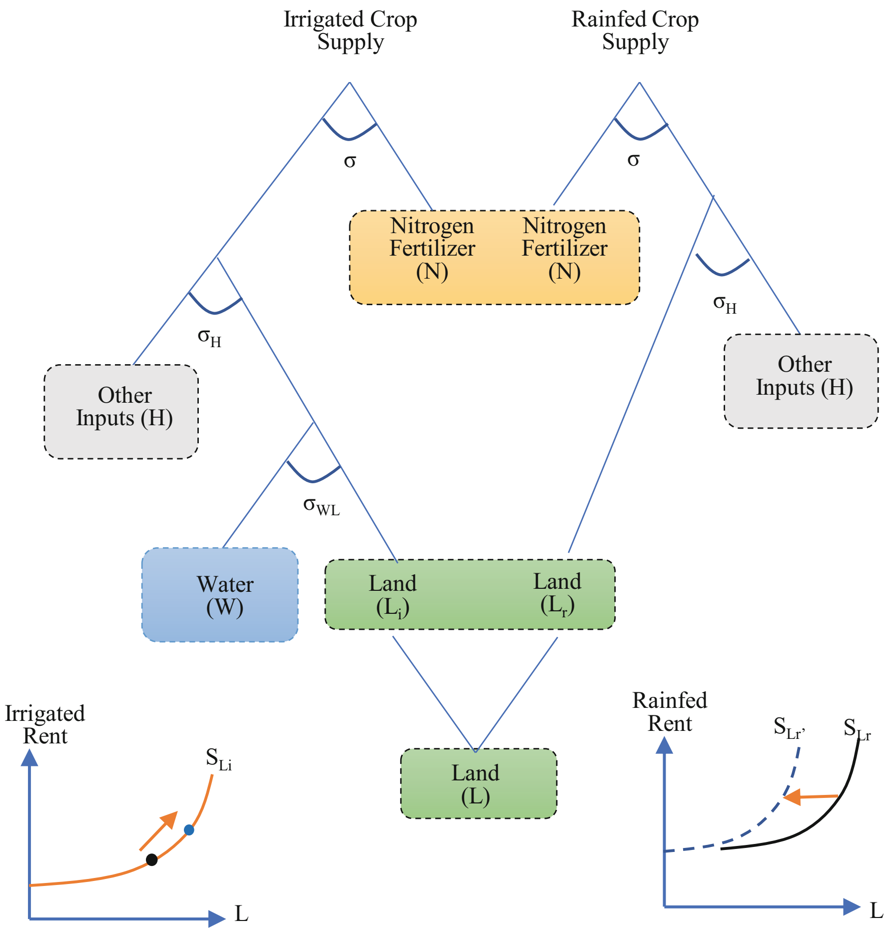

Fig. 5.4 Basic production structure with land allocation
The presence of two different practices in each grid cell allows for cropland for to be converted from one use to another based on the CET (constant elasticity of transformation) parameter

movement between land uses determined by a constant elasticity of transformation function (Ahmed et al. 2008). The key parameter in this function is the elasticity of transformation between irrigated and rainfed cropland. This elasticity measures the responsiveness of the rainfed–irrigated crop mix ratio to changes in relative returns to the two activities. A larger absolute value of this elasticity indicates an easier transformation of cropland between irrigated and rainfed categories. The conversion of land from rainfed to irrigated cropping is heavily influenced by institutional and biophysical constraints. In the United States, these constraints depend critically on state and local water laws, which vary greatly by region.

We use a quantity-preserving CET function for this particular application (van der Mensbrugghe and Peters 2020). A QCET function is a function that can be used to allocate land to different land cover types (e.g., agriculture, forestry, urban development) or agricultural practices (e.g., irrigated and rainfed land). This function allows us to account for the different characteristics of land, such as yield. In the context of land allocation, the CET function is used to model the conversion of various land uses. The QCET function is important because it preserves the total quantity of land. The traditional CET does not have this feature. Instead, it imposes a constraint on the overall economic value of the land. This quantity-preserving feature is important when tracking total physical units of land area is of central importance.

The multiactivity model introduces a new index for each variable. Here, we introduce l for land types, and the demand for land type l is written as follows:

$$q^L_{g,l} + a^L_{g,l} = q^{LW}_{g,l} - \sigma^{LW}_{g,l}\left(p^L_{g,l} - a^L_{g,l} - p^{LW}_{g,l}\right). \tag{5.18}$$

Equation 5.18 is similar to the land demand in Model 3, with the addition of the land type index. We also introduce new composite inputs: $q^{LW}_{g,l}$, which is a land and water composite, and $q^{HLW}_{g,l}$, which a composite of human input, land, and water. The default model has two land types: irrigated and rainfed. To allocate land to different land types, we need to introduce three new equations:

$$q^L_{g,l} = q^L_g - \tau^L_g\left(p^L_{g,l} - p^{\dagger L}_{g}\right), \tag{5.19}$$

$$p^{\dagger L}_{g} = \sum_l \theta^{\dagger L}_{g,l} p^L_{g,l}, \tag{5.20}$$

$$p^L_g + q^L_g = \sum_l \theta^L_{g,l}\left(p^L_{g,l} + q^L_{g,l}\right), \tag{5.21}$$

where $q^L_{g,l}$ is the percentage change in supply of land to different land types; τ^L_g is the transformation elasticity representing the flexibility in land conversion; $p^{\dagger L}_{g}$ is the weighted average change in land rents, where the weights are defined as quantity shares; $\theta^{\dagger L}_{g,l}$ is the quantity share; and $\theta^L_{g,l}$ is the value share in the land allocation structure. Together, Eqs. 5.19–5.21 comprise the QCET supply system.

4.1 The Advantage of Splitting Rainfed and Irrigated Production

Irrigation water is another focal point of SIMPLE-G. Irrigation water supply and demand are endogenously determined for each grid cell, but they are linked to exogenous environmental factors. For example, heat stress may increase the water requirements of crops grown in a grid cell, or a drought may reduce the environmental supply of water. Hydrological dynamics are not directly modeled and are treated exogenously. However, SIMPLE-G can be readily paired with a hydrological model to shed light on, for example, the economic consequences of changing basin-level water scarcity or interbasin transfers of water (Liu et al. 2017; Woo et al. 2022).

Water withdrawals are endogenously determined through the interaction of supply constraints and irrigation demand for crop production. Demand for water depends on the irrigation area, production levels, technology, and relative prices. This includes likely adaptation channels and adjustment mechanisms. We consider change in irrigation extent (Haqiqi and Hertel 2019), crop production location, change in irrigation technology, change in water intensity, and trade (Haqiqi and Hertel 2016).

4.2 Using the QCET to Govern Cropland Supply

Just as cropland is allocated across irrigated and rainfed production, the allocation of total land area between cropland and other uses (e.g., pastureland, forests, and other land cover types) can also be governed by a quantity-preserving CET (QCET) function. Cropland supply is a function of the CET parameter and the share of cropland in the grid cell and varies with the cropland rental rate relative to the returns to other uses. In SIMPLE-G, the other land uses are not explicitly modeled, so those land rental rates are deemed exogenous. However, they may be shocked to simulate the impact of intensified competition for land. An important feature of the QCET function is that as cropland expands and covers more of the available land in the grid cell, the cropland supply response diminishes and eventually falls to zero when the entire grid cell is devoted to crop production. The CET parameter can be calibrated to estimated elasticities of current cropland supply (see Chap. 7). In some of the applications featured in Part IV, the QCET function is replaced by a simple cropland supply elasticity, which does not explicitly account for the origin of this cropland or the use to which unused cropland might revert. Table 5.5 summarizes the production-side equations for Model 4 (SIMPLE-G-2.4.1).

5 Model 5: Introducing Product Differentiation

In economics, product differentiation refers to the existence of different varieties of a product that are perceived by consumers as being imperfect substitutes, due to a variety of factors including physical attributes, brand reputation, or quality. Product differentiation opens the possibility of commodity prices varying by product origin and composition, with significant implications for gridded modeling in the output market and input markets. This approach is widely used in international trade (Armington 1969), and it has proven effective for modeling the flows of goods

Table 5.5 Gridded equations for SIMPLE-G-2.4.1

Variable[a]	Equation	Description
p_g^L	$q_g^L = \eta_g^L p_g^L + s_g^L$	Gridded supply of land input
p_g^W	$q_g^W = \eta_g^W p_g^W + s_g^W$	Gridded supply of water input
p_r^N	$q_r^N = \eta_r^N p_r^N + s_r^N$	Regional supply of fertilizer input
p_z^H	$q_z^H = \eta_z p_z^H + s_z^H$	Supply of human input
$q_{g,l}^L$	$q_{g,l}^L + a_{g,l}^L = q_{g,l}^{LW} - \sigma_{g,l}^{LW}\left(p_{g,l}^L - a_{g,l}^L - p_{g,l}^{LW}\right)$	Gridded demand for land input
$q_{g,l}^W$	$q_{g,l}^W + a_{g,l}^W = q_{g,l}^{LW} - \sigma_{g,l}^{LW}\left(p_{g,l}^W - a_{g,l}^W - p_{g,l}^{LW}\right)$	Gridded demand for water input
$q_{g,l}^N$	$q_{g,l}^N + a_{g,l}^N = q_{g,l} - a_{g,l} - \sigma_{g,l}\left(p_{g,l}^N - a_{g,l}^N - p_{g,l} - a_{g,l}\right)$	Gridded demand for fertilizer input
$q_{g,l}^H$	$q_{g,l}^H + a_{g,l}^H = q_{g,l}^{HLW} - \sigma_{g,l}^{HLW}\left(p_{g,l}^H - a_{g,l}^H - p_{g,l}^{HLW}\right)$	Gridded demand for other inputs
$q_{g,l}^{LW}$	$q_g^{LW} + a_{g,l}^{LW} = q_{g,l}^{HLW} - \sigma_{g,l}^{HLW}\left(p_{g,l}^{LW} - a_{g,l}^{LW} - p_{g,l}^{HLW}\right)$	Gridded demand for land–water composite (*LW*)
$q_{g,l}^{HLW}$	$q_{g,l}^{HLW} = q_{g,l} - a_{g,l} - \sigma_{g,l}\left(p_{g,l}^{HLW} - p_{g,l} - a_{g,l}\right)$	Gridded demand for composite *HLW*
$p_{g,l}^{LW}$	$p_{g,l}^{LW} = \beta_{g,l}^L p_{g,l}^L + \beta_{g,l}^W p_{g,l}^W$	Gridded price for *LW*
$p_{g,l}^{HLW}$	$p_{g,l}^{HLW} = \beta_{g,l}^H p_{g,l}^H + \beta_{g,l}^{LW} p_{g,l}^{LW}$	Gridded price for composite *HLW*
$q_{g,\ l}$	$q_{g,l} - a_{g,l} = \theta_{g,l}^L\left(q_{g,l}^L + a_{g,l}^L\right) + \theta_{g,l}^W\left(q_{g,l}^W + a_{g,l}^W\right) + \theta_{g,l}^N\left(q_{g,l}^N + a_{g,l}^N\right) + \theta_{g,l}^H\left(q_{g,l}^H + a_{g,l}^H\right)$	Gridded zero profits
$p_{g,l}^L$	$q_{g,l}^L = q_g^L - \tau_g^L\left(p_{g,l}^L - p^{\dagger L}_g\right)$	QCET land allocation
$p^{\dagger L}_g$	$p^{\dagger L}_g = \sum_l \theta^{\dagger L}_{g,l} p_{g,l}^L$	QCET price index
q_g^L	$p_g^L + q_g^L = \sum_l \theta_{g,l}^L\left(p_{g,l}^L + q_{g,l}^L\right)$	Ensure revenue fully exhausted
q_z^H	$Q_z^H = \sum_{g\in z}\sum_l Q_{g,l}^H$ [b]	Total subregional input supply
q_r^N	$Q_r^N = \sum_{g\in r}\sum_l Q_{g,l}^N$ [b]	Total subregional input supply
q_r	$Q_r = \sum_{g\in r}\sum_l Q_{g,l}$ [b]	Total regional crop supply
$p_{g,l}^H$	$p_{g,l}^H = p_z^H$	Gridded price of mobile input
$p_{g,l}^N$	$p_{g,l}^N = p_r^N$	Gridded price of fertilizer
$p_{g,\ l}$	$p_{g,\ l} = p_r$	Gridded price of crops

[a]This column shows matching variables that are determined by each equation (usually price or quantity) with matching dimension

[b]These equations are written in levels, not in percentage change

between countries and regions (Tinbergen 1962; Anderson 1979; Hillberry and Hummels 2003, 2008; Chaney 2008, 2018; Hillberry and Hummels 2013).

In SIMPLE-G, the composite crop output from any given grid cell is likely unique in its product composition. While we call them all "crops" and assume that their returns move in tandem over the long run, they do in fact differ. Indeed, the Food and Agriculture Organization of the United Nations (FAO) identifies 175 different crop categories; even at this fine level of detail, there are still some composite crop types. In short, it is not possible—particularly in a SIMPLE model—to distinguish all the different crop types. However, many of these differences are likely to matter for consumers. For example, within the product category of wheat, hard red winter wheat is used for baking purposes and durum wheat is used to make pasta. Both are listed as wheat, but they are, in fact, different products in the eyes of food manufacturers and consumers, generally grown in different locations. For this reason, the SIMPLE model allows the prices of crops from different locations to differ. In some versions (e.g., Models 1–4), this differentiation is applied only at the regional level and therefore governs international trade. However, Model 5 also applies this differentiation regionally to capture differences in product composition across grid cells and individual countries (Haqiqi et al. 2022, 2023).

5.1 Modeling Product Differentiation

The equilibration of supply and demand for crops occurs at the level of market regions. Within the market regions in SIMPLE, crop demands are an aggregate of four end uses: direct consumption, food processing, livestock demand, and biofuels. Demands may be satisfied from either domestic or global markets, depending on relative prices, following the method suggested by Armington (1969), which results in imperfect substitution between domestic and foreign products. Symmetrically, on the supply side, producers "transform" their products imperfectly between domestic and global markets. They do not shift completely from the domestic to the foreign market when a small price difference emerges. This permits us to calibrate the model to observed data in which similar products are both imported and exported from the same country.

Accommodating product differentiation in the crop market requires four new equations. The main subregional Armington equations are expressed as

$$q_g = q_s - \varphi_s (p_g - p_s), \tag{5.22}$$

$$q_s = q_r - \kappa_r (p_s - p_r), \tag{5.23}$$

where q_g is the demand for crops produced in grid cell g; q_s is the demand for all the crops produced in subregion s; φ_s is the substitution elasticity parameter that determines the substitutability based on preferences; q_r is total demand for crops produced in region r; and κ_r is a parameter that determines the substitutability between subregional crop composites. For example, in Fig. 4.2, the parameter φ determines the degree of similarity of crops within each subregion. In Fig. 4.2, the parameter κ illustrates the imperfect substitutability across subregions and between products coming out of subregions.

5.2 Modeling Differentiated Inputs

Model 5 also introduces differentiated inputs, particularly for irrigation water, reflecting the fact that groundwater and surface water have different physical and economic characteristics. Differences in physical characteristics (e.g., salinity and mineral content) can affect their suitability for different uses. For example, groundwater is often preferred for drinking water because it is typically cleaner than surface water. However, surface water is often preferred for irrigation because it may be more readily available. In addition to their physical characteristics, groundwater and surface water also have different economic characteristics. Groundwater extraction is typically more expensive than surface water extraction, but it is also more reliable. Surface water is typically less expensive to extract than groundwater, but it is also more susceptible to contamination and drought. The different physical and economic characteristics of groundwater and surface water lead to product differentiation in the market for water. Model 5 captures this differentiation by introducing the two distinct sources of irrigation water and treating them as imperfect substitutes (Fig. 5.5).

5.3 Nested CES Production Function for Model 5

Figure 5.5 presents the two nested CES production functions for each grid cell in Model 5. As in Model 4, we distinguish between rainfed and irrigated activities, and the top nest determines the yield response to fertilizer applications. The "other inputs" are a composite of water, land, and the remaining variable inputs. This is followed by a CES nest combining land and irrigation water. If crop output is strictly proportional to the amount of irrigation water delivered, then the elasticity of substitution between land and irrigation water is zero (i.e., no deficit irrigation). On the other hand, if a reduction in water delivered to the crop is not accompanied by a proportionate reduction in output, then this elasticity is greater than zero and it captures the potential for deficit irrigation (i.e., achieving the same output level with less water but more land). The next CES nest in Fig. 5.5 combines irrigation water with equipment. The associated elasticity of substitution at the bottom of this production tree reflects the potential for conserving irrigation water through investments in, for example, drip irrigation to replace sprinkler or canal-based irrigation capital. This is a key sustainability parameter discussed in Chap. 7.

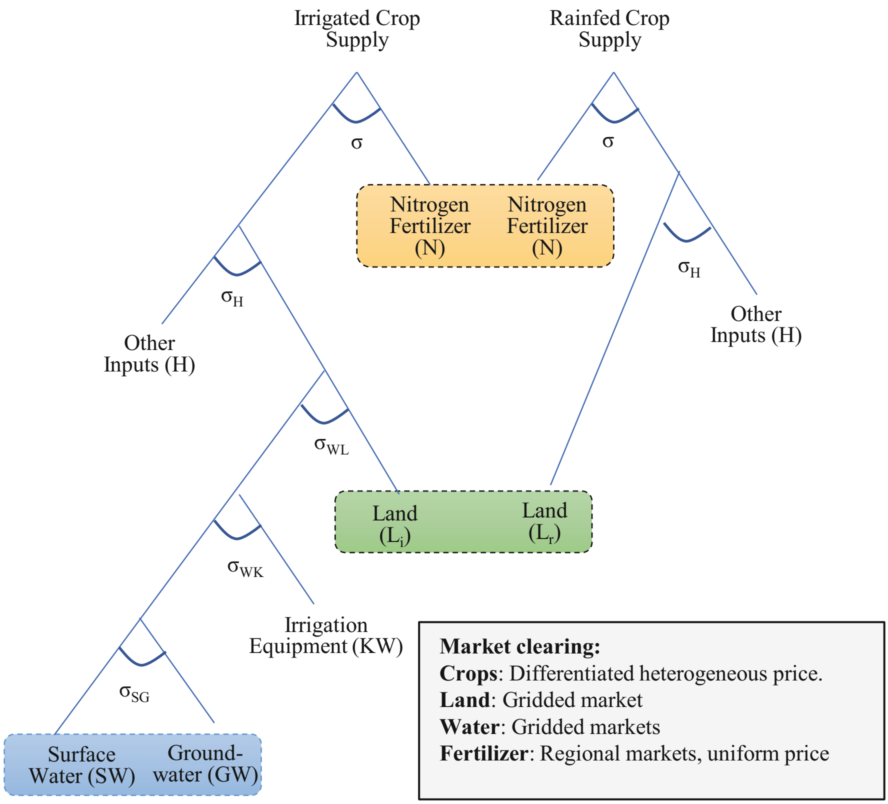

Fig. 5.5 Structure of crop production at each grid cell
Elasticities of substitution are denoted by σ. The equilibrium quantity and price of land and water are determined at the grid-cell level. Irrigated and rainfed practices compete for land. Land supply depends on total cropland supply and the elasticity of transformation between irrigated and rainfed land

The final CES nest in Fig. 5.5 combines surface and groundwater to create an irrigation water composite. The rationale for this nesting is that surface and groundwater extractions often coexist in a given grid cell, despite differences in cost. As previously noted, the two sources of water offer farmers different characteristics. Groundwater, for example, is available on demand and is largely independent of current weather conditions. Surface water must be delivered to the farm, often by infrastructure that is out of the farmer's control. The equations in Table 5.6 describe the substitution possibilities at each level in the production "tree."

Crop production is the result of representative producers' profits maximization, subject to technology, prices, policies, and resource constraints. The crop production technologies (both rainfed and irrigated production) in each grid cell allow for substitution between N fertilizer, water, land, and other inputs (an aggregate of capital, labor, other chemicals, and energy). The particular mix of inputs employed

Table 5.6 Gridded equations for SIMPLE-G-2.6.1

	Equation	Description
Old equations		
p_g^W	$q_g^W = \eta_g^W p_g^W + s_g^W$	Gridded supply of water input
$p_{g,l}$	$Q_r = \sum_{g \in r} \sum_l Q_{g,l}{}^b$ $p_{g,l} = p_r$	Regional crop markets and price index
New water		
p_g^{GW}	$q_g^{GW} = \eta_g^{GW} p_g^{GW} + s_g^{GW}$	Supply of groundwater
p_g^{SW}	$q_g^{SW} = \eta_g^{SW} p_g^{SW} + s_g^{SW}$	Supply of surface water
p_z^{KW}	$q_z^{KW} = \eta_z^{KW} p_z^{KW} + s_z^{KW}$	Supply of irrigation equipment and inputs
p_g^W	$p_g^W = \beta_g^{SW} p_g^{SW} + \beta_g^{GW} p_g^{GW} + \beta_g^{KW} p_g^{KW}$	Irrigation price index
q_g^{SW}	$q_g^{SW} = q_g^{SG} - \sigma_g^{SG}\left(p_g^{SW} - p_g^{SG}\right)$	Demand for surface water
q_g^{GW}	$q_g^{GW} = q_g^{SG} - \sigma_g^{SG}\left(p_g^{GW} - p_g^{SG}\right)$	Demand for groundwater
q_g^{KW}	$q_g^{KW} = q_g^{W} - \sigma_g^{W}\left(p_g^{KW} - p_g^{W}\right)$	Demand for irrigation inputs
q_g^{SG}	$q_g^{SG} = q_g^{W} - \sigma_g^{W}\left(p_g^{SG} - p_g^{W}\right)$	Demand for total surface water and groundwater
q_z^{KW}	$Q_z^{KW} = \sum_{g \in z} Q_g^{KW}$	Market clearing for irrigation inputs
p_g^{KW}	$p_g^{KW} = p_r^{KW}$	Price index for irrigation inputs
New product differentiation		
q_g	$q_g = q_s - \varphi_s(p_g - p_s)$	Subregional Armington (bottom layer)
q_s	$q_s = q_r - \kappa_r(p_s - p_r)$	Subregional Armington (top layer)
p_s	$p_s = \sum_{g \in s} \sum_l \theta_{g,l} p_{g,l}$	Subregional crop price index
p_r	$p_r = \sum_{s \in r} \theta_s p_s$	Regional crop price index

in a grid cell depends on relative prices, government policies, and production possibilities. Output levels expand or contract to ensure zero pure economic profits over the long run. Thus, unlike downscaling approaches to the gridded analysis of land use, the spatial pattern of production is endogenously determined. Crop producers within a given grid cell are price takers, as they are assumed to have no market power.

Table 5.6 summarizes the equations that introduce product differentiation and differentiated water sources. The equations for supply and demand for irrigation inputs follow an approach similar to that introduced earlier. The substitution elasticities are key parameters in demand and are derived from the nested CES functions; supply elasticities are also key parameters in supply functions. Here, *GW*, *SW*, and *KW* represent groundwater, surface water, and irrigation equipment, respectively. While supply of *GW* and *SW* are grid-cell-specific, the supply of *KW* occurs at the regional level, assuming that irrigation capital is a mobile factor of production.

6 Consumer Demand and Trade

The one common module across all of the SIMPLE-G specifications in Models 1–5 is the demand system, which follows the (nongridded) SIMPLE model (Baldos and Hertel. 2013; Hertel and Baldos 2016). This section describes the relevant equations taken from the SIMPLE regional model. We have modified the notation slightly but kept the demand structure intact. Table 5.7 summarizes these equations for quick reference.

6.1 Equations for Socioeconomic Determinants of Food Demand

At the regional scale, consumer demand for different commodities is a function of population, income, and prices. Equilibrium prices are determined endogenously as a function of supply and demand, while population and income changes are exogenous to the model, with increases in per capita income driving diet changes. Total demand for crops in a given region comes from four sources. The direct crop demand is calculated by multiplying per capita demand for crop consumption by population in each region. Then, we sum the total direct demand for crops in final consumption with the indirect (derived) demands in the livestock, food processing, and biofuel sectors. The demand for crops in biofuel production is assumed to be exogenously determined by government mandates. The livestock and food processing sectors' demands for crops are endogenous and modeled using CES production functions

Table 5.7 Major equations for the demand side of SIMPLE-G

Equation	Description
$q_{r,f} = \varepsilon_f^p p_{r,f} + \varepsilon_f^y y_{r,f}$ $\varepsilon_{r,f}^y = \alpha_{r,f}^y + \beta_{r,f}^y \ln Y_r$** $\varepsilon_{r,f}^p = \alpha_{r,f}^p + \beta_{r,f}^p \ln Y_r$**	Consumer demand for food and its elasticities
$qc_r^V + ac_r^V = qo_r^V - ao_r^V - \sigma_r^V\left(pc_r^V - ac_r^V - p_r^V - ao_r^V\right)$ $p_r^V = \theta_r^V pc_r^V + \left(1 - \theta_r^V\right) pn_r^V$	Livestock (V) demand for crops as feed
$qc_r^F + ac_r^F = qo_r^F - ao_r^F - \sigma_r^F\left(pc_r^F - ac_r^F - p_r^F - ao_r^F\right)$ $p_r^F = \theta_r^F pc_r^F + \left(1 - \theta_r^F\right) pn_r^F$	Processed food (F) demand for crops
$qm_{r,b} = qc_{r,b} - \omega_r(pm_r - pc_r)$ $qd_{r,b} = qc_{r,b} - \omega_r(pd_r - pc_r)$ $pc_r = \theta_r^m pm_r + \theta_r^d pd_r$ $pm_r = pw$	Demand for imported and domestic crops
$qx_r = q_r - \psi_r(px_r - p_r)$ $qd_r = q_r - \psi_r(pd_r - p_r)$ $p_r = \theta_r^x px_r + \theta_r^s pd_r$ $px_r = pw$	Exports and domestic supply

that combine the raw crop input with other inputs used in livestock or processed food production.

One of the best-understood patterns of economic development is Engel's Law, which states that as per capita income rises, the share of income devoted to food will decrease (Clements and Chen 1996). The SIMPLE framework captures this relationship by allowing the income elasticity of demand for food (ε_i^y, the propensity to spend incremental income on food) to evolve with per capita income (Y) based on the estimated parameters α_i^y and β_i^y. The price elasticity of food demand (ε_i^p) is derived similarly. In a general form,

$$\varepsilon_i^y = \alpha_i^y + \beta_i^y \ln Y; \tag{5.24}$$

$$\varepsilon_i^p = \alpha_i^p + \beta_i^p \ln Y. \tag{5.25}$$

Equations 5.24 and 5.25 are indexed by food type (i). SIMPLE distinguishes between direct consumption of crops and indirect consumption through either livestock product consumption or processed food consumption. This results in the following equations describing the evolution of per capita demand for each type of food product:

$$q_i = \varepsilon_i^p p_i + \varepsilon_i^y y. \tag{5.26}$$

Specifically, for each region r and commodity i, we can write consumption demand as

$$qcons_{r,i} = \varepsilon_{r,i}^p p_{r,i} + \varepsilon_{r,i}^y y_r, \tag{5.27}$$

where $qcons$ shows the demand for commodities that include crops, livestock, processed food, and nonfood. While the income elasticity of demand for food is typically positive (i.e., the demand for food increases as income increases), it can also be negative, showing a dietary shift with income growth. The price elasticity of demand for food is negative, which means that the demand for food decreases as the price of food increases.

For each food commodity, we introduce a production function to show different uses of crops in the food sectors. For each use (activity), the production function combines crops as one input with other inputs and supplies a food commodity to the final consumer. In each food sector, demand for crops is determined assuming a CES function. The linearized form of demand can be written as

$$\begin{aligned} qc_r^V + ac_r^V &= qo_r^V - ao_r^V - \sigma_r^V\left(pc_r^V - ac_r^V - p_r^V - ao_r^V\right) \\ p_r^V &= \theta_r^V pc_r^V + \left(1 - \theta_r^V\right) pn_r^V \end{aligned} \tag{5.28}$$

$$qc_r^F + ac_r^F = qo_r^F - ao_r^F - \sigma_r^F\left(pc_r^F - ac_r^F - p_r^F - ao_r^F\right)$$
$$p_r^F = \theta_r^F pc_r^F + \left(1 - \theta_r^F\right)pn_r^F$$

where qc is the percentage change in crop demand for use in processed food (F) or livestock (V) production region r; ac is related to partial factor productivity; qo is the percentage change in activity output; ao is the total factor productivity in activity i; σ is the substitution elasticity, which differ by region and use; pc is the regional crop price index; and p is the regional food commodity price index. The equilibrium condition is that the commodity supplied from each activity should be the quantity demanded, given the absence of trade for these processed products in this model:

$$qo_r^f = qcons_{r,f}. \tag{5.29}$$

6.2 Equations for Global Trade Flows

As with consumer demand, SIMPLE-G follows the SIMPLE model in its specification of international trade. It relies on product differentiation by origin (Armington 1969), identifying domestic and international goods separately. Heterogeneous consumers are assumed to choose between domestic and imported goods, based on relative prices and preferences; when aggregated, the demand for imports and domestic goods follows a CES function (Anderson et al. 1989). The more homogeneous consumers are (both in preferences and market access), the greater the elasticity of substitution and the more readily that aggregate demand shifts between imported and domestic crop products. Producers are also assumed to be heterogeneous, with different access to international markets. Analogous to what occurs among consumers, this results in a CET market supply function for domestic and export markets. In addition to the elasticities of substitution and transformation between domestic and international goods, the initial penetration of exports and imports into any given market will play a key role in determining the extent of price transmission from global to domestic markets. This can differ between consumers and producers. Finally, it is important to note that SIMPLE-G does not model international trade in livestock or processed food products. These commodities are more lightly traded than are crops, with much of the food processing occurring locally, often facilitated by foreign direct investment. Nonetheless, this remains an important limitation of SIMPLE-G. A proposal for relaxing this is explored in Part V of this book, where SIMPLE-G is nested with a general equilibrium model in which all commodities are traded.

The demand for imports depends on the prices of domestically produced crops and import prices, as well as the Armington elasticity of substitution:

$$qm_{r,b} = qc_{r,b} - \omega_r(pm_r - pc_r), \tag{5.30}$$

$$qd_{r,b} = qc_{r,b} - \omega_r(pd_r - pc_r), \tag{5.31}$$

$$pc_r = \theta_r^m pm_r + \theta_r^d pd_r, \tag{5.32}$$

$$pm_r = pw, \tag{5.33}$$

where *m* is imports and *d* is domestic markets; *qm* is the percentage change in demand for imported crops; *qd* is percentage change in demand for domestic crops; *qc* is total demand for crops by buyer; *pc* is average buyers' price index; *pm* is the imports price index; *pw* is the global average price index of all crops; θ is the value share in price index; and ω is the Armington substitution elasticity parameter.

Imports are obtained from the global market for crops, which is fulfilled through exports. Producers in each region face a choice between selling their goods within their country or exporting them to the global market. This decision involves a trade-off, which is captured by the (negative-valued) CET parameter. This parameter indicates how easily producers can switch between domestic and global markets. A higher absolute value for this elasticity shows that producers are more responsive to price changes, (i.e., they are more likely to focus on exporting their goods when foreign prices are comparatively attractive). In contrast, a lower absolute value for this elasticity suggests that producers have less access to global markets and are therefore less affected by the price differentials that may emerge between domestic and global markets:

$$qx_r = q_r - \psi_r(px_r - p_r), \tag{5.34}$$

$$qd_r = q_r - \psi_r(pd_r - p_r), \tag{5.35}$$

$$p_r = \theta_r^x px_r + \theta_r^s pd_r, \tag{5.36}$$

$$px_r = pw, \tag{5.37}$$

where *x* is exports and *d* is domestic markets; *qx* is the percentage change in supply for exports to global markets; *qd* is percentage change in supply to domestic markets; *q* is total regional supply of crops; *p* is average producer price index; *px* is the exports price index; *pw* is the global average price index of all crops; θ is the value share in price index; and ψ is the (negative) CET transformation elasticity parameter.

7 Summary

This chapter presented mathematical equations for five specifications of the SIMPLE-G gridded production and markets. In Model 1, agricultural production is a function of human and environmental inputs with different degrees of mobility across space. This framework is useful for understanding the tradeoffs and synergies between food security and environmental sustainability as well as evaluating spill-over effects. In Model 2, land and water inputs are explicitly introduced, enabling detailed modeling of land and water use at each grid cell. Model 3 adds fertilizer inputs and their markets to enable water quality assessments. In Model 4, multiple local activities are introduced at each grid cell, showing competition over local land and water resources. Finally, in Model 5, groundwater and surface water resources are separated, enabling analysis of specific groundwater sustainability policies. These models are examples of the SIMPLE-G framework. While they provide a solid foundation for sustainability analysis, it is important to keep in mind that additional dimensions may need to be introduced to ensure a comprehensive analysis, depending on the research question. By doing so, we can enhance our understanding and arrive at more accurate conclusions. The models introduced in this chapter are both flexible and scalable. With appropriate data preparation, they can be applied to various grid cell sizes, ranging from very fine to coarse resolutions, as needed for multiscale analysis. As you learn more about data preparation and parameter selection in the upcoming chapters, we invite you to consider how these tools can be harnessed to uncover new insights and drive meaningful discoveries in your research.

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