

Water Supply Management under Uncertainty: A Stochastic CGE Model with Application to Egypt

Abdelradi Mohammed

Abstract

Volatility in water supply is a main challenge in water management. This paper develops a stochastic dynamic general equilibrium approach for modeling water supply in a water-scarce economy. Water is used for production and final consumption and is supplied by stochastic freshwater flows or through desalination. The stochastic general equilibrium model is developed as a quadratic system without an explicit objective function using recursive dynamic programming techniques. The model is calibrated to the SAM data for Egypt 2019 and then solved numerically. The results show that uncertainty reduces investment in the desalination sector and increases investment in water-substituting capital instead. As a result, production becomes less water-intensive with increasing uncertainty. Water storage is much higher under uncertainty than in a deterministic model, and consumption is lower.

1 Introduction

Water from conventional sources is becoming more scarce, due to rising demand, and less reliable due to climate change. For the Nile basin region, for example, hydrological models predict increasing variability in water supplies in the future ([Siam and Eltahir, 2017](#); [Mengistu et al., 2021](#)). Transboundary disputes and upstream activities also affect downstream countries' water supplies ([Heggy et al., 2021](#); [Digna et al., 2018](#)). This requires that the water planner takes this into account in order to avoid welfare losses under adverse future events such as reduced water supplies. In this context, exploring non-conventional water resources such as desalination and wastewater reuse becomes more urgent. However, using water from these sources requires costly investment in capacity building.

The importance of water for economic growth is emphasized in numerous studies ([Barbier, 2004](#); [Hertel and Liu, 2019](#)). The study of uncertainty and its relation to the management of water resources is also established in the literature ([Burt, 1964](#); [Knapp and Olson, 1996](#); [Howitt et al., 2005](#); [Li and Swain, 2016](#)). A growing literature focuses on optimal freshwater management as an adaptation mechanism to climate change and to meet rising demand ([Beça et al., 2023](#); [Garrote, 2017](#)). However, most of these studies look at the water management system in isolation from the rest of the economy or consider only one source of water. Studying the optimal freshwater management by integrating conventional and non-conventional water sources under a stochastic dynamic general equilibrium setup is still lacking in the literature.

The main tool for water supply management is the construction of dams and the storage of water in artificial lakes. Dams and in addition to providing hydropower and protection from extreme precipitation events regulate freshwater flows and make water available at times of low natural flows, helping to sustain the availability of this resource, particularly in water-constrained regions. Worldwide, there are more than 14000 dams ([AQUASTAT, n.d.](#)). In the case of Egypt, for example, the water management system relies more than 15 dams on the Nile with a storage capacity of more than 170 billion cubic meters (bcm). The major one is the High Aswan dam with a storage capacity of 162 bcm. They play the central role in managing the water supply and control floods in addition to power generation. The current reservoir management scheme (lake Nasser) can be improved to reduce evaporation losses that are estimated at 10 bcm annually.¹

Another possibility is to increase water supply from non-conventional sources. This could provide unlimited amounts of water. The technological improvement in the desalination sector

¹[National Water Resources Plan – 2017](#) (n.d.)

has led to a secular increase in the utilization of this technology that establishes the potential of desalination to provide a reliable source of water for different uses (Ghaffour et al., 2013; Jones et al., 2019). As a result, the amount of desalination water has expanded from around 4.5 bcm in 2000 to 22.5 bcm worldwide in 2022, most of it in the Gulf states (29%) and China (56%) (AQUASTAT, 2020). However, desalination production is still lagging behind in many water-scarce regions, including the current case, Egypt. Faced with an increase in the demand for water, better management of reservoirs in addition to investing in the desalination capacity, which is expected to increase, are identified as the main points of the national water strategy (Elkholy, 2021).

This article provides a framework for modeling uncertainty in dynamic general equilibrium and calibrating the model to national accounting data. The approach is used to investigate the impact of uncertainty in freshwater flows caused by climate change and how this affects the investment in non-conventional water production, e.g., desalination sector as well as the production pattern in a water-scarce economy. To do this, a stochastic dynamic general equilibrium model with two different types of capital is developed. One type is used in desalination, and the other is used in the remaining sectors. Water is a homogeneous good produced from freshwater and desalination. The randomness originates from the natural freshwater supplies. The model is then calibrated to the SAM data for Egypt in 2019. Egypt is an interesting case, as it is an arid region with limited water supplies and potential for an expanding desalination sector (Elsaie et al., 2023).

The model is solved as a stochastic Bellman equation by deriving the optimality conditions, the market clearing conditions, and state evolution equations. To obtain a numerical solution of the model, a square system of stochastic equations is solved in GAMS. This is made possible by estimating an AR(1) process for the Nile flow data (the source of freshwater supplies) and applying the Tauchen method to derive the probability transition matrix to approximate the underlying continuous distribution of the flows (Tauchen, 1986). Four different scenarios regarding the future are simulated; A. deterministic scenario, B. stochastic model with current level of volatility, C. stochastic model with increased volatility, D. stochastic model with increased volatility and reduced mean annual flows.

The numerical solution shows that uncertainty in water flows results in higher levels of storage and lower levels of water extraction, resulting in larger artificial lakes. It reduces consumption and encourages a higher level of investment in the water-substituting capital. Consequently, production becomes less water-intensive. In addition, uncertainty discourages investment in the desalination sector. If the mean annual flows are considered by the social

planner and uncertainty is ignored – the deterministic case – then substantial investment in desalination is made, and a less restrictive water management regime is adopted (low storage).

The rest of the paper is organized as follows, section 2 sets up the model and characterizes the steady state, section 3 outlines the data and the parameter calibration, section 4 describes the scenarios, section 5 presents the results and discusses their implications and section 6 concludes.

2 Methods

2.1 The Model

To address the research question, I present here a tree-sector, two-commodities growth model. The commodities are the water commodity and the Y commodity, which is an aggregation of all other commodities in the economy. The water commodity is produced from freshwater or desalinated water, D. The Y sector produces good Y using capital, labor and a produced water commodity in constant returns to scale Cobb Douglas production function:

$$Y = A_1 Q^{\alpha_1} K_1^{\alpha_2} L^{\alpha_3}, \quad \alpha_1 + \alpha_2 + \alpha_3 = 1 \quad (1)$$

Where Q is the water input, K_1 is capital type 1 (capital used in sector Y) and L is labor. A_1 is the productivity shifter that is assumed to remain constant over time. The desalination sector uses only capital (type 2) for production:

$$D = A_2 K_2^\theta, \quad 0 < \theta < 1 \quad (2)$$

where the productivity shifter A_2 is a constant calibrated from the SAM data for a given θ . The capital used in desalination is different from the capital used in sector Y in that the desalination capital entails adjustment cost. Freshwater is produced using freshwater as a resource and a portion of good Y to extract the resource and provide it to users. Desalination water is a perfect substitute for the freshwater supplied by nature as renewable surface water flows.

The difference between these two types of capital is the partial irreversibility. K_1 is completely reversible, meaning that the capital good can be divested and used for consumption or converted to K_2 . Desalination capital, K_2 , on the other hand, is partially irreversible, meaning that when it is divested, some of the value is lost.

The adjustment cost of investment in desalination capital is assumed to be proportional to the stock of capital in that sector, see, for instance, [Bloom \(2009\)](#). The adjustment cost reflects the partial irreversibility aspect of some investment and highlights the idea that some types of capital have value only if used in production. Converting this capital to another type of capital or to a consumption good implies squandering some of its value. A quadratic form of the adjustment cost is applied ([Romer, 2001](#)):

$$G(I_2, K_2) = \frac{b}{2} \frac{I_2^2}{K_2}, \quad b > 0 \quad (3)$$

where the cost of installing one additional unit of capital in the desalination sector increases with the ratio of new capital I_2 to the stock K_2 in proportion b . Consequently, the investment cost in type 2 capital is $I_2 + G(I_2, K_2)$.

The freshwater endowment is a renewable annual flow that reaches the country in year t , and it is stochastic. Some of this water can be withdrawn (extracted), W , for use, and some can be used to build a stock S in the form of an artificial lake or to recharge groundwater. Artificial lakes play a major role in water management (hedging against floods and drought) and in the generation of hydropower. As mentioned above, there are 14000 dams worldwide and about 2000 of them are in Africa ([AQUASTAT, n.d.](#)). In this article, the focus lies on the water management role of dam reservoirs.

Hydrologists distinguish between water withdrawal (extraction) and water consumption, some of the water used might be retrieved to the original source, and some of it is consumed irretrievably ([Barbier, 2004](#)). Here, it is assumed that all the water withdrawn is consumed irreversibly. The freshwater system evolves according to:

$$S_{t+1} = (1 - v)S_t + Z_t - W_t \quad , \quad 0 < W_t < S_t \quad (4)$$

$$Z_t = a + \rho Z_{t-1} + \epsilon_t, \quad \epsilon_t \stackrel{\text{iid}}{\sim} N(0, \sigma^2) \quad (5)$$

where S_t denotes the water stock (stored in lakes) in year t , stochastic annual flows are given by Z_t , the water withdrawn for use in final consumption or as input in production is W , v denotes the rate of evaporation, and ϵ is an iid normally distributed innovation term with mean zero and constant variance σ^2 . The decision about the withdrawal and all other model decisions are made at the beginning of each period and before the realization of Z .

Withdrawing water from the reservoir and providing it to the end-user imply some cost. Following [Burt \(1964\)](#) the share of output that is used to extract water is a function of the

stock available, let us specify:

$$\varphi(S) = a/S^\eta, \quad a > 0, \eta > 0, S > 0, \quad \lim_{S \rightarrow 0} \varphi(S) = \infty \quad (6)$$

$\varphi(S)$ is the amount of output (of good Y) used to extract one unit of water and deliver it to end-users. This means that when the stock is low, it becomes more costly to extract and pump it to the end user.

The representative household consumes an amount X of good Y and an amount H of total water supply W+D, with a substitution parameter σ and a constant absolute risk aversion (CARA) utility. The total expected utility over an infinite horizon is:

$$\mathbb{E}[U] = \mathbb{E}\left[\sum_0^\infty -\beta^t e^{-\gamma C_t}\right], \quad \gamma > 0, 0 < \beta < 1 \quad (7)$$

$$C = a_c (\lambda X^\rho + (1 - \lambda)H^\rho)^{1/\rho}, \quad \sigma_c = \frac{1}{1 - \rho} > 0$$

where γ is the coefficient of absolute risk aversion. The special property of this utility form is precautionary savings.² The parameters a_c and λ denote the shifter and the value share, respectively, and are calibrated from the data for a given σ_c .

In addition to its use in consumption, good Y is used as an investment good in both types of capital and also for the extraction of freshwater resources. The market clearing conditions for the two goods Y and the water commodity, respectively, are given by:

$$Y = X + I_1 + I_2 + \frac{b}{2} \frac{I_2^2}{K_2} + \varphi(S)W \quad (8)$$

$$W + D = Q + H \quad (9)$$

Where commodity Y is used for consumption X, investment I_1 and I_2 , and extraction of freshwater. The values of all goods and factors in the model are measured in terms of good Y, the model numeraire. The quantity of water commodity used for consumption by the representative household is H, and Q is the quantity used for production in sector Y. Together, they cannot exceed the total supply of water $W + D$. Both capital types evolve according to:

$$K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t}, \quad i = 1, 2 \quad (10)$$

²See (Mas-Colell et al., 1995; Caballero, 1990)

with the same depreciation rate δ for both types of capital. Labor is used in sector Y only and its supply is exogenously given according to:

$$L_{t+1} = (1 + n)L_t \quad (11)$$

where n is the growth rate of L . The quantity L represents the population and the labor force and is fully employed. The optimization criterion of the social planner is to maximize the total utility of the representative household over an infinite horizon, equation 7, subject to the rest of the equations 1-11. Let us start by setting up the stochastic dynamic optimization problem. The Bellman equation is:

$$V(K_1, K_2, S) = \max_{X, Q, W, I_2} \left\{ -e^{-\gamma C} + \beta \mathbb{E} \left[V(K'_1, K'_2, S') \right] \right\} \quad (12)$$

where the prime symbol on the variables denotes the $t + 1$ period, $\mathbb{E}$ is the expectation at time t and V is the value function. Equation 12 is the main equation in which the problem is reduced to a problem of two periods, the utility from consumption at t and the discounted expected value of the remaining assets at $t + 1$, the assets being the two types of capital and the water stock.³ The control variables in the current setup are X (the quantity of Y used for private consumption), the input of water in the Y sector, Q , the withdrawal of water from the reservoir, W , and the investment in the desalination sector I_2 . The rest of the model variables; H, I_1 and Y are determined using the market clearing conditions for commodities and factors. The evolution of the state variables, K_1, K_2 and S , is driven by the initial conditions and the control variables. For example, S_1 depends on S_0 , which is given, and the control W_0 , which is chosen optimally, in addition to exogenous stochastic flow Z . The same is true for all periods and the other state variables. Using these relationships, the Bellman equation can be written in terms of the control variables as follows:

$$V(K_1, K_2, S) = \max_{X, Q, W, I_2} \left\{ -e^{-\gamma a_c(\lambda X^\rho + (1-\lambda)(W+D-Q)^\rho)^{1/\rho}} + \beta \mathbb{E} \left[V \left((1-\delta)K_1 + A_1 Q^{\alpha_1} K_1^{\alpha_2} L^{\alpha_3} - X - I_2 - \frac{b}{2} \frac{I_2^2}{K_2} - \varphi(S)W, \right. \right. \right. \quad (13)$$

$$\left. \left. \left. (1-\delta)K_2 + I_2, (1-\nu)S + Z - W \right) \right] \right\}$$

³For a treatment of simple problems of dynamic programming see [Sydsæter et al. \(2008\)](#), Ch. 12.

Now I proceed by differentiating eq. 13 with respect to (w.r.t.) each of the controls. The full derivation of the model is given in appendix A.1:

$$\frac{d}{dX} : \quad \gamma\lambda \left(\frac{C}{X} \right)^{1-\rho} e^{-\gamma C} = \beta \mathbb{E} [V_{K'_1}] \quad (14a)$$

$$\frac{d}{dQ} : \quad \gamma(1-\lambda)(W+D-Q)^{\rho-1} C^{1-\rho} e^{-\gamma C} = \beta \frac{\alpha_1 Y}{Q} \mathbb{E} [V_{K'_1}] \quad (14b)$$

$$\frac{d}{dW} : \quad \gamma(1-\lambda)(W+D-Q)^{\rho-1} C^{1-\rho} e^{-\gamma C} = \beta \varphi(S) \mathbb{E} [V_{K'_1}] + \beta \mathbb{E} [V_{S'}] \quad (14c)$$

$$\frac{d}{dI_2} : \quad \left[1 + b \frac{I_2}{K_2} \right] \mathbb{E} [V_{K'_1}] = \mathbb{E} [V_{K'_2}] \quad (14d)$$

where $\mathbb{E} [V_{K'_1}]$ is the derivative of the expected value function w.r.t. K_1 in period $t+1$, and the same for K_2 and S . Equation 14a is the familiar FOC that equates the marginal utility of consumption with the discounted expected value of saving one unit of output to be used to accumulate capital, which will then become productive in the next period. Thus, this relationship captures the intertemporal opportunity cost of consumption/saving. Equation 14b sets the condition for the optimal choice of water input in production and 14c sets the optimality condition for the withdrawal of water from the reservoir. Further manipulation is needed for these two equations to see what kind of optimality conditions they imply, which is done below. Finally, the condition for optimality of the investment in both types of capital is given implicitly by the equation 14d. This condition states that the ratio of the capital cost of both types of capital is determined by the marginal adjustment cost. This is the Tobin's q relationship (Romer, 2001). To see this, suppose that the unit cost of K_1 is 1, then the unit cost of K_2 is $1 + G'(I_2, K_2)$, which is exactly Tobin's q . That is, an additional unit of investment in K_2 would be made only if the expected value generated by it is greater than the additional expected value generated from K_1 plus the marginal adjustment cost.

In addition to the unknown values of the parameters, the system of equations has three further unknowns, which are the value functions of the three state variables $V_{K'_1}$, $V_{K'_2}$ and $V_{S'}$. To solve the model, these have to be eliminated; this is done using the envelope theorem. The Bellman equation in 13 is differentiated w.r.t. each one of the state variables (see the steps in appendix A.1). In doing this, four Euler equations are obtained. The first is the optimality condition for consumption C_t (which is a CES combination of goods) between any

two subsequent periods, given by:

$$\left(\frac{C}{X}\right)^{1-\rho} e^{-\gamma C} = \beta \mathbb{E} \left[\left(1 - \delta + \frac{\alpha_2 Y'}{K_1'}\right) \left(\frac{C'}{X'}\right)^{1-\rho} e^{-\gamma C'} \right] \quad (15)$$

This is the Euler equation for consumption of the aggregate commodity in two periods. The equation relates the intertemporal marginal utility at t (the left hand side) to the discounted expected marginal utility augmented by the net marginal product of capital K_1 and $t + 1$. In the deterministic case, this gives the ratio of marginal utility of consumption over any two subsequent periods as the discounted marginal product of capital net of depreciation. The Euler equation for water use is given as:

$$\frac{H}{X} = \left(\frac{1 - \lambda}{\lambda} \frac{\alpha_1 Y}{Q} \right)^\sigma \quad (16)$$

This equation implicitly determines the allocation of water into final consumption H and production Q in the same period. Thus, it is an allocation rule for water among different uses and is governed by the marginal product of water in production. The optimality rule (Euler Equation) for reservoir management is:

$$\begin{aligned} (1 - \lambda) \left(\frac{C}{W + D - Q} \right)^{1-\rho} e^{-\gamma C} &= \beta \lambda \varphi(S) \mathbb{E} \left[\left(1 - \delta + \alpha_2 \frac{Y'}{K_1'}\right) \left(\frac{C'}{X'}\right)^{1-\rho} e^{-\gamma C'} \right] \\ &+ \beta \lambda \mathbb{E} \left[\left((1 - \nu) \left(\alpha_1 \frac{Y'}{Q'} - \varphi(S') \right) - \varphi'(S') W' \right) \left(\frac{C'}{X'}\right)^{1-\rho} e^{-\gamma C'} \right] \end{aligned} \quad (17)$$

This equation determines the intertemporal relation between withdrawing water now or keeping it in the reservoir for the next period, in which case it evaporates at rate v . The left hand side is the marginal utility of water use in consumption, and the right hand side reflects the opportunity cost of keeping the water in the reservoir. Keeping water in the reservoir makes it available for production (or consumption) next period, while the change in the reservoir stock influences the extraction cost $\varphi(S)$. I will return to this point in the next section when studying the steady state. The last relationship is the Euler equation for investment allocation between the two types of capital after eliminating the unknown value functions, which is written as:

$$\left(1 + b \frac{I_2}{K_2}\right) \left(\frac{C}{X}\right)^{1-\rho} e^{-\gamma C} = \beta \mathbb{E} \left[\left(\frac{b}{2} \left(\frac{I_2'}{K_2'} \right)^2 + (1 - \delta) \left(1 + b \frac{I_2'}{K_2'}\right) \right) \left(\frac{C'}{X'}\right)^{1-\rho} e^{-\gamma C'} \right] \quad (18)$$

This equation determines the allocation of investment between the two sectors. It reflects the trade off between the two types of investment; invest more in type 1 capital to replace the water commodity or invest in type 2 capital to produce desalinated water which can then be used to replace type 1 capital in the production of the main commodity in this economy, good Y .

The Euler equations (15), (16), (17), and (18) in addition to the law-of-motion equations, the market clearing conditions and the boundary conditions (initial levels and steady-state levels) determine the path of the model variables between 0 and ∞ . It builds a square system of equal unknowns and equations which gives a unique solution of the system. In order to solve the system of equations, one needs first to determine the steady state level of the model variables and impose this as terminal condition. So let us now turn to studying the steady state case.

2.2 The steady state

The steady state is a deterministic case of the current model where the stochastic freshwater flows are replaced by their mean. Further, the infinite horizon model is approximated by a finite horizon model by imposing certain constraints on the terminal period. In this regard, eating up all the stocks in the final period is not optimal as would be in a problem of finite horizon. Instead, the following conditions are imposed ([Lau et al., 2002](#)):

$$\lim_{t \rightarrow \infty} K_{1,t} = K_1^* \quad \lim_{t \rightarrow \infty} K_{2,t} = K_2^* \quad \lim_{t \rightarrow \infty} S_t = S^* \quad (19)$$

By doing this, the steady state level of each state variable in the infinite horizon problem is imposed as the level of that variable in the terminal period in the finite horizon version. The steady state of the current model defines the per capita growth rate of all quantities to be zero for all variables (except water stock S). That is, the level of these variables increases at the same rate as the population n . For the water stock, the steady state condition is that the growth rate of the stock (and not per capita growth) is zero. Appendix A.2 shows the derivation of the steady state of the model. For both types of capital, the steady state levels are given, by:

$$K_1^* = \psi Q^{\frac{\alpha_1}{1-\alpha_2}} L^{\frac{\alpha_3}{1-\alpha_2}} \quad \text{where } \psi = \left(\frac{\alpha_2 A_1}{\frac{1}{\beta(1+n)} - 1 + \delta} \right)^{\frac{1}{1-\alpha_2}} \quad (20)$$

$$K_2^* = \left[\frac{\alpha_1 A_2 \theta \psi}{\alpha_2 [1 + b(n + \delta)]} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1-\alpha_2}} \right]^{\frac{1}{1-\theta}} \quad (21)$$

It is notable that the steady state of both types of capital depend on the steady state level of the control variable Q (water input in production) and the exogenous labor level in the end period.

An important relationship in the current model is how optimal management of the freshwater reservoir responds to changes in desalination production. By substituting the left hand side in eq. 17 by eq 16, the steady state relationship of the total consumption of good X is defined by:

$$\frac{u'(X_t)}{\beta u'(X_{t+1})} f_Q = \bar{u} f_Q = \varphi(S)R + f_Q - \varphi(S) - \varphi'(S)W \quad (22)$$

where f_Q is the marginal product of water, $\bar{u}$ is the ratio between the marginal utilities in the steady state and R is the gross return on capital used in Y net of depreciation,. This equation implicitly defines the relationship between water storage and water withdrawal, and thus total water supply in the steady state. Using the implicit function theorem and substituting W from the market clearing condition for water commodity and differentiating:

$$\begin{aligned} f(S, D) &= \varphi(S)[R - 1] + f_Q - \varphi(S) - \varphi'(S)(H + Q - D) \\ df &= f_S dS + f_D dD = 0 \\ \frac{dS}{dD} &= \frac{\varphi'(S)}{\varphi'(S)[1 - \gamma] + \varphi''(S)(H + Q - D)} < 0 \end{aligned} \quad (23)$$

where $f(S, D)$ is the augmented ratio of marginal utilities in the steady state. In Eq. 23 the numerator is negative. The first term in the denominator is positive because R is larger than 1, and the first derivative of φ is negative.⁴ The second term is also positive since desalination is less than the total demand for water $H + Q$ and the second derivative is positive due to the convexity of φ . Thus, the whole expression is negative. This result states that for a given level of economic activity, the stock of freshwater in the reservoir is smaller, the larger the desalination production. Desalination provides an additional source of water, reducing the need for water conservation. In contrast, if desalination becomes more expensive and thus D decreases, more

⁴Since the value of the marginal product of capital must exceed the depreciation rate, note also that in the current model df is a positive constant, but this does not change anything in the analysis.

water must be stored in the reservoir and S increases. Another interesting relationship implied in eq. 22 is the effect of extraction cost on stock management. The term $f_Q - \varphi(S) = P$ is the difference between the marginal product of water in production and the unit extraction cost, both in units of output of good Y , thus P gives the net marginal product of water. After rearranging and differentiating P w.r.t. S and R , respectively:

$$\frac{\partial P}{\partial S} = -\varphi'(S)R + \varphi''(S)W > 0 \quad (24)$$

$$\frac{\partial P}{\partial R} = -\varphi'(S) > 0 \quad (25)$$

Equation 24 states that the net value of the marginal product of water is increasing in the stock. When the stock is high, the extraction costs are low. Thus, the residual value P increases and vice versa. That is why the term $\varphi'(S)$ (the derivative of the extraction cost w.r.t. the stock) appears in the optimality condition in eq. 17 which reflects the effect of withdrawal today on future extraction cost. The motivation for punishing stock extraction might also be motivated by the uncertainty aspect (Solow and Wan, 1976). Equation 25 states that P increases in the productivity of K_1 . Not surprisingly, water has a higher value, the more productive the economy is. For the numerical model, the steady state level of reservoir storage, eq.17 can also be written as:

$$\begin{aligned} \alpha_1 A_1 \psi^{\alpha_2} \left(\frac{Q^*}{L} \right)^{\frac{\alpha_3}{\alpha_2-1}} &= (1+n)\beta \left[\varphi(S^*) (1 - \delta + \alpha_2 A_1 \psi^{\alpha_2-1}) \right. \\ &\quad \left. + (1-\nu) \left(\alpha_1 A_1 \psi^{\alpha_2} \left(\frac{Q^*}{L} \right)^{\frac{\alpha_3}{\alpha_2-1}} - \varphi(S^*) \right) - \varphi'(S^*) W^* \right] \end{aligned} \quad (26)$$

where W^* is determined by imposing the steady state condition that the total stock of the reservoir is constant $S_{t+1} - S_t = 0$ as t becomes large enough:

$$W^* = \bar{Z} - \nu S^* \quad (27)$$

$\bar{Z}$ is the mean annual flow of surface freshwater (an exogenous quantity).

Finally, the steady state for the water input Q^* is given implicitly by the equation:

$$A_1 \psi^{\alpha_2} K_2^* \left(\frac{Q^*}{L^{-\alpha_3}} \right)^{\frac{1}{1-\alpha_2}} = \left[\frac{1-\lambda}{\lambda} \frac{1}{\alpha_1 A_1} \psi^{-\alpha_2} \left(\frac{Q^*}{L} \right)^{\frac{\alpha_3}{1-\alpha_2}} \right]^{\frac{1}{\rho-1}} (W^* + D^* - Q^*) + (\delta + n) K_1^* + \left(\delta + n + \frac{b}{2} (\delta + n)^2 \right) K_2^* + \varphi(S^*) W^* \quad (28)$$

The equations 20, 21, 26, 27, and 28 construct a quadratic system of five non-linear equations with five unknowns – the three state variables in addition to Q^* and W^* , which need to be determined as well, since the steady state level of the state variables depend on them. Appendix A.2 shows the derivation of the steady state model. Using these steady state levels as terminal conditions in addition to the given initial conditions of the three state variables, the model now can be solved numerically to find the whole path of each endogenous variable in the model.

The first step towards solving the model numerically is to use the data to calibrate the model parameters. Unfortunately, the SAM data are not enough to do this for all parameters. Therefore, for some parameters, the values have to be assumed. The second step is to solve for the deterministic steady state. The third step is solving the deterministic version of the model path (solution for all periods). The last step is to solve the stochastic model.

3 Data and Model calibration

The model is calibrated to the social accounting matrix for Egypt 2019, [Serag et al. \(2021\)](#). These data are a detailed account of all transactions in the economy for that year. The SAM comprises 69 activities and 73 commodities.

Multiple adjustments are made to the original SAM to align it with the structure of the current model, keeping the total value of the economy unchanged, table 1. First, the water sector is split into two sectors (activities), freshwater activity and desalination activity. The rest of the sectors were aggregated into one sector, the Y sector. There are two commodities; Y-c is produced by sector Y-a, whereas the water commodity, water-c, is produced by both the freshwater sector and the desalination sector. The column entries give the value of the input in the production of the output (rows). For households, the row entries represent income and the column entries represent consumption/saving. The original SAM includes 10 households, which are aggregated to one representative household. The SAM is complemented by obtaining the physical quantities from administrative data. Data for physical quantities of water use are compiled from [Report \(2016\)](#) and [AQUASTAT \(2020\)](#), while the values are taken from the

Table 1: Social Accounting Matrix (SAM) for Egypt (2019). This is an aggregation of the original SAM with 73 sectors and commodities – a denotes the activities, c the commodities, f the primary factors, s-i is investment. The values are in billion EGP (2019 prices).

	freshw-a	des-a	y-a	water-c	y-c	K_1 -f	K_2 -f	labor-f	freshw-f	household	s-i	total
freshw-a				509.935								509.935
des-a				12								12
y-a					4874.52							4874.52
water-c			487.46									487.46
y-c	24.8484									34.4753		521.935
K_1 -f			2915.36							4456.55	393.123	4874.52
K_2 -f		12										2915.36
labor-f			1471.7									1472.7
freshw-f	485.087											485.087
household						2915.36	12	1471.7	485.087			4884.15
s-i										393.123		393.123
total	509.935	12	4874.52	521.935	4874.52	2915.36	12	1472.7	485.087	4884.15	393.123	

Table 2: Physical values and model parameters. The values for water variables are in billion cubic meter.

Physical quantities in base year	S_0 150	D_0 0.3	W_0 55	Q_0 49.8	H_0 5.3			
Calibrated Parameters	a_c 1.3	a_y 12.4	a_d 0.07	α_1 0.1	α_2 0.6	α_3 0.3	λ 0.99	a 0.03
Exogenous parameters	δ 0.05	β 0.97	v 0.1	n 0.01	σ_c 0.7	θ 0.6	η 1.5	b 2

Note: The exogenous parameters δ and β are similar to values used in the literature, v and n are country-specific, the real values for σ_c , θ , b , and η are unknown and thus the current values are chosen to ensure numerical stability of the model. The calibrated parameters are not exogenously chosen, but 'calibrated' to ensure that the model equations and relationships are satisfied in the base year.

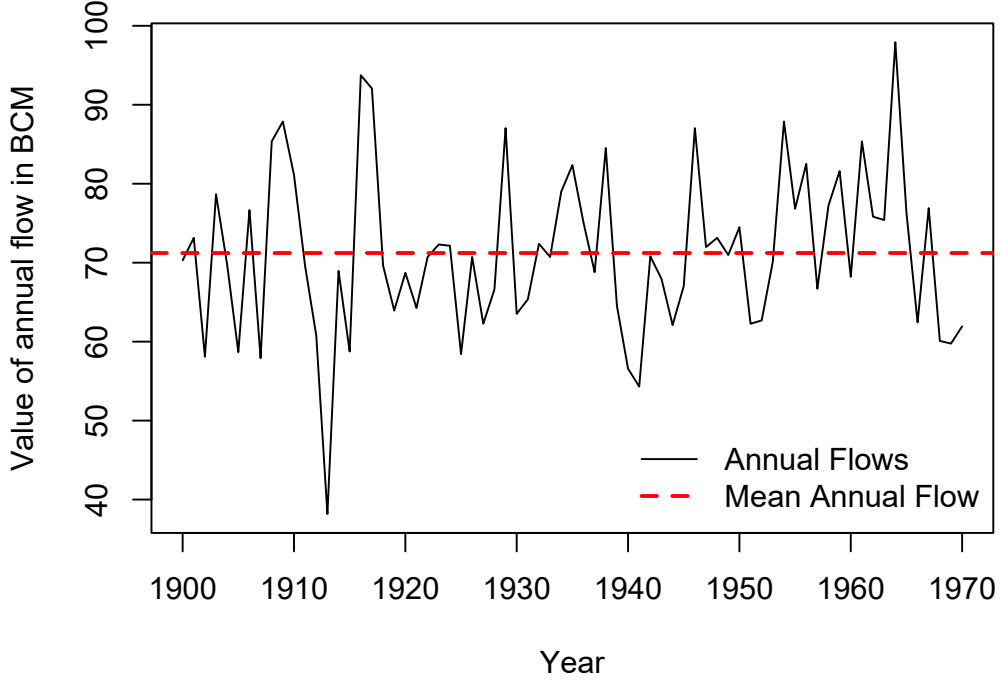
SAM accounts for water and sewage, see Table 2.

The SAM does not include a desalination sector. To fix this, the external data on the value of inputs, which are mostly capital and energy, are assigned to capital as the only input in this sector. The quantity of desalination in 2019 is 0.3 bcm and costs 12 billion EGP, which represents the rental value of capital ([Elsaie et al., 2023](#); [Lamei et al., 2008](#)).

A further limitation of the current SAM is the lack of information on the contribution of water to GDP. Numerous studies have shown that reducing water supplies would result in a substantial drop in GDP ([Strzepek et al., 2008](#); [Heggy et al., 2021](#); [Mohammed et al., 2026](#)). This suggests that the real value of water is much higher than the value used in the SAM. Therefore, the SAM is adjusted to account for that as well. As a result, a value share of 10% of GDP is assigned to the water input by subtracting the same amount from the other two inputs (capital and labor) proportionally.

Finally, the observed base year values of the controls are used to calibrate the model parameters before the model is solved. For example, the base year values for W_0 and S_0 from table 2 are used to calibrate the parameter a in the extraction cost function φ before solving the model. The model solution then returns the optimal values of the controls starting in period 0, which means that the optimal solution for the first period could deviate from the observed values for consumption, water withdrawal, etc.

Figure 1: The Annual Nile flows at Aswan 1900-1970



The data for the annual Nile flows are taken from (Sindhu, n.d.). The annual mean is estimated at 71 bcm and the standard deviation is 10.5. Figure 1 shows the annual Nile flows from 1900-1970.

4 Simulations

There are at least two main channels through which the changing climate would impact the water supply in the Nile Basin. The first is the impact on the precipitation and thus the river stream flows. The second is that the rising temperature changes the evaporation losses in the stream and reservoirs.⁵

Using climate data, hydrological models try to predict the impact on precipitation and evapotranspiration. The impact of climate change on stream flow in the Nile river basin is inconclusive. Some studies find increases in stream flow until the end of the 21st century (Tariku and Gan, 2018; Amognehegn et al., 2023). Others report that climate change would reduce

⁵The impact of climate change on the water demand side is mainly due to the effect on the water requirement of the crops, which is expected to increase (Gabr, 2023).

stream flow (Worqlul et al., 2018; Roth et al., 2018). Even though the hydrological models do not agree on the direction of the change in the precipitation, there is consensus about increased seasonal and inter-annual variability of these variables. Siam and Eltahir (2017) find that the long term (2000-2100) standard deviation of the annual Nile flow increases by 50%, compared to the 1900-2000 period.

Regarding the evaporation losses specific to Egypt and its largest reservoir, the lake Nasser Gado and El-Agha (2021) report that the evaporation losses in lake Nasser (the main reservoir in Egypt) increase by 0.47, 0.88 and 1.66 bcm by 2030, 2050 and 2100, respectively. A review is provided in Teklesadik et al. (2017). Another element in the impact of climate change on the hydrology of the Nile basin is extreme events. Floods and drought are expected to occur more frequently, which is not explored further here (Elhaddad et al., 2025; Amin et al., 2025).

To construct the scenarios, these predictions of the hydraulic models are used to specify the path of the parameters of annual flows. Four scenarios are simulated. Scenario 1 (Sc.1) is the deterministic scenario where the planner considers only the long-term expected value of Z in her decision making. It is as if there is a water flow equal to the historical mean available for every period, and the model is solved under certainty. In addition to that, there are 3 stochastic scenarios in which Z enters the model as a random flow. Scenario 2 (Sc.2) simulates the default uncertainty using the historical distribution of the flows (figure 1) and assumes that the current mean and the variance persist in the future. Scenario 3 simulates an increase in uncertainty while the mean annual flows remain unchanged. Scenario 4. is the (pessimistic) climate change scenario in which the mean annual flows increase by 10% and the standard deviation increases by 50%. The changes that occur in Sc.3 and Sc.4 are assumed to start in the year 2040 and continue afterwards. Before that, the three stochastic scenarios are identical. Table 3 summarizes the scenarios.

5 Results and Discussion

For the stochastic scenarios, a set of 10 finite states of nature is defined, and the resulting Markov transition probabilities are used to calculate the expectations of the Euler equations as well as the evolution of the stocks in the different states. See appendix A.3 on how the data in figure 1 are used to define the states and construct the probability transition matrix. The solution of the stochastic model defines a contingent plan for each variable in each state of nature, and the model is solved for each possibility of the stochastic variable. The average value for 10 states taken over 10,000 Monte Carlo simulations is reported for each variable.

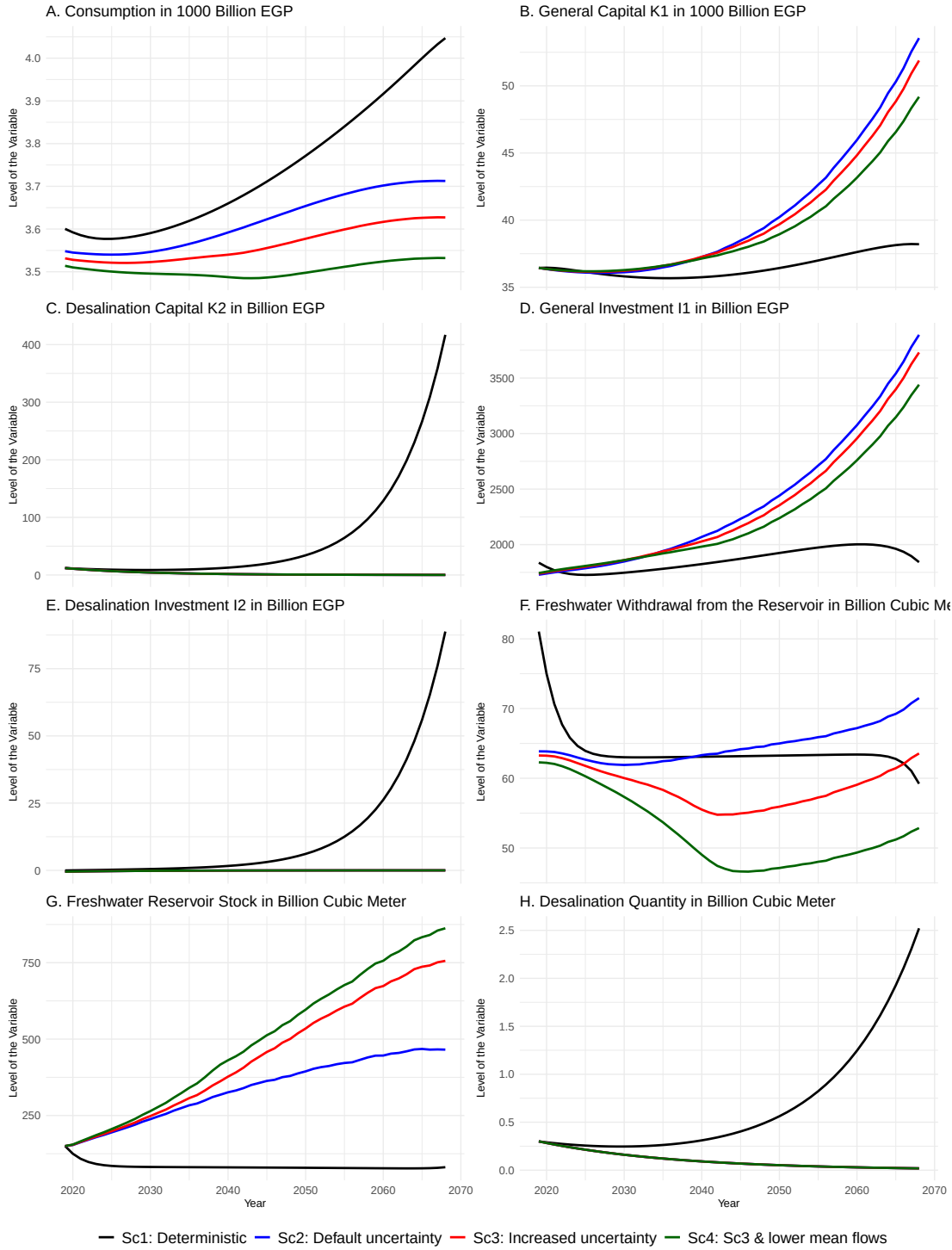
Table 3: Simulated Scenarios

Scenario	Description
Sc.1 Deterministic Baseline without climate change	This is the optimal path of the model under deterministic water flows and without climate change
Sc.2 Stochastic Baseline without climate change	This is the optimal path of the model without climate change under stochastic freshwater flows (current uncertainty)
Sc.3 Stochastic baseline with climate change I	The mean of the annual flow is fixed at current level. The Standard deviation of the annual flows increases by 50% starting in 2040. Before 2040 same std. dev. as in Sc.2
Sc.4 Stochastic Baseline with climate change II (Drought Scenario)	10% drop in the water flows and 50% increase in standard deviation both starting in 2040. Before 2040, the same mean and std. dev. of the flows as in Sc.2.

Since the path under each scenario (as in any numerical analysis) depends on parameters for which it is hard to get a grip on their empirical estimation, it might be reasonable to view these results in terms of inter-path comparison. For another set of assumed parameters, one would get a different path. So I compare the path of each variable under the four different scenarios to understand how the optimal path is impacted by the change in the mean or the standard deviation of the stochastic variable Z . It is important to note here that the deterministic steady state is not imposed as a boundary condition under stochastic scenarios. Instead, the steady state is used as a starting point for the solution ([Li and Swain, 2016](#)). That is why the deterministic model and the stochastic models would have different end-period states even though they share the same initial point.

Figure 2 plots for the main variables of the model the whole path under four different scenarios as described in table 3. Starting with aggregate consumption (subfigure A), the path under the deterministic baseline (Sc.1) is higher than all other paths under uncertainty. The uncertainty clearly reduces consumption. In Sc.2, in which the mean flows are the same as in Sc.1, the consumption is lower. Comparing Sc.3 with Sc.4 – where both simulate the same

Figure 2: The path of model variables under four different scenarios. Sc.1 is the deterministic scenario with mean, $\mu_Z = 71$ and zero variance. Sc.2 is the default uncertainty with the same mean, and $\sigma = 10.5$. Sc.3. has same mean and $\sigma = 15.75$ (50% increase in volatility). Sc.4 has mean, $\mu_Z = 63.9$ (10% drop in the mean) and $\sigma = 15.75$.



uncertainty level and differ only with regard to the mean – reveals that the decline in the mean annual flows by 2040 reduces consumption long before the decline in the annual mean occurs. The two scenarios exhibit parallel paths where the vertical distance between these two lines is due to the different mean annual flow in Sc.4 compared to Sc.3.

To put this into perspective, the decline in consumption occurs to the advantage of capital stock (subfigure B) and investment in Capital K_1 (subfigure D). Both are higher under uncertainty than under the deterministic path. The reason is that capital K_1 is a substitute for water in production and the drop in flows mean or increased uncertainty motivates increases in investment and stocks of type 1 capital are larger. Even though the initial capital is the same under the 4 scenarios, the steady state differs as mentioned above.

Interestingly, the story is reversed for the desalination capital (Subfigure C) and the investment in the desalination capital (Subfigure E), where uncertainty reduces the investment in the desalination sector and thus the capital stock of K_2 is much lower under uncertainty. The investment level I_2 is almost zero for stochastic scenarios. As a consequence, the production of desalinated water is very low under uncertainty (Subfigure H). In contrast, the quantity of desalination in the deterministic case is eight times its initial level. The current model assumes an investment adjustment cost for this sector and it seems that this has a large effect on the decision to invest in the desalination sector when freshwater flows are stochastic. In the deterministic case, investment in desalination is made despite the adjustment cost. Thus, I conclude that it is the combination of partial irreversibility of investment and uncertainty that make the investment in desalination sector economically infeasible.

Considering the optimal freshwater management system, in subfigures F and G, we see that more water is stored in the reservoir under uncertainty than in the deterministic case. The higher the uncertainty, the higher the freshwater storage level (compare Sc.2 with Sc.3) for a given mean flows. Even when the mean is lower, Sc.4, the storage level is close to Sc.3 with the same uncertainty level. In the deterministic case the storage and withdrawal level exhibit a smooth path after initial adjustments. This strong response of reservoir storage is partly due to the specific property of the CARA utility function, which exhibits precautionary behavior. Avoiding the risk of flow volatility leads to excessive water storage. Since uncertainty increases storage, water withdrawal is lower under stochastic scenarios (Subfigure F). The impact of climate change shock which starts in 2040 on the different variables including the freshwater management starts from period one (since the shock is known to be happening at that year and continues afterwards as a recurrent independent shock from the same distribution). Increases in water stocks (mainly ground water stocks) due to uncertainty are reported by many authors,

but to a much lower degree (Li and Swain, 2016; Knapp and Olson, 1996). The storage of water has two main goals in the current model. The first is to avoid water shortages in drought periods, which would reduce utility. Second, the storage level affects the extraction cost which is higher when the storage level is low. These two channels cause the model to react strongly to the volatility of the freshwater flows. Further investigation might be needed to test the sensitivity of these results to the specification of the utility function and the extraction cost function. With regard to the first aspect, a constant relative risk aversion (CRRA) utility form might be more appropriate.

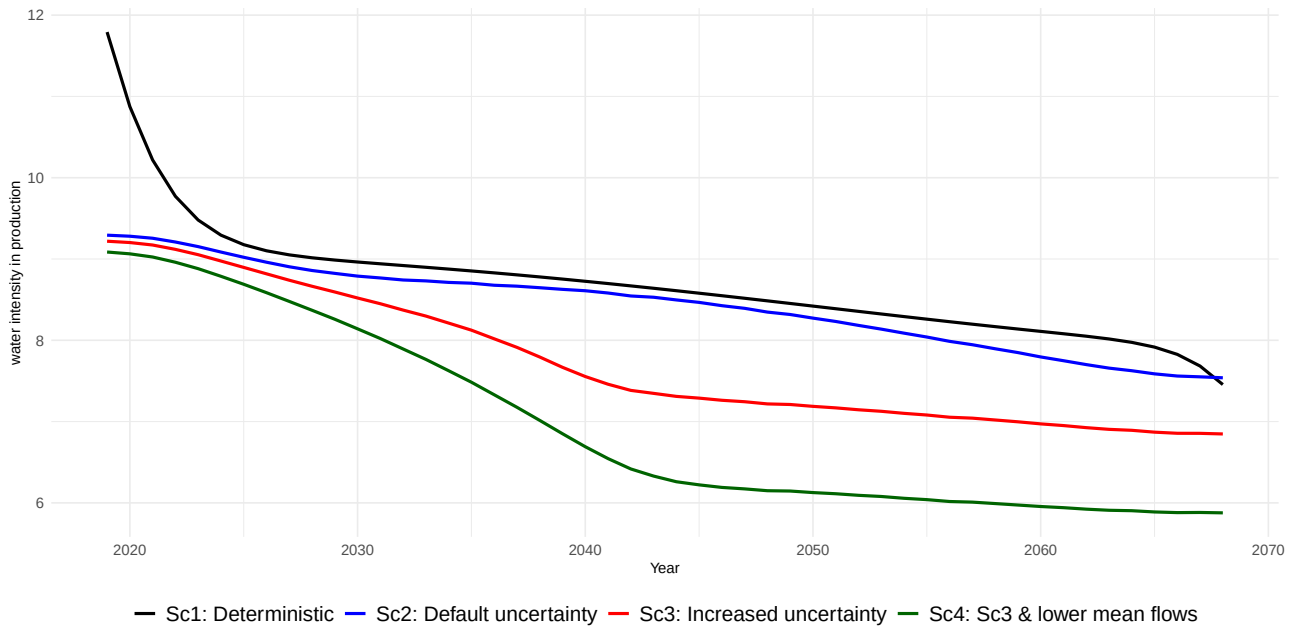
An interesting result here is that in the deterministic case, the storage is low when the desalination is high. A result that was derived analytically in eq. 23. In the stochastic case, these findings reveal that storing water is cheaper than investing in desalination. Overall, the current analysis shows that increased volatility imposes additional challenges on the economy compared to a baseline where current volatility remains unchanged in the future. The increased uncertainty and the resulting high level of water storage require in practice an expansion of the current storage capacity. The level of investment in desalination is high in the deterministic case and very low in all stochastic scenarios. The main caveat here is that a key aspect in reservoir management is left out, namely the flood risk. A more realistic modeling would need to account for the trade off between the drought risk and flood risk, where the latter increase as the stock level approaches the storage capacity limit.

A key question in the current analysis is how the production side is affected by changes in water availability and volatility. Figure 3 plots the water intensity (amount of water per unit of output) for the four different scenarios. One can clearly see that uncertainty leads to a drop in water intensity in production. There is a substitution towards capital. More capital and less water are used in production. This substitution effect is higher, the higher the volatility. In addition, the endowment effect matters (Sc.4) encouraging a stronger shift towards less water-intensive production patterns. Again, this adjustment begins much earlier than the occurrence of the shock which starts in year 2040. Even in the deterministic case, the economy becomes less water-intensive over time. That is because population growth increases the demand for water in final consumption and thus less water would be available for production over time.

6 Conclusion

This article studies the impact of uncertainty of renewable freshwater flows on the economy as a whole and its impact on water management including the investment in non-conventional

Figure 3: The water intensity in production (Q/Y) by scenario . Sc.1. is the deterministic scenario with mean, $\mu_Z = 71$ and zero variance. Sc.2. is the default uncertainty with same mean, and $\sigma = 10.5$. Sc.3. has same mean and $\sigma = 15.75$ (50% increase in volatility). Sc.4. has mean, $\mu_Z = 63.9$ (10% drop in the mean) and $\sigma = 15.75$.



water production (e.g., desalination). The model is solved recursively as a stochastic Bellman equation subject to the model constraints. The numerical solution shows that uncertainty results in higher levels of storage and lower levels of investment in the desalination sector. The uncertainty reduces consumption, but increases the investment in water-substituting capital. If the mean annual flows are considered by the social planner and uncertainty is ignored – the deterministic case – then investment in desalination is undetracted and a less restrictive water management regime is adopted. This article introduces a framework for modeling structural (recurrent) uncertainties in general equilibrium models that are suitable for numerical solutions. It contributes to the discussion on the climate change and adaptation policies in water availability and water management, including the costly investment in non-conventional water technologies. However, the framework is general and can be applied to study a wide range of multi-sectoral optimal control problems.

References

- Amin, D., Keshta, E., Ragab, R., Chakraborty, R. and Elaksher, A. F. (2025), ‘Impact of climate change on hydrological extremes (floods and droughts) in the upper blue Nile basin’, Earth Systems and Environment pp. 1–20.
- Amognehegn, A. E., Nigussie, A. B., Ayalew, D. W., Abera, F. F. and Ayana, M. (2023), ‘Evaluating climate change impact on the hydrology of Kessie watershed, upper blue Nile basin, Ethiopia’, Applied Water Science **13**(7), 148.
- AQUASTAT (2020), ‘Aquastat main database’. Accessed: 2026-05-19.
URL: <https://data.apps.fao.org/aquastat/?lang=en>
- AQUASTAT, D. (n.d.).
URL: <https://www.fao.org/aquastat/en/databases/dams/>
- Barbier, E. B. (2004), ‘Water and economic growth’, Economic Record **80**(248), 1–16. doi:10.1111/j.1475-4932.2004.00121.x.
- Beça, P., Rodrigues, A. C., Nunes, J. P., Diogo, P. and Mujtaba, B. (2023), ‘Optimizing reservoir water management in a changing climate’, Water Resources Management **37**(9), 3423–3437.
- Bertsekas, D. and Tsitsiklis, J. N. (2008), Introduction to probability, Vol. 1, Athena Scientific.

- Bloom, N. (2009), 'The impact of uncertainty shocks', Econometrica **77**(3), 623–685.
- Burt, O. R. (1964), 'Optimal resource use over time with an application to ground water', Management Science **11**(1), 80–93.
- Caballero, R. J. (1990), 'Consumption puzzles and precautionary savings', Journal of Monetary Economics **25**(1), 113–136.
- Digna, R. F., Mohamed, Y. A., van der Zaag, P., Uhlenbrook, S., van der Krogt, W. and Corzo, G. (2018), 'Impact of water resources development on water availability for hydropower production and irrigated agriculture of the Eastern Nile Basin', Journal of Water Resources Planning and Management **144**(5), 05018007. doi:10.1061/(asce)wr.1943-5452.0000912.
- Elhaddad, H., Sultan, M., Yan, E., Tran, D., Torres-Urbe, H. E. and Karimi, H. (2025), 'Nile basin flow regimes under 21st century climate variability', Communications Earth & Environment **6**(1), 880.
- Elkholy, M. (2021), Assessment of water resources in egypt: Current status and future plan, in 'Groundwater in Egypt's Deserts', Springer, pp. 395–423.
- Elsaie, Y., Ismail, S., Soussa, H., Gado, M. and Balah, A. (2023), 'Water desalination in egypt; literature review and assessment', Ain Shams Engineering Journal **14**(7), 101998.
- Gabr, M. E. (2023), 'Impact of climatic changes on future irrigation water requirement in the middle east and north africa's region: a case study of upper egypt', Applied Water Science **13**(7), 158.
- Gado, T. A. and El-Agha, D. E. (2021), Climate change impacts on water balance in egypt and opportunities for adaptations, in 'Agro-environmental sustainability in MENA regions', Springer, pp. 13–47.
- Garrote, L. (2017), 'Managing water resources to adapt to climate change: facing uncertainty and scarcity in a changing context', Water Resources Management **31**(10), 2951–2963.
- Ghaffour, N., Missimer, T. M. and Amy, G. L. (2013), 'Technical review and evaluation of the economics of water desalination: Current and future challenges for better water supply sustainability', Desalination **309**, 197–207.

- Heggy, E., Sharkawy, Z. and Abotalib, A. Z. (2021), ‘Egypt’s water budget deficit and suggested mitigation policies for the grand ethiopian renaissance dam filling scenarios’, Environmental Research Letters **16**(7), 074022.
- Hertel, T. and Liu, J. (2019), Implications of water scarcity for economic growth, Springer. doi:10.1007/978-981-13-6101-2.
- Howitt, R. E., Msangi, S., Reynaud, A. and Knapp, K. C. (2005), ‘Estimating intertemporal preferences for natural resource allocation’, American Journal of Agricultural Economics **87**(4), 969–983.
- Jones, E., Qadir, M., van Vliet, M. T. H., Smakhtin, V. and Kang, S.-m. (2019), ‘The state of desalination and brine production: A global outlook’, Science of the Total Environment **657**, 1343–1356.
- Knapp, K. C. and Olson, L. J. (1996), ‘Dynamic resource management: Intertemporal substitution and risk aversion’, American Journal of Agricultural Economics **78**(4), 1004–1014.
- Lamei, A., Van der Zaag, P. and Von Münch, E. (2008), ‘Basic cost equations to estimate unit production costs for ro desalination and long-distance piping to supply water to tourism-dominated arid coastal regions of egypt’, Desalination **225**(1-3), 1–12.
- Lau, M. I., Pahlke, A. and Rutherford, T. F. (2002), ‘Approximating infinite-horizon models in a complementarity format: A primer in dynamic general equilibrium analysis’, Journal of Economic Dynamics and Control **26**(4), 577–609.
- Li, C.-Z. and Swain, R. B. (2016), ‘Growth, water resilience, and sustainability: a dsge model applied to south africa’, Water Economics and Policy **2**(4), 1650022.
- Mas-Colell, A., Whinston, M. D., Green, J. R. et al. (1995), Microeconomic theory, Vol. 1, Oxford university press New York.
- Mengistu, D., Bewket, W., Dosio, A. and Panitz, H.-J. (2021), ‘Climate change impacts on water resources in the upper blue Nile (Abay) river basin, Ethiopia’, Journal of Hydrology **592**, 125614.
- Mohammed, A., Zabel, F. and Delzeit, R. (2026), ‘Improving water policy analysis in CGE models: Deriving substitution elasticities from biophysical crop data in a case study for Egypt’, Journal of Water Economics .

National Water Resources Plan – 2017 (n.d.), Technical report, Ministry of Water Resources and Irrigation, 2005.

URL: <https://shorturl.at/iyYqC>

Report (2016), national water resources managment plan 2050, Technical report, Ministry of water resources and irrigation, Egypt (published in Arabic).

Romer, D. (2001), ‘Advanced macroeconomics 2nd ed’.

Roth, V., Lemann, T., Zeleke, G., Subhatu, A. T., Nigussie, T. K. and Hurni, H. (2018), ‘Effects of climate change on water resources in the upper blue Nile basin of Ethiopia’, Heliyon **4**(9), e00771.

Serag, E., Ibrahim, F., El Araby, Z., Abd El Latif, M., El Sarawy, M., El Zababawy, D., El Dib, S. A., Salem, K., Breisinger, C. and Raouf, M. (2021), A 2019 Nexus Social Accounting Matrix for Egypt, Vol. 35, Intl Food Policy Res Inst. doi:10.2499/p15738coll2.134544.

Siam, M. S. and Eltahir, E. A. (2017), ‘Climate change enhances interannual variability of the Nile river flow’, Nature Climate Change **7**(5), 350–354.

Sindhu, L. (n.d.), ‘Flow of the river Nile’.

URL: <https://www.kaggle.com/datasets/lsind18/flow-of-the-river-nile>

Solow, R. M. and Wan, F. Y. (1976), ‘Extraction costs in the theory of exhaustible resources’, The Bell Journal of Economics pp. 359–370.

Stokey, N. L. and Lucas Jr, R. E. (1989), Recursive methods in economic dynamics, Harvard University Press.

Strzepek, K. M., Yohe, G. W., Tol, R. S. and Rosegrant, M. W. (2008), ‘The value of the high Aswan Dam to the Egyptian economy’, Ecological Economics **66**(1), 117–126. doi:10.1016/j.ecolecon.2007.08.019.

Sydsæter, K. and Hammond, P. J. (2008), Essential mathematics for economic analysis, Pearson Education.

Sydsæter, K., Seierstad, A. and Strom, A. (2008), Further mathematics for economic analysis, Pearson Education.

- Tariku, T. B. and Gan, T. Y. (2018), ‘Regional climate change impact on extreme precipitation and temperature of the Nile river basin’, Climate Dynamics **51**(9), 3487–3506.
- Tauchen, G. (1986), ‘Finite state markov-chain approximations to univariate and vector autoregressions’, Economics Letters **20**(2), 177–181.
- Teklesadik, A. D., Alemayehu, T., Van Griensven, A., Kumar, R., Liersch, S., Eisner, S., Tecklenburg, J., Ewunte, S. and Wang, X. (2017), ‘Inter-model comparison of hydrological impacts of climate change on the upper blue Nile basin using ensemble of hydrological models and global climate models’, Climatic Change **141**(3), 517–532.
- Worqlul, A. W., Dile, Y. T., Ayana, E. K., Jeong, J., Adem, A. A. and Gerik, T. (2018), ‘Impact of climate change on streamflow hydrology in headwater catchments of the upper blue Nile basin, Ethiopia’, Water **10**(2), 120.

Appendix

A.1 Derivation of the model

The instantaneous utility function of the representative household over the consumption of the aggregate good C is assumed to exhibit a constant absolute risk aversion equal to γ . Thus, the total expected utility of the infinitely living household viewed in period 0 is:

$$\mathbb{E} \left[\sum_0^{\infty} -\beta^t e^{-\gamma C_t} \right]$$

subject to:

$$\begin{aligned} K'_1 &= (1 - \delta)K_1 + I_1 \\ K'_2 &= (1 - \delta)K_2 + I_2 \\ S' &= (1 - \nu)S + Z - W \\ Y &= A_1 Q^{\alpha_1} K_1^{\alpha_2} L^{\alpha_3} = X + I_1 + I_2 + \frac{b}{2} \frac{I_2^2}{K_2} + \varphi(S)W \\ D &= A_2 K_2^{\theta} = Q + H - W \\ C &= a_c (\lambda X^{\rho} + (1 - \lambda)H^{\rho})^{1/\rho} \\ L_{t+1} &= (1 + n)L_t \end{aligned} \tag{A.1}$$

where the prime over the variable refer to the $t + 1$ period. The stochastic Bellman equation is the sum of the utility in the current period plus the discounted expected value of the assets (endowments) in the next period. By substituting from the definitions of C and the state variables, the Bellman equation is written in terms of the control X, Q, W and I_2 (What these variables describe is defined in the main text):

$$\begin{aligned} V(K_1, K_2, S) = \max_{X, Q, W, I_2} & \left\{ -e^{-\gamma a_c (\lambda X^{\rho} + (1 - \lambda)(W + D - Q)^{\rho})^{1/\rho}} \right. \\ & + \beta \mathbb{E} \left[V \left((1 - \delta)K_1 + A_1 Q^{\alpha_1} K_1^{\alpha_2} L^{\alpha_3} - X - I_2 - \frac{b}{2} \frac{I_2^2}{K_2} - \varphi(S)W, \right. \right. \\ & \left. \left. (1 - \delta)K_2 + I_2, (1 - \nu)S + Z - W \right) \right] \left. \right\} \end{aligned} \tag{A.2}$$

An optimal solution has to satisfy the following First order conditions FOCs:

$$\frac{d}{dX} : \quad \gamma\lambda \left(\frac{C}{X}\right)^{1-\rho} e^{-\gamma C} = \beta \mathbb{E} [V_{K'_1}] \quad (\text{A.3})$$

$$\frac{d}{dQ} : \quad \gamma(1-\lambda)(W+D-Q)^{\rho-1} C^{1-\rho} e^{-\gamma C} = \beta \frac{\alpha_1 Y}{Q} \mathbb{E} [V_{K'_1}] \quad (\text{A.4})$$

$$\frac{d}{dW} : \quad \gamma(1-\lambda)(W+D-Q)^{\rho-1} C^{1-\rho} e^{-\gamma C} = \beta \varphi(S) \mathbb{E} [V_{K'_1}] + \beta \mathbb{E} [V_{S'}] \quad (\text{A.5})$$

$$\begin{aligned} \frac{d}{dI_2} : \quad & \beta \mathbb{E} \left[V_{K'_1} \left(-1 - b \frac{I_2}{K_2} \right) + V'_{K_2} \right] = 0 \\ \Rightarrow \quad & \left[1 + b \frac{I_2}{K_2} \right] \mathbb{E} [V_{K'_1}] = \mathbb{E} [V_{K'_2}] \end{aligned} \quad (\text{A.6})$$

Combining equations A.3 and A.4 to get:

$$\begin{aligned} \gamma(1-\lambda)(W+D-Q)^{\rho-1} C^{1-\rho} e^{-\gamma C} &= \frac{\alpha_1 Y}{Q} \gamma\lambda \left(\frac{C}{X}\right)^{1-\rho} e^{-\gamma C} \\ \Rightarrow \quad (1-\lambda)(W+D-Q)^{\rho-1} &= \lambda \frac{\alpha_1 Y}{Q} X^{\rho-1} \end{aligned} \quad (\text{A.7})$$

combine A.4 and A.5 (note that the lhs in these equations are identical):

$$\begin{aligned} \beta \frac{\alpha_1 Y}{Q} \mathbb{E} [V_{K'_1}] &= \beta \varphi(S) \mathbb{E} [V_{K'_1}] + \beta \mathbb{E} [V_{S'}] \\ \Rightarrow \quad \left[\frac{\alpha_1 Y}{Q} - \varphi(S) \right] \mathbb{E} [V_{K'_1}] &= \mathbb{E} [V_{S'}] \end{aligned} \quad (\text{A.8})$$

Now combine A.3 and A.8 to get:

$$\left[\frac{\alpha_1 Y}{Q} - \varphi(S) \right] \gamma\lambda X^{\rho-1} C^{1-\rho} e^{-\gamma C} = \beta \mathbb{E} [V_{S'}] \quad (\text{A.9})$$

The system of equations still includes more unknowns than equations. The three additional unknowns are the value functions, where the optimality condition of the system depends on them. In order to replace the value functions $V(K_1)$, $V(K_2)$ and $V(S)$ with known quantities,

I first differentiate A.2 w.r.t. K_1, K_2 and S :

$$V_{K_1} = \beta \mathbb{E} \left[V_{K'_1} \left(1 - \delta + \frac{\alpha_2 Y}{K_1} \right) \right] = \beta \left(1 - \delta + \frac{\alpha_2 Y}{K_1} \right) \mathbb{E} [V_{K'_1}] \quad (\text{A.10})$$

where V_{K_1} and $V_{K'_1}$ denote the derivative of the value function w.r.t. K_1 in periods t and $t+1$, respectively. In this calculation I used the envelope result that the derivative of the value function w.r.t. consumption C_t is zero, or more formally:

$$V_{K_1}(K_1, K_2, S) = V_{K_1}(C^*(K_1), K_1, K_2, S) = \frac{\partial V}{\partial C^*} \frac{dC^*}{dK_1} + \beta \frac{\partial V}{\partial K_1} \mathbb{E} [V(K'_1, K'_2, S')] \quad (\text{A.11})$$

and since the value function V already obtains the maximum consumption we must have $\partial V(C^*(K_1), K_1, K_2, S)/\partial C^* = 0$, where the impact of any change in capital on optimal consumption is negligible and only the direct effect of the change in capital on the value function matters.⁶ It follows that:

$$\frac{\partial V(C^*(K_1), K_1, K_2, S)}{\partial K_1} = \beta \frac{\partial V}{\partial K_1} \mathbb{E} [V(K'_1, K'_2, S')] \quad (\text{A.12})$$

The same applies for K_2 and S :

$$\begin{aligned} V_{K_2} &= \beta \mathbb{E} \left[V_{K'_1} \frac{b}{2} \left(\frac{I_2}{K_2} \right)^2 + V_{K'_2} (1 - \delta) \right] \\ &= \beta \frac{b}{2} \left(\frac{I_2}{K_2} \right)^2 \mathbb{E} [V_{K'_1}] + \beta (1 - \delta) \mathbb{E} [V_{K'_2}] \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} V_S &= \beta \mathbb{E} [V_{K'_1} (-\varphi'(S)W) + V_{S'} (1 - \nu)] \\ &= \beta (1 - \nu) \mathbb{E} [V_{S'}] - \varphi'(S)W \mathbb{E} [V_{K'_1}] \end{aligned} \quad (\text{A.14})$$

Combine A.3 and A.10 to get:

⁶See [Sydsæter and Hammond \(2008\)](#), Ch. 13, for a neat discussion of the envelope theorem and its applications.

$$V_{K_1} = \left(1 - \delta + \frac{\alpha_2 Y}{K_1}\right) \gamma \lambda \left(\frac{C}{X}\right)^{1-\rho} e^{-\gamma C} \quad (\text{A.15})$$

iterate this one step forward, then plug it back into A.3:

$$\left(\frac{C}{X}\right)^{1-\rho} e^{-\gamma C} = \beta \mathbb{E} \left[\left(1 - \delta + \frac{\alpha_2 Y'}{K_1'}\right) \left(\frac{C'}{X'}\right)^{1-\rho} e^{-\gamma C'} \right] \quad (\text{A.16})$$

Equation A.16 is the stochastic Euler equation for the intertemporal substitution of consumption and corresponds to eq. 15 in the main text. Using A.8 in A.14:

$$\begin{aligned} V_S &= \beta(1 - \nu) \left[\frac{\alpha_1 Y}{Q} - \varphi(S) \right] \mathbb{E} [V_{K_1'}] - \beta \varphi'(S) W \mathbb{E} [V_{K_1'}] \\ &= \beta \left[(1 - \nu) \left(\frac{\alpha_1 Y}{Q} - \varphi(S) \right) - \varphi'(S) W \right] \mathbb{E} [V_{K_1'}] \end{aligned} \quad (\text{A.17})$$

$$V_S = \left[(1 - \nu) \left(\frac{\alpha_1 Y}{Q} - \varphi(S) \right) - \varphi'(S) W \right] \gamma \lambda X^{\rho-1} C^{1-\rho} e^{-\gamma C} \quad (\text{A.18})$$

Now I can use these results to eliminate the value function expression from the system. Starting with eq. A.5, I replace V_{K_1} with its equivalent from A.15 iterated one step forward and replace V_S with its equivalent from A.18 iterated one step forward to obtain:

$$\begin{aligned} &(1 - \lambda)(W + D - Q)^{\rho-1} C^{1-\rho} e^{-\gamma C} \\ &= \beta \lambda \varphi(S) \mathbb{E} \left[\left(1 - \delta + \frac{\alpha_2 Y'}{K_1'}\right) \left(\frac{C'}{X'}\right)^{1-\rho} e^{-\gamma C'} \right] \\ &\quad + \beta \lambda \mathbb{E} \left[\left((1 - \nu) \left(\frac{\alpha_1 Y'}{Q'} - \varphi(S') \right) - \varphi'(S') W' \right) X'^{\rho-1} C'^{1-\rho} e^{-\gamma C'} \right] \end{aligned} \quad (\text{A.19})$$

Equation A.19 is the Euler equation for intertemporal freshwater reservoir management and corresponds to eq. 17 in the main text. To eliminate the expressions of the value function from Eq. A.6, first I combine A.6 and A.13:

$$\begin{aligned} V_{K_2} &= \beta \frac{b}{2} \left(\frac{I_2}{K_2} \right)^2 \mathbb{E} [V_{K_1'}] + \beta(1 - \delta) \left(1 + b \frac{I_2}{K_2} \right) \mathbb{E} [V_{K_1'}] \\ &= \beta \left[\frac{b}{2} \left(\frac{I_2}{K_2} \right)^2 + (1 - \delta) \left(1 + b \frac{I_2}{K_2} \right) \right] \mathbb{E} [V_{K_1'}] \end{aligned} \quad (\text{A.20})$$

then substituting A.3 into A.20 to obtain:

$$V_{K_2} = \beta \left[\frac{b}{2} \left(\frac{I_2}{K_2} \right)^2 + (1 - \delta) \left(1 + b \frac{I_2}{K_2} \right) \right] \gamma \lambda \left(\frac{C}{X} \right)^{1-\rho} e^{-\gamma C} \quad (\text{A.21})$$

Finally, iterating eq. A.21 one step forward and using this and eq A.3 into A.6 yields:

$$\begin{aligned} \left[1 + b \frac{I_2}{K_2} \right] \gamma \lambda \left(\frac{C}{X} \right)^{1-\rho} e^{-\gamma C} &= \beta \mathbb{E} \left[\left(\frac{b}{2} \left(\frac{I'_2}{K'_2} \right)^2 + (1 - \delta) \left(1 + b \frac{I'_2}{K'_2} \right) \right) \gamma \lambda \left(\frac{C'}{X'} \right)^{1-\rho} e^{-\gamma C'} \right] \\ \Rightarrow \left(1 + b \frac{I_2}{K_2} \right) \left(\frac{C}{X} \right)^{1-\rho} e^{-\gamma C} &= \beta \mathbb{E} \left[\left(\frac{b}{2} \left(\frac{I'_2}{K'_2} \right)^2 + (1 - \delta) \left(1 + b \frac{I'_2}{K'_2} \right) \right) \left(\frac{C'}{X'} \right)^{1-\rho} e^{-\gamma C'} \right] \end{aligned} \quad (\text{A.22})$$

Equation A.22 defines the optimal investment allocation between the Y sector and the desalination sector and corresponds to eq. 18 in the main text.

A.2 Steady state

Now let us turn our attention to studying the stationary version of the deterministic model, i.e., the steady state. Starting with eq. A.16 and imposing the condition that total consumption increases at the rate of population growth n – which implies that the growth rate of per capita consumption is zero. The same is true for X and H . Moving the marginal utility on one side, eq A.16 can be written as :

$$\frac{u'(C_{t+1})}{u'(C_t)} = \frac{1}{\beta \left[1 - \delta + \alpha_2 \frac{Y^*}{K_1^*} \right]} \approx 1 + n \quad (\text{A.23})$$

$$\Rightarrow 1 - \delta + \alpha_2 A_1 (Q^*)^{\alpha_1} (K_1^*)^{\alpha_2 - 1} L^{\alpha_3} = \frac{1}{\beta(1+n)} \quad (\text{A.24})$$

Since C_{t+1} is equal to $(1+n)C_t$ in the steady state, the ratio of marginal utility of total consumption, $u'(C)$ and $u'((1+n)C)$ in the steady state is $(1+n) \exp(-\gamma n C^*)$. Taking the Taylor approximation around $n_0 = 0$, the ratio can be written as $1 + n - \gamma n C^*$. For small γ the term $1+n$ is a reasonable (and convenient) approximation of the change in the marginal utilities

of total consumption. Small γ means that the curvature of the utility function, which is equal to $-\gamma^2 \exp(-\gamma C)$, approaches zero and resembles more and more a linear utility function. In the current model γ is set to 0.5 (table 2). For higher values of γ , using the expression $1 + n - \gamma n C^*$ would be better. In general, The use of the term $1 + n$ is rather inferior because it states that the marginal utility increases by the same rate as consumption, which is not exactly true for concave utilities like the one we have here (this occurs due to the exclusion of the term $-\gamma n C$ which accounts for exactly this).

Solving equation A.24 for the steady state level of K_1 to get:

$$K_1^* = \psi (Q^*)^{\frac{\alpha_1}{1-\alpha_2}} L^{\frac{\alpha_3}{1-\alpha_2}} \quad \text{where } \psi = \left(\frac{\alpha_2 A_1}{\frac{1}{\beta(1+n)} - 1 + \delta} \right)^{\frac{1}{1-\alpha_2}} \quad (\text{A.25})$$

Equation A.25 corresponds to eq. 20 in the main text. Now, let us define some useful expressions that will be used in determining the steady state of the remaining model variables:

$$\begin{aligned} \frac{Y^*}{Q^*} &= A_1 (Q^*)^{\alpha_1-1} (K_1^*)^{\alpha_2} L^{\alpha_3} \\ &= A_1 \psi^{\alpha_2} (Q^*)^{\frac{\alpha_1+\alpha_2-1}{1-\alpha_2}} L^{\frac{\alpha_3}{1-\alpha_2}} \\ &= A_1 \psi^{\alpha_2} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1-\alpha_2}} \end{aligned} \quad (\text{A.26})$$

and:

$$\frac{Y^*}{K_1^*} = A_1 (Q^*)^{\alpha_1} \psi^{\alpha_2-1} (Q^*)^{-\alpha_1} L^{-\alpha_3} L^{\alpha_3} = A_1 \psi^{\alpha_2-1} \quad (\text{A.27})$$

The first steady state condition for the water management system is calculated from A.19, after using A.7 to substitute the left hand side in A.19:

$$\begin{aligned} \alpha_1 \frac{Y}{Q} X^{\rho-1} C^{1-\rho} e^{-\gamma C} &= \beta \left[\left(\varphi(S) \left(1 - \delta + \alpha_2 \frac{Y}{K_1} \right) \right. \right. \\ &\quad \left. \left. + (1 - \nu) \left(\alpha_1 \frac{Y}{Q} - \varphi(S) \right) - \varphi'(S) W \right) \left(\frac{C'}{X'} \right)^{1-\rho} e^{-\gamma C'} \right] \end{aligned} \quad (\text{A.28})$$

Now I impose the same condition on the ratio of marginal utilities in the steady state as in A.23 :

$$\frac{\left(\frac{C'}{X'}\right)^{1-\rho} e^{-\gamma C'}}{\left(\frac{C}{X}\right)^{1-\rho} e^{-\gamma C}} = 1 + n = \frac{\alpha_1 Y^*}{\beta Q^*} \left[\varphi(S^*) \left(1 - \delta + \alpha_2 \frac{Y^*}{K_1^*} \right) + (1 - \nu) \left(\alpha_1 \frac{Y^*}{Q^*} - \varphi(S^*) \right) - \varphi'(S^*) W^* \right]^{-1}$$

$$\implies \alpha_1 \frac{Y^*}{Q^*} = (1 + n) \beta \left[\varphi(S^*) \left(1 - \delta + \alpha_2 \frac{Y^*}{K_1^*} \right) + (1 - \nu) \left(\alpha_1 \frac{Y^*}{Q^*} - \varphi(S^*) \right) - \varphi'(S^*) W^* \right] \quad (\text{A.29})$$

Eq. A.29 is the first steady state equation of the freshwater management system. Using the steady state ratios defined in A.27 and A.26:

$$\alpha_1 A_1 \psi^{\alpha_2} \left(\frac{Q^*}{L} \right)^{\frac{\alpha_3}{\alpha_2 - 1}} = (1 + n) \beta \left[\varphi(S^*) \left(1 - \delta + \alpha_2 A_1 \psi^{\alpha_2 - 1} \right) + (1 - \nu) \left(\alpha_1 A_1 \psi^{\alpha_2} \left(\frac{Q^*}{L} \right)^{\frac{\alpha_3}{\alpha_2 - 1}} - \varphi(S^*) \right) - \varphi'(S^*) W^* \right] \quad (\text{A.30})$$

This equation corresponds to eq. 26 in the main text.

The second steady state condition of the freshwater management system is that the total reservoir quantity S is constant (this is the only model variable for which the steady state growth rate is zero). From the state evolution equation 4:

$$W^* = \bar{Z} - \nu S^* \quad (\text{A.31})$$

where $\bar{Z}$ is the mean of the annual flows. Now, it remains to determine Q^* and K_2^* .

To determine K_2^* , I use the relationship in A.6 as well as the definition of marginal productivity of capital types K_1 and K_2 :

$$Y = A_1 Q^{\alpha_1} K_1^{\alpha_2} L^{\alpha_3} = A_1 (A_2 K_2^\theta - H + W)^{\alpha_1} K_1^{\alpha_2} L^{\alpha_3} \quad (\text{A.32})$$

$$\begin{aligned} \frac{dY^*}{dK_1^*} &= \alpha_2 A_1 (Q^*)^{\alpha_1} (K_1^*)^{\alpha_2 - 1} L^{\alpha_3} \\ &= \alpha_2 A_1 (Q^*)^{\alpha_1} \left(\psi (Q^*)^{\frac{\alpha_1}{1 - \alpha_2}} L^{\frac{\alpha_3}{1 - \alpha_2}} \right)^{\alpha_2 - 1} L^{\alpha_3} \\ &= \alpha_2 A_1 \psi^{\alpha_2 - 1} \end{aligned} \quad (\text{A.33})$$

$$\begin{aligned}
\frac{dY^*}{dK_2^*} &= \frac{\partial Y^*}{\partial Q^*} \cdot \frac{dQ^*}{dK_2^*} = A_1 \alpha_1 (Q^*)^{\alpha_1 - 1} (K_1^*)^{\alpha_2} L^{\alpha_3} A_2 \theta (K_2^*)^{\theta - 1} \\
&= A_1 \alpha_1 A_2 \theta \psi^{\alpha_2} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1 - \alpha_2}} (K_2^*)^{\theta - 1}
\end{aligned} \tag{A.34}$$

Now I can use A.6 in this in addition to the fact that the (value of) marginal product for each capital type must be equal to its expected value, $MPK_i = \mathbb{E}[V_{K_i}]$:⁷

$$\begin{aligned}
\left[1 + b \frac{I_2^*}{K_2^*} \right] \alpha_2 A_1 \psi^{\alpha_2 - 1} &= A_1 \alpha_1 A_2 \theta \psi^{\alpha_2} (Q^*)^{-\frac{\alpha_3}{1 - \alpha_2}} L^{\frac{\alpha_3}{1 - \alpha_2}} (K_2^*)^{\theta - 1} \\
1 + b \frac{I_2^*}{K_2^*} &= \frac{\alpha_1 A_2 \theta \psi}{\alpha_2} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1 - \alpha_2}} (K_2^*)^{\theta - 1} \\
\frac{K_2^* + b I_2^*}{K_2^*} \cdot (K_2^*)^{1 - \theta} &= \frac{\alpha_1 A_2 \theta \psi}{\alpha_2} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1 - \alpha_2}} \\
(K_2^* + b I_2^*) (K_2^*)^{-\theta} &= \frac{\alpha_1 A_2 \theta \psi}{\alpha_2} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1 - \alpha_2}}
\end{aligned} \tag{A.35}$$

I_2^* can be directly obtained from the state equation for K_2 :

$$\begin{aligned}
K_2' - K_2 &= -\delta K_2^* + I_2^* = n K_2^* \quad (\text{total capital grows at rate } n) \\
\Rightarrow \quad I_2^* &= (\delta + n) K_2^*
\end{aligned} \tag{A.36}$$

using A.36 into A.35 we obtain:

$$\begin{aligned}
(K_2^* + b(n + \delta) K_2^*) (K_2^*)^{-\theta} &= \frac{\alpha_1 A_2 \theta \psi}{\alpha_2} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1 - \alpha_2}} \quad : \text{ divide by } K_2 \\
(1 + b(n + \delta)) (K_2^*)^{1 - \theta} &= \frac{\alpha_1 A_2 \theta \psi}{\alpha_2} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1 - \alpha_2}} \\
K_2^* &= \left[\frac{\alpha_1 A_2 \theta \psi}{\alpha_2 [1 + b(n + \delta)]} \left(\frac{Q^*}{L} \right)^{-\frac{\alpha_3}{1 - \alpha_2}} \right]^{\frac{1}{1 - \theta}}
\end{aligned} \tag{A.37}$$

⁷ V_{K_i} is the change in the value function due to a one unit change in capital, which is the marginal value of capital.

Eq. A.37 corresponds to eq. 21 in the main text. The last quantity that needs to be determined is Q^* . For this, I use the market clearing condition given by eq. 8:

$$\begin{aligned} Y^* &= A_1(Q^*)^{\alpha_1}(K_1^*)^{\alpha_2}L^{\alpha_3} = A_1(Q^*)^{\alpha_1} \left(\psi(Q^*)^{\frac{\alpha_1}{1-\alpha_2}} L^{\frac{\alpha_3}{1-\alpha_2}} \right)^{\alpha_2} L^{\alpha_3} \\ &= A_1 \psi^{\alpha_2} (Q^*)^{\frac{\alpha_1}{1-\alpha_2}} L^{\frac{\alpha_3}{1-\alpha_2}} \\ &= X^* + (\delta + n)K_1^* + (\delta + n)K_2^* + \frac{b}{2}(\delta + n)^2 K_2^* + \varphi(S^*)W^* \end{aligned}$$

The only unknown in this expression is X^* , using A.7 to replace it yields:

$$\begin{aligned} A_1 \psi^{\alpha_2} (Q^*)^{\frac{\alpha_1}{1-\alpha_2}} L^{\frac{\alpha_3}{1-\alpha_2}} &= \left[\frac{1-\lambda}{\lambda} (W^* + D^* - Q^*)^{\rho-1} \frac{1}{\alpha_1 A_1} \psi^{-\alpha_2} \left(\frac{Q^*}{L} \right)^{\frac{\alpha_3}{1-\alpha_2}} \right]^{\frac{1}{\rho-1}} \\ &\quad + (\delta + n)K_1^* + (1 + \frac{b}{2}(\delta + n))(\delta + n)K_2^* + \varphi(S^*)W^* \\ \Rightarrow A_1 \psi^{\alpha_2} K_2 \left(\frac{Q^*}{L^{\alpha_3}} \right)^{\frac{1}{1-\alpha_2}} &= \left[\frac{1-\lambda}{\lambda} \frac{1}{\alpha_1 A_1} \psi^{-\alpha_2} \left(\frac{Q^*}{L} \right)^{\frac{\alpha_3}{1-\alpha_2}} \right]^{\frac{1}{\rho-1}} (W^* + D^* - Q^*) \\ &\quad + (\delta + n)K_1^* + \left(\delta + n + \frac{b}{2}(\delta + n)^2 \right) K_2^* + \varphi(S^*)W^* \quad (\text{A.38}) \end{aligned}$$

This equation completes the steady state system and corresponds to eq. 28 in the main text.

A.3 Estimating the transition matrix from the AR(1) process of water flows

The Bellman equation 12 is a function of the stochastic water flows Z_t through S and the expectation can be calculated using the density function of Z . However, the discrete model used here would require a discrete set of states that this stochastic variable can take. Therefore, the density function is replaced by a transition function (see [Stokey and Lucas Jr \(1989\)](#), Ch. 8). The transition function P_{ij} describes the probability that the flows in year $t+1$ would move to a state j given that they are in state i in year t .⁸ The Markov property of the process is that for predicting the next period realization, all information about the previous flows is summarized

⁸If Z for instance has two states, low and high, the transition function tells us about the probability that the flows in the next period would remain low, become high given that they are low, for example this year, likewise if they are high in year t .

in the current period (Bertsekas and Tsitsiklis, 2008). First, time series data for annual water flows are used to estimate an AR(1) model with a drift. The model is parameterized as :

$$Z_t = a + \rho Z_{t-1} + \epsilon_t, \quad \epsilon_t \stackrel{\text{iid}}{\sim} N(0, \sigma^2) \quad (\text{A.39})$$

where a is the intercept and the unconditional mean μ is equal to $a/(1 - \rho)$, ρ is a persistence parameter and ϵ is a series of independent annual shocks drawn from the same distribution for every period.⁹

The estimated parameters are as follows: The unconditional mean (long run average) of Z is estimated at 71 bcm, ρ is 0.2, and the unconditional standard deviation σ is estimated at 10.5 bcm. The continuous distribution of the shocks can be approximated by a set of finite states around the unconditional mean according to (Tauchen, 1986). Using these data, a finite set of 10 states is defined where the draws are taken from the area of 2 standard deviations to the right and left of the mean with equal spacing d . The value of Z in state j is given by:

$$z_j = \mu - m\sigma_z + (j - 1)d, \quad j = 1, 2, \dots, 10 \quad (\text{A.40})$$

where m is an interval-defining coefficient set to 2, $\sigma_z^2 = \sigma^2/(1 - \rho^2)$ is the unconditional variance of the flows (long run variance). Table A.1 presents the resulting values of the Z by scenario. The probability that in year $t + 1$ the annual flow is in state j given that it is in state i in year t is given by:

$$\begin{aligned} P_{ij} &= Pr \left[Z_j - \frac{d}{2} \leq a + \rho Z_i + \epsilon_t \leq Z_j + \frac{d}{2} \right] \\ &= Pr \left[Z_j - a - \rho Z_i - \frac{d}{2} \leq \epsilon \leq Z_j - a - \rho Z_i + \frac{d}{2} \right] \end{aligned} \quad (\text{A.41})$$

Finally, imposing the normality of the shock, the transition probability is defined by:

$$P_{ij} = \Phi \left(\frac{Z_j - a - \rho Z_i + d/2}{\sigma} \right) - \Phi \left(\frac{Z_j - a - \rho Z_i - d/2}{\sigma} \right) \quad (\text{A.42})$$

for interior states $j = 2, 3, \dots, n - 1$

⁹That is because an AR(1) in R is parametrized as $Z_t = (1 - \rho)\mu + \rho Z_{t-1} + \epsilon_t$ which makes the intercept equal to $(1 - \rho)\mu$

where Φ is the cumulative distribution function of the standard normal distribution.
For the boundary states:

$$\begin{aligned} P_{i1} &= \Phi\left(\frac{Z_1 - a - \rho Z_i + d/2}{\sigma}\right) \\ P_{in} &= 1 - \Phi\left(\frac{Z_n - a - \rho Z_i - d/2}{\sigma}\right) \end{aligned} \quad (\text{A.43})$$

The resulting values of the probability transition matrix and are given in table A.2:

Table A.1: The values of the stochastic flow in 10 different states of nature by scenario. Sc.1 is the deterministic scenario with mean, $\mu_Z = 71$ in each state. Sc.2 is the default uncertainty with same mean, and $\sigma = 10.5$. Sc.3 has same mean and $\sigma = 15.75$ (50% increase in volatility). Sc.4 has mean, $\mu_Z = 63.9$ (10% drop in the mean) and $\sigma = 15.75$.

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
Sc.2	49.49	54.27	59.05	63.83	68.61	73.39	78.17	82.95	87.73	92.51
Sc.3	38.74	45.91	53.08	60.25	67.42	74.58	81.75	88.92	96.09	103.26
Sc.4	31.64	38.81	45.98	53.15	60.32	67.48	74.65	81.82	88.99	96.16

As shown in table A.2, due to the low level of persistence in the data, the estimated ρ is equal to 0.2, there is more concentration around the inner states 4,5 and 6.

Table A.2: The Markov probability transition matrix estimated from the data using Tauchen method

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
z_1	0.084	0.094	0.142	0.175	0.176	0.144	0.097	0.053	0.023	0.012
z_2	0.070	0.084	0.132	0.170	0.179	0.153	0.107	0.061	0.028	0.015
z_3	0.058	0.074	0.122	0.164	0.180	0.161	0.118	0.070	0.034	0.019
z_4	0.047	0.064	0.111	0.156	0.179	0.168	0.128	0.080	0.041	0.025
z_5	0.038	0.056	0.101	0.148	0.177	0.173	0.138	0.090	0.048	0.031
z_6	0.031	0.048	0.090	0.138	0.173	0.177	0.148	0.101	0.056	0.038
z_7	0.025	0.041	0.080	0.128	0.168	0.179	0.156	0.111	0.064	0.047
z_8	0.019	0.034	0.070	0.118	0.161	0.180	0.164	0.122	0.074	0.058
z_9	0.015	0.028	0.061	0.107	0.153	0.179	0.170	0.132	0.084	0.070
z_{10}	0.012	0.023	0.053	0.097	0.144	0.176	0.175	0.142	0.094	0.084