

The Specification of Price and Income Elasticities in Computable General Equilibrium Models: An Application of Latent Separability¹

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Abstract :

This paper contributes to the ongoing debate about the specifications of price and income effects in CGE models. We detail in this paper a procedure which allows to implement in such models any regular configuration of price and income effects. This procedure exploits the advantages of latent separability. By allowing some overlapping in the grouping of commodities, this separability concept offers much more flexibility than other separability structures since substitution between goods runs through many channels. This paper also provides an empirical illustration which demonstrates the applicability of our procedure and which highlights, once again, the substantial bearing of these specifications on CGE results.

Keywords : CGE models, Flexible Functional Forms, Latent Separability

Introduction

Computable General Equilibrium (CGE) models are now widely used in order to examine a wide array of economic issues (trade reform, economic integration, environmental policy,...). The popularity of these economic tools can be partly attributed to their ability to take fully into account inter-sectoral effects of economic shocks. These inter-sectoral effects mainly occur through prices (of goods, primary factors of production, ...) and income, reflecting competition for scarce resources, limited disposable income, ... Accordingly, the specification of price and income effects is a crucial factor for the relevance of CGE models.

This specification of price and income effects is directly connected with the choice of functional forms used to represent production technologies of firms, preferences of households, ... Several papers already highlight the substantial bearing of this choice upon CGE results. Let's mention four papers illustrating four different Flexible Functional Forms (FFF). The first paper by Hertel (1985) considers a CGE model of the New York State economy in order to examine the impact of a system of partial factor subsidies. In this framework, Hertel tests two specifications of production technologies. The first one is based on the Cobb Douglas (CD) functional form, which can be easily introduced in the model but embodies restrictive hypotheses. The second uses the translog (TL)

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functional form, which is much less “convenient” but more flexible and adequate to capture patterns of substitution between production factors.² As expected, this analysis shows the huge impacts of these two specifications on results. Moreover, Hertel compares these results with those obtained using a flexible partial equilibrium model. The simulations’ results indicate that the flexible partial equilibrium model dominates its CD, general equilibrium counterpart, yielding a more accurate approximation to the TL, general equilibrium “base-line” model. This latter result obviously depends on the particular problem under consideration but leads the author to conclude that general equilibrium analysis with restrictive specifications may be of little value for some policy analysis. The second paper by Despotakis and Fisher (1998) focuses on the energy sectors in the California economy. Again two specifications of production technologies are contemplated. The first one uses the Generalized Leontief (GL) functional form while the second relies on fixed coefficients. The authors then simulate the long-run impacts of a doubling of oil price under these two specifications. It comes as no surprise that this experiment leads to a much larger drop in oil use with the GL specification (-34%) than is obtained with the fixed-coefficients version of the model (-11%). More interesting are the differences on aggregate variables and, as a result, policy recommendations. This experiment leads to a strong decrease of gross domestic output with the GL specification (-4%) and a small increase with the alternative one (+0,2%). The third paper by Robinson et al. (1991) investigates the role of functional forms for the specification of import demand functions.³ Using a three-country CGE model, they contrast the standard constant elasticity of substitution (CES) import-aggregation function with the almost ideal demand system (AIDS) formulation. Their analysis also demonstrates that, depending on simulation experiments, the choice of a particular specification has a strong impact on model results. Specifically, for experiments involving growth and tariff protection and thus generating significant income effects, the standard CES specification yields unrealistic terms-of-trade and trade-volume effects while the new AIDS specification does not. Finally, the fourth paper by McKittrick (1998) provides another robust and recent demonstration of the substantial role of functional forms. He developed a CGE model for the Canadian economy. Here the comparison is between the CES and the Normalised Quadratic (NQ) functional forms adopted to represent production technologies as well as consumers’ preferences. Three fiscal experiments are simulated, reflecting “small”, “medium” and “large” policy shocks. It is again found that the choice of functional forms affects not only industry-specific results, but aggregate results as well, even for the small policy shock.

Despite these well-perceived results warning CGE modelers against the use of convenient/restrictive functional forms, many CGE applications still rely on them. Here we make the assumption that two main reasons explain this unsatisfactory situation. Firstly, knowledge of substitution/price/income elasticities is limited. This justification may be relevant in some cases, but clearly inappropriate in other cases. The second reason is more fundamental and is the core subject of this paper. Few modellers have adopted FFFs for the reason that they generally exhibit poor global properties. For instance, Caves and Christensen (1980) demonstrate that the TL and GL FFFs do not satisfy the restrictions of monotonicity and convexity of indirect utility function over all possible range of prices and income. Imposing global properties on FFF is not advantageous because this destroys their flexibility property (cf. Diewert and Wales (1987) on the TL). This lack of global regularity is problematic for equilibrium models (either CGE or partial equilibrium), as it may cause numerical solution methods to fail. Aforementioned papers using FFFs in their CGE models acknowledge this potential issue. In practice, however, it seems that McKittrick only has to deal with the lack of global monotonicity in his experiments. Hence he imposes some parameter

² Flexibility is defined here as the ability to represent production technologies (or consumers’ preferences) without placing any prior restrictions on the full set of price (as well as income) elasticities at a base point.

³ To our knowledge, this paper has never been published in an academic journal but is often quoted in subsequent papers using an AIDS specification (Robinson et al., 1993; Pogany, 1996; Weyerbrock, 1998).

restrictions that “reduce” the flexibility of its CGE model. But the “restricted” model still remains much more flexible compared to CGE models using traditional specifications.

The critical issue of the introduction of FFFs in CGE models was first recognized by Whalley (1986), who considers this as a great challenge to CGE modellers. Thirteen years after, Hertel (1999) revisits Whalley and emphasizes that parameter specification is still an unmet challenge to applied general equilibrium analysis. Very recent works have been initiated to resolve this apparent trade-off between global regularity and flexibility of functional forms specified in CGE models. To our knowledge, these efforts are now mainly oriented towards the introduction of the AIDADS (An Implicitly Direct Additive Demand System) non homothetic demand system pioneered by Rimmer and Powell (1996). This new demand system is globally regular and more flexible in its treatment of income effects than commonly demand systems used in CGE models, having Engel curve of rank three instead of, at most, two. Yu et al. (2003) detail the introduction of this demand system in the well-known GTAP model and then compare it to several traditional demand systems in the context of projections for disaggregated global food demand. As anticipated, they show that the AIDADS demand system represents a substantial improvement, particularly for the rapidly growing developing countries, by allowing to capture changes in income elasticities of demand. However, this new demand system is based on the assumption of implicitly additive preference and therefore is not second-order flexible in its treatment of price effects.

Our overall purpose in this paper is to contribute to this literature by suggesting to resort to the notion of latent separability, recently formalized by Blundell and Robin (2000) but applied for a long time. In a very general way, latent separability generalizes weak separability by allowing some overlapping in the grouping of commodities into different “sub-functions” of the main one (utility function, production function, expenditure function, profit function, ...). Thus latent separability offers much more flexibility than weak separability since substitution between goods runs through more than one channel. This advantage has been exploited in various econometric studies, performed both at the supply and demand side. Perroni and Rutherford (hereafter P&R) (1995) demonstrate theoretically that latent separability applied with CES-like functions overcomes the previous trade-off between flexibility and global regularity. More precisely, they define a class of Regular-Flexible Functional Forms (R-FFFs) which allows the introduction in an equilibrium model of any regular configuration of price and income elasticities. They furthermore illustrate, in the context of a three-input constant returns to scale production function, the relative performance of this family of R-FFFs over well-known FFFs (NQ, TL, GL) (P&R, 1998). Then our contribution stands within this line of research which, to our knowledge, never really reaches CGE modelling. Our practical objectives in this paper are twofold. Firstly, we detail the implementation of a R-FFF in a typical CGE model (static, perfect competition, ...), which allows us to capture any regular pattern of price/income effects. We focus our presentation on the representation of households’ preferences. Extensions to production technologies, factor mobility, production differentiation by sources are straightforward. Secondly, we once again illustrate the substantial bearing of the specifications of price as well as income effects on CGE results by conducting a carefully designed set of experiments. In that respect, we make use of the simple and well-known CGE model of Harrison et al. (1997). This model is implemented using version 4 GTAP database, and we adopt a commodity/region disaggregation that allows us to introduce robust estimates of price and income elasticities.

This paper is organized as follows. The first section briefly reviews the notion of latent separability and some econometric applications of this notion. The second section, which is the core of this paper, details the different steps required to implement a R-FFF. The third section provides an illustration of the applicability of this approach and of its usefulness. Conclusions and qualifications are offered in the final section.

1. The concept of latent separability

1.1. Definition

The concept of latent separability has been theoretically formalised by Blundell and Robin (2000) but was present in many previous economic papers.⁴ For instance, this concept is implicit in Gorman's papers on the theory of aggregation for capital inputs (Gorman, 1978) or the theory of household production (1980). To present this concept, it is useful to start from the well-known notion of weak separability. Suppose that households' preferences are defined over goods $q_i, i = 1, \dots, n$ and suppose that these can be arranged into $m(< n)$ groups. In this case, weak separability would imply that preferences can be expressed as

$$\Psi(q_1, \dots, q_n) = F(U_1(q^1), \dots, U_m(q^m)) \quad (1)$$

in which the q^k vectors of group specific purchased goods describe a mutually exclusive and exhaustive separation: $q^T = (q_1, \dots, q_n) = (q^{1T}, \dots, q^{mT})$ where T is the transpose vector. $\Psi(\cdot)$ is the direct utility function, F is a regular aggregator utility function and $U_m(\cdot)$ are regular intermediate utility functions.

Latent separability relaxes this partition by allowing some goods to enter more than one group. As these separate inputs cannot be directly observed, we denote the latent input of good i in group k as $\tilde{q}_i^k$ with $\tilde{q}^k = (\tilde{q}_1^k, \dots, \tilde{q}_n^k)$. Under latent separability, preferences can be expressed as :

$$\Psi(q_1, \dots, q_n) = F(U_1(\tilde{q}^1), \dots, U_m(\tilde{q}^m)) \quad s.t. \quad \sum_{k=1}^m \tilde{q}^k = q \quad (2)$$

The allocation problem faced by the representative household with such preferences is then to :

$$\max_{\tilde{q}^1, \dots, \tilde{q}^m} \Psi(q_1, \dots, q_n) = F(U_1(\tilde{q}^1), \dots, U_m(\tilde{q}^m)) \quad s.t. \quad \sum_{k=1}^m \tilde{q}^k = q \quad and \quad p^T q = x \quad (3)$$

where p is the price vector of goods and x represents total expenditure on all goods.

1.2. Properties

Practically, latent separability can be seen within an intermediate production process. Goods are first used to produce "commodities" which are the true arguments of the utility function and not the goods. In other words, market goods are not desired for their own sake but only as inputs into the production of commodities. In the context of production, we can think of a sequential production process that first uses primary inputs to produce intermediate inputs and then uses these intermediates inputs to produce final output. The intuitive appeal of this concept of latent separability is thus clear when one works with aggregate data, like in CGE models.

The concept of latent separability also offers other valuable properties. Latent separability exhibits more flexibility than weak separability since the demand relationship for any good runs through more than one channel. Even if each intermediate utility function is homothetic, there is a wide spectrum of possible income and substitution effects for purchased goods generated from the combination of different groups to which each good belongs.

The notion of latent separability is also interesting in the context of price/commodity aggregation. Because the number of goods in the "real" economy is vast, empirical economic analysis is

⁴ Some of them talk about nonseparability (for instance, Pollak and Wales, 1987). We prefer to use in this paper the latent separability terminology as we believe it more explicit.

commonly carried out at some level of aggregation. This common practice is theoretically correct if one of the two following conditions is satisfied. The first, known as the Hicks-Leontief composite commodity theorem, requires that the prices of those goods which are included in the same aggregate move in parallel. The second, known as the Gorman conditions, imposes separability assumptions that are very restrictive and thus unlikely in practice. In the context of demand, these Gorman conditions state that, either each intermediate utility function is homothetic or the aggregator utility function is strongly separable and each indirect intermediate utility function is of the generalized Gorman polar form. In other words, the Gorman conditions severely restrict substitution possibilities between goods. Under latent separability, prices can be perfectly aggregated without restricting these substitution possibilities (cf. above) and even if they are not collinear, as latent separability represents a property of preferences rather a feature of price data.

The notion of latent separability thus exhibits desirable theoretical properties. This obviously comes with some costs that must be weighted against these theoretical benefits. As far as we are aware, two main related criticisms have been addressed to this notion and can be described as follows in the context of final demand. Only purchased goods are directly observable. Since these can be used by the consumers in more than one group, the latent allocations by the consumers are not directly observable to the econometrician. This obviously complicates the econometric analysis. The second point is that, even if econometric results can be obtained, the resulting composition of groups may be counter-intuitive in the sense that some substitution/complementarity relationships were completely unexpected. From the available literature, it seems that these costs are quite low in comparison to the benefits. In fact, econometric applications using latent separability reveal the usefulness of this concept. For instance, Pollak and Wales (1987) estimate six systems of input demands using three specifications. The first one is the usual TL and serves as the benchmark. The two others make use of latent separability, assume a two-stage technology and use CES intermediate factor technologies. These two specifications only differ in the specification of the aggregator function which can be Leontief or Cobb Douglas. Econometric results show that the Leontief-CES latent specification, which satisfies global regularity conditions, is the preferred specification. Moreover, these two authors conclude their paper by arguing that the notion of latent separability provide a promising approach to specifying interesting and empirically tractable input demand systems. Another strong demonstration is obviously provided by Blundell and Robin (2000) who compare two specifications. The first one assumes weak separability and uses the quadratic almost ideal demand system (QUAIDS). The second assumes latent separability using QUAIDS at the upper level and Cobb Douglas function for intermediate expenditure function. In their empirical application to the demand for 22 nondurables and services commodities using the British Family Expenditure Survey data, they show that latent separability can provide a powerful tool for improving the precision of substitution elasticities while imposing a structure on preferences that is considerably less restrictive than weak separability.

Therefore latent separability offers both theoretical and empirical advantages. The non-observation of latent allocations does not prevent econometric analysis. In our calibration approach, the knowledge of price/income elasticities will allow us to recover these latent allocations and the pattern of substitutions between latent inputs. This is what we explain now.

2. The implementation of any regular configuration of price and income elasticities

2.1. Analytical framework

Our objective in this section is to detail the implementation of any regular configuration of price and income elasticities in a standard CGE model. To ease the presentation, we adopt a highly stylised CGE model of a closed economy, assuming mono-product, constant returns to scale

production technologies, perfect competition, fixed amounts of primary factors of production, one representative household, no taxes/subsidies, no dynamics. This simplified economy can be represented by the six following equations:

$$X_{f,i} = X_{f,i}(Y_i, PF) \quad \forall f = 1, \dots, N_F; \forall i = 1, \dots, N_I \quad (4)$$

$$P_i = P_i(PF) \quad \forall i = 1, \dots, N_I \quad (5)$$

$$\sum_{i=1}^{N_I} X_{f,i} = \bar{X}_f \quad \forall f = 1, \dots, N_F \quad (6)$$

$$Q_i = Q_i(P, R) \quad \forall i = 1, \dots, N_I \quad (7)$$

$$R = \sum_{f=1}^{N_F} PF_f \cdot \bar{X}_f \quad (8)$$

$$Y_i = Q_i \quad \forall i = 1, \dots, N_I \quad (9)$$

with the notation : $i = 1, \dots, N_I$ the index of goods, $f = 1, \dots, N_F$ the index of primary factors of production, $X_{f,i}$ the derived demand of primary factor f by sector i , Y_i the production level of good i , PF_f the market price of primary factor f , P_i the market price of good i , $\bar{X}_f$ the fixed amount of primary factor f , Q_i the final demand of good i and R the income level of the representative household. Equation (4) expresses the derived demands of primary factors of production. Equation (5) is the zero profit condition for each production sector. Equation (6) clears primary factor markets. Equation (7) expresses the final demands of goods by the representative household. Equation (8) describes the income level of this household. Finally, equation (9) is the product market clearing equation.

We focus our analysis on the specification of the final demand equation (7) and assume that initial levels of quantities, prices and income are available (for instance, from a social accounting matrix). Moreover, we assume throughout this paper that income elasticities (η_i), compensated price elasticities (ε_{ij}^h) and/or uncompensated price elasticities (ε_{ij}^m) are also known (for instance, from a literature survey). The common practice is to adopt convenient functional forms (like CD, CES, Linear Expenditure System (LES)) which can be easily implemented but do not reflect the information contained in all price/income elasticities. For example, if a CD is adopted, then equation (7) takes the form:

$$Q_i = \alpha_i R / P_i$$

where α_i is the share of good i in consumer expenditures, income elasticities are all equal to one, uncompensated own price elasticities are all equal to minus one and uncompensated cross price elasticities are all equal to zero.

Our goal is thus the specification of the final demand equations that allows the incorporation of the elasticities' information. As already mentioned, P&R (1995) define a class of R-FFFs, which combines latent separability and CES-like functions, and which allows to capture any regular configuration of price and income elasticities. This class only relies on linearly homogeneous functions which are not directly able to represent nonhomothetic preferences or, in other words, non-unitary income effects. Here we follow the technical suggestions of Perroni (1992) who provides a device in order to get a homothetic representation of regular non-homothetic preferences.

Our implementation procedure may thus be divided in two steps. In the first step, we convert non-homothetic demand functions into homothetic ones. In the second step, we calibrate the new homothetic demand system with one particular R-FFF. These two steps are now explained.

2.2. The implementation of income effects

Perroni (1992) describes a methodology through which well-behaved, non-homothetic preferences can be locally approximated by means of a globally regular, homogeneous expenditure function. The technical trick is to assume that there exists a “hidden” consumed good, in addition to “observable” goods, which is available in a fixed amount for the representative consumer. Perroni furthermore assumes that preferences are homothetic over all goods, including the hidden one. In this setting, non-homotheticity of observable demands arises from differences in the degree of substitutability and complementarity between this hidden good and all other goods.

This idea of the existence of a hidden good is not simply a theoretical curiosity. It also has an appealing economic interpretation which can be attributed to Muth (1966) who stipulates that the production function for any commodity in a household production model is homogeneous of degree one in all the relevant inputs, including fixed labour used within the household. Precisely, Muth argues that income elasticities of demand are the same for all goods used to produce one commodity. He furthermore underlines that inconsistent empirical results, in terms of estimates of income elasticities, might be explained by recognizing the composite nature of demand for many goods and also the importance of household labour in the production of commodities.

In order to implement this idea in our stylised CGE model, we need to make the following modifications. We introduce in our economy a new good (indexed hi for hidden good) and a new primary factor of production (indexed hf for hidden primary factor). We assume that this new good is only consumed by the representative household and is produced with the sole new primary factor of production, according to a fixed proportion technology. Concerning the market of the new primary factor of production, we assume that the total demand is equal to the derived demand for the production of the hidden good and that the total supply is fixed. Our original equations are then replaced by the following ones:

$$X_{hf,hi} = Y_{hi} \quad , \quad X_{f,i} = X_{f,i}(Y_i, PF) \quad \forall f = 1, \dots, N_F; \forall i = 1, \dots, N_I \quad (4')$$

$$P_{hi} = PF_{hf} \quad , \quad P_i = P_i(PF) \quad \forall i = 1, \dots, N_I \quad (5')$$

$$X_{hf,hi} = \bar{X}_{hf} \quad , \quad \sum_{i=1}^{N_I} X_{f,i} = \bar{X}_f \quad \forall f = 1, \dots, N_F \quad (6')$$

$$Q_i = \tilde{Q}_i(P, \tilde{R}, P_{hi}) \quad \forall i = 1, \dots, N_I, hi \quad (7')$$

$$\tilde{R} = R + PF_{hf} \cdot \bar{X}_{hf} = \sum_{f=1}^{N_F} PF_f \cdot \bar{X}_f + PF_{hf} \cdot \bar{X}_{hf} \quad (8')$$

$$Y_i = Q_i \quad \forall i = 1, \dots, N_I, hi \quad (9')$$

Equations (4'), (5'), (6') and (9') are very similar to the original ones, the main difference being the introduction of the new good and/or primary factor. On the other hand, the final demand and income equations are different. Total income $\tilde{R}$ is now equal to the observable income increased by the earning of the new primary factor (equation 8'). This augmented income is now allocated between the observed goods and the new good (equation 7'). The rationale for this procedure is that

it is now possible to express initial income elasticities of the observable goods in terms of compensated price elasticities between observable goods and the hidden good (Perroni, 1992) :

$$\tilde{\varepsilon}_{i,hi}^h = \tilde{\varepsilon}_{hi,hi}^h \left(1 - \frac{\tilde{R}}{R} \eta_i \right) \quad \forall i = 1, \dots, N_I \quad (10)$$

Note that to implement this formulae, one need an initial value for the own compensated price elasticity of the hidden good and one initial value for the expenditure on this hidden good (which is equal to the difference between the augmented income and observable income). These two values can be arbitrarily selected without affecting the local approximation⁵. Original compensated elasticities on observable goods need also to be adapted as follows:

$$\tilde{\varepsilon}_{i,j}^h = \varepsilon_{i,j}^h + \frac{\tilde{\varepsilon}_{i,hi}^h \cdot \tilde{\varepsilon}_{j,hi}^h}{\tilde{\varepsilon}_{hi,hi}^h} \cdot \frac{R}{\tilde{R} - R} \cdot \alpha_j \quad \forall i, j = 1, \dots, N_I \quad (11)$$

where α_i remains the share of observable good i in observable income. One can check that the new system of compensated price elasticities over all goods, including the hidden one, satisfies the theoretical requirements of demand systems (in particular, the homogeneity, symmetry and concavity conditions). In this augmented system, all income elasticities are equal to one:

$$\tilde{\eta}_i = \frac{\partial \ln Q_i}{\partial \ln \tilde{R}} = 1 \quad \forall i = 1, \dots, N_I, hi \quad (12)$$

This first step allows to move the problem from the implementation of price and income elasticities to the implementation of price elasticities only. In other words, all information in terms of elasticities is now available in an augmented matrix of price elasticities.

Before turning to the second phase of our procedure, we note that the addition of these new good/primary factor of production makes the computation of welfare effect more complex. Typical measures of welfare reported with CGE analysis are the Equivalent Variation (EV) and/or the Compensating Variation (CV). Let's focus on EV which is defined by:

$$EV = E(P^0, U^1) - E(P^0, U^0) \quad (13)$$

where $E(P, U)$ is the minimal income which is necessary to reach the utility level U at given good prices P . If an economic change would bring about an increase in welfare, EV represents the minimum amount that the household requires to accept foregoing the change. For an economic project entailing a welfare loss, EV is the maximum amount the household will be willing to pay to avoid the change. The expenditure function is traditionally computed on observable goods and do not include the hidden one. We show how we can get this welfare measure in the presence of a hidden good.

In that respect, we first underline that the utility level of the representative household is unaffected by the addition of this hidden good, because we make explicit something that is implicit in the original modelling. Mathematically, we have:

$$\tilde{V}(P, \tilde{R}, P_{hi}) = V(P, R) = U \quad (14)$$

where the functions $V(\cdot)$ and $\tilde{V}(\cdot)$ are indirect utility functions. With appropriate inversion, we then obtain:

$$\tilde{E}(P, U, P_{hi}) = \tilde{R} = R + P_{hi} \cdot \bar{X}_{ih} = E(P, U) + P_{hi} \cdot \bar{X}_{ih} \quad (15)$$

⁵ Obviously, the own elasticity must be negative and the expenditure positive.

In this equation, it is important to note that the shadow price of the hidden good is an endogenous variable which depends on the prices of observable goods, the utility level and the fixed level of the hidden primary factor. Combining equations (15) and (13), we obtain the EV expression in our modified CGE model:

$$EV = \left(\tilde{E}(P^0, U^1, P_{hi}^1) - P_{hi}^1(P_0, U^1, \bar{X}_{hi}) \bar{X}_{hi} \right) - \left(\tilde{E}(P^0, U^0, P_{hi}^0) - P_{hi}^0(P_0, U^0, \bar{X}_{hi}) \bar{X}_{hi} \right) \quad (16)$$

with $P_{hi}^1(\cdot)$ such that $Q_{ih}(P_0, U^1, P_{hi}^1) = \bar{X}_{ih}$

Accordingly, the computation of EV requires to solve the new demand system (and not the whole new CGE model) in order to obtain the shadow price of the hidden good.⁶

2.3. The implementation of price elasticities

The purpose of the second step of our implementation procedure is to specify the demand equation (7') such that the information available in the new matrix of compensated price elasticities ($\tilde{\varepsilon}$) is introduced in the model. In that respect, we take advantage of the concept of latent separability and follow P&R analysis (1995). We choose the Lower-Triangular Leontief N-stage Nonseparable CES function (LTL-NNCES) among all members of the wide class of R-FFFs. This choice is motivated by the fact that it has just enough free parameters in order to incorporate elasticities' information. For completeness, we briefly present this form in the case of $\tilde{N} = N_I + 1$ goods. Figure 1 represents the nesting structure of this form and allows us to introduce some new notations.

This functional form assumes a $\tilde{N} - 1$ level structure. Each level l contains only two nests which are combined according to a CES function. The “left” subnest (noted QLA^l for Quantity of Leontief Agregate, with composite price PLA^l) is a Leontief aggregate of $\tilde{N} - l$ latent goods (noted LQ_i^l). The “right” subnest (noted QCA^l for Quantity of CES Agregate, with composite price PCA^l) is an intermediate nest for $l < \tilde{N} - 1$. The last right nest ($l = \tilde{N} - 1$) includes only one latent good (noted $LQ_i^{\tilde{N}} = QLA^{\tilde{N}}$). The number of latent goods is given by $1 + \sum_{l=1}^{\tilde{N}-1} l$ and the number of substitution

elasticities is $\tilde{N} - 1$. The number of parameters to be calibrated in order to implement this functional form is therefore equal to $\tilde{N} \cdot (\tilde{N} + 1) / 2$, which is just equal to the sum of the number of independent compensated price elasticities $\tilde{N} \cdot (\tilde{N} - 1) / 2$ and the number of good demands $\tilde{N}$. The calibration of these parameters is detailed in P&R (1995;1998) and is not repeated here. We just note that this calibration is done sequentially, starting from the top of the tree structure. In this calibration procedure, moving from the level l to the level $l + 1$ reduces the number of free parameters by $\tilde{N} - 1 + l$, because each calibration step determines $\tilde{N} - l$ latent allocations and one substitution elasticity.

In order to implement the LTL-NNCES in our stylised CGE model, the demand equation (7') is substituted by the six followings:

⁶ At this stage, let's note that the specification of the non-homothetic Constant Difference of Elasticities (CDE) demand system in the GTAP model suffered from one main defect, which have been resolved only recently (McDougall, 2001). In a very general way, this defect was to assume the fixity of the elasticity of expenditure with respect to utility ($\varepsilon_U^E = \partial \ln E / \partial \ln U$). McDougall proposes a new modelling which allows to take the endogeneity of this elasticity into account in the particular CDE case. We believe here that our approach is more general in that it can be applied to any non-homothetic demand system. In our approach, the issue of endogeneity of this elasticity is “tackled” by the endogeneity of the shadow price of the hidden good.

$$Q_i = \sum_{l=1}^{\tilde{N}} LQ_i^l \quad \forall i = 1, \dots, \tilde{N} \quad (7a)$$

$$LQ_i^l = io_{i,l} \cdot QLA^l \quad \forall i = 1, \dots, \tilde{N}; \forall l = 1, \dots, \tilde{N} \quad (7b)$$

$$PLA^l = \sum_{i=1}^{\tilde{N}} io_{i,l} P_i \quad \forall l = 1, \dots, \tilde{N} \quad (7c)$$

$$\begin{aligned} QLA^l &= QCA^l \cdot CES_{PLA^l}(PLA^l, PCA^{l+1}) \quad \forall l = 1, \dots, \tilde{N} - 2 \\ &= QCA^l \cdot CES_{PLA^l}(PLA^l, PLA^{l+1}) \quad \text{for } l = \tilde{N} - 1 \\ &= QCA^{l-1} \cdot CES_{PLA^l}(PLA^{l-1}, PLA^l) \quad \text{for } l = \tilde{N} \end{aligned} \quad (7d)$$

$$\begin{aligned} QCA^l &= QCA^{l-1} \cdot CES_{PCA^l}(PLA^{l-1}, PCA^l) \quad \forall l = 2, \dots, \tilde{N} - 1 \\ &= \tilde{R} / PCA^l \quad \text{for } l = 1 \end{aligned} \quad (7e)$$

$$\begin{aligned} PCA^l &= CES(PLA^l, PCA^{l+1}) \quad \forall l = 1, \dots, \tilde{N} - 2 \\ &= CES(PLA^l, PLA^{l+1}) \quad \text{for } l = \tilde{N} - 1 \end{aligned} \quad (7f)$$

Equation (7a) simply states that the demand for one good, either observable or hidden, is equal to the sum of the demand for its latent allocation. Equation (7b) determines the level of latent allocation, according to a fixed proportion relationship. Equations (7c), (7d), (7e) and (7f) determine the quantities and prices of “right” and “left” composite aggregates, where the $CES(.)$ notation stands for the CES form and $CES_p(.)$ its first-order derivative with respect to the market price of the considered composite aggregate. One can check from these equations that the demand functions are all linearly homogeneous with respect to total income. This completes our implementation of regular configurations of price and income elasticities in a standard CGE model. We now turn to an empirical application.

3. Illustration

Our objective in this section is to illustrate with a concrete case 1) the practical feasibility of our implementation procedure and 2) the substantial bearing of the specification of the price and income effects in one CGE model. We first describe our empirical framework before conducting illustrative experiments.

3.1. Empirical framework

We use the Harrison, Rutherford and Tarr (HRT) model as the core of our empirical framework. This model is a relatively standard multi-sector CGE model which is, in its simplest version, static, perfectly competitive, and constant returns to scale. This model differs from our highly stylised CGE model in the following aspects. HRT is a multi-region model where bilateral trade is modelled with a nested Armington structure. All distortions, including trade instruments, are represented as ad-valorem price wedges. Production entails the use of intermediate inputs and primary factors of production. These primary factors are mobile across sectors within a region but are internationally immobile. Production technologies are specified according to a CES function for value added, and Leontief functions for intermediates and the value added composite. Total government demand in each region is fixed. Finally, each region has a single representative consumer who allocates her

income across goods so as to maximise welfare. Preferences of this representative consumer are specified with a CD function. We adopt this simple base model as a benchmark.

The core database is GTAP version 4. The aggregation we adopt includes two regions (The United Kingdom and The Rest of the World), and eight sectors (food, alcohol, fuel, clothing, transport, services, other non-durables and other goods). Price and income elasticities of UK final demands are obtained from Blundell et al. (1993). They are reproduced in tables 1.⁷ These estimates are clearly in contradiction with a CD representation of households' preferences. For instance, own-price uncompensated elasticities are far from being all equal to minus one, many cross-price uncompensated elasticities are strictly positive or negative and finally income elasticities range from 0.547 to 1.855. Therefore, we develop another model, hereafter labelled flexible model, which captures these elasticities. This is the sole difference with the base model. In order to implement this flexible model, we follow the two-step procedure described above. As already indicated, we need to specify two arbitrary values in the first step, the value of the hidden good and its own compensated price elasticity. We try different values and effectively observe that they have no impacts on the local approximation of the price/income elasticities. On the other hand, they have some impacts on the evolution of these elasticities or, in other words, on the third order curvature properties. Table 2a) reports one sensitivity analysis on the income effects with three different values of the own price uncompensated elasticity of the hidden good (-1.75, -1 and -0.25). In this sensitivity analysis, the share of the hidden good in total income is fixed at 5% and the prices of all observable goods are kept fixed. When the observable income increases by 10%, the effects on demand are nearly independent of the value of the own price compensated elasticity of the hidden good. As expected, this is no longer the case when the observable income increases by 50% but one must note that, whatever the value of this elasticity is, the initial calibration information is preserved. For instance, food expenditure still increases less than observable income, while services or transport increase by higher percentages. Furthermore, we observe that the income elasticities are nearly constant with the -1.75 choice. In the forthcoming experiments, we adopt this high value and accordingly put the emphasis on the preservation of the information about income elasticities. The results of the second step of our implementation procedures are available in table 2b. This table reports the substitution elasticities at the seven levels and the latent allocations. Precisely, we provide the shares of each good accruing to each level (LQ_i^l / Q_i).

3.2. Simulations

We perform four experiments with the base and flexible models. These experiments are purely illustrative and designed in order to demonstrate the usefulness of our approach. The first experiment, labelled the food experiment, considers the removal of all trade instruments (export and import taxes) in the food sector. The second experiment, labelled the trade experiment, considers full liberalisation, i.e. the removal of all trade instrument in all sectors. The third experiment, labelled the income experiment, considers a 5% increase of the supply of three primary factors of production (skilled labour, unskilled labour and capital). Finally, experiment four, labelled the full experiment, combines the trade and income experiments. This sequence of experiments will allow us to progressively appreciate the benefits/weaknesses of both specifications.

For all experiments, we provide UK market effects (production, final demand, imports and market prices) as well as UK macro-economic effects (observable income and welfare measured by EV).

⁷ The price elasticities reported in Blundell et al. (1993) have been slightly adjusted in order to respect symmetry and concavity conditions. We also slightly modify the initial levels of UK final demands in the GTAP database in order to get the expenditure shares of Blundell et al. Practically, we shift parts of the initial final demands to the government demands for the first seven goods and shift all final demand to government demand for the other durable good (non included in the Blundell et al. estimation). These modifications of the GTAP database are also applied in the benchmark model.

Results of the food experiment are reported in table 3. This experiment “mainly” induces a price effect, i.e. a decrease of the food market price. It is interesting to remark that price and income variations are very similar in both models whereas the impact of the food experiment on final consumption differs widely across models. For instance, food consumption increases by 3.4% with the base model and by “only” 1.8% with the flexible model. In relative terms, the difference between these two results is roughly 50%. This difference is clearly related to the specifications of own price and income elasticities of food, as the following decompositions suggest it:

$$\dot{Q}_{foo} \approx (-1)(-4.27) + (1)(-1) = 3.27 \quad \text{with the base model}$$

$$\dot{Q}_{foo} \approx (-0.54)(-4.29) + (0.55)(-1) = 1.77 \quad \text{with the flexible model.}$$

Moreover, inter-sectoral effects of this experiment vary significantly with the considered model. All goods are net substitutes in the base model, so that final demand of goods other than food decreases. On the other hand, the pattern of substitution between food and all other goods is very general in the flexible model. For instance, food and other non durables are net complements. Therefore, the final demand of other non durables increases by 1% in the flexible model, while it decreases by -0.9% in the base model. Another good example is alcohol. Food and alcohol are strong net substitutes and therefore the food experiment leads to a strong decrease of alcohol consumption (-1.6%) in the flexible model, compared to a much more limited decrease (-0.2%) in the base model. These different effects on final consumption obviously translate into the other market variables. At first sight, it seems astonishing that impacts on imports do not differ significantly between the two models. This simply results from the small shares of imports in total consumption. Accordingly, differences in demand impacts mainly lead to the same differences in production impacts. Finally, it is worth noting that welfare impacts are very similar and, as expected, positive.

The trade experiment generate more price variations, notably with the decrease of alcohol and clothing market prices (cf. tables 4). The main interesting message of this experiment is that results are much more comparable between the two models. There still exist some differences but they are much more muted compared to the food experiment. This simply illustrates that the issue of the specification of prices and income effects in CGE models depends to a large extent to the problem under consideration.

As expected, the income experiment highlights the substantial bearing of income elasticities (cf. table 5). This income experiment leads, in both models, to a strong increase of observable income (around 28%). With the base model, we then get an increase of final demand of around 28% for all goods. With the flexible model, food demand increases by only 15.6% while services demand increase by more than 50%.

Finally, the last experiment combines many price and income variations. This full experiment, which is typical in CGE analysis, clearly supports the general conclusion that the specification of price and income effects do matter for the evaluation of market effects. The critical role of these specifications for the measure of welfare effects is however less clear with our contemplated experiments.

Conclusion

The starting point of this research is the stimulating debate concerning the specifications of price and income effects in CGE models. We detail in this paper a procedure which allows to implement in such models any regular configuration of price and income effects. This procedure exploits the advantages of latent separability. By allowing some overlapping in the grouping of commodities, this separability concept offers much more flexibility than other separability structures since substitution between goods runs through many channels. This paper also provides an empirical illustration which demonstrates the applicability of our procedure. This illustration focuses on the

demand side of the economy, and can be easily extended to production, trade or primary factors blocks. We contrast our proposed specification to a traditional one (Cobb Douglas representation of preferences) with different experiments. In a general way, these experiments underline the substantial bearing of the specification on sectoral results. The results also suggest that the usefulness of the proposed specification depends on the simulations.

Accordingly, we do not consider our approach as a panacea. For example, if one contemplates one simulation with very strong income variation and very little price variations, then a rank three demand system, such as the AIDADS, may be more appropriate. The main difficulty here is to anticipate what will be the main effects and for many economic analysis, these are only revealed *ex post*. Therefore, we do believe that our proposed approach is potentially a good candidate. Last, but not least, it allows to reconcile CGE models with results of econometric analysis.

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Figure1. The nested structure of the LTL-NNCES functional form

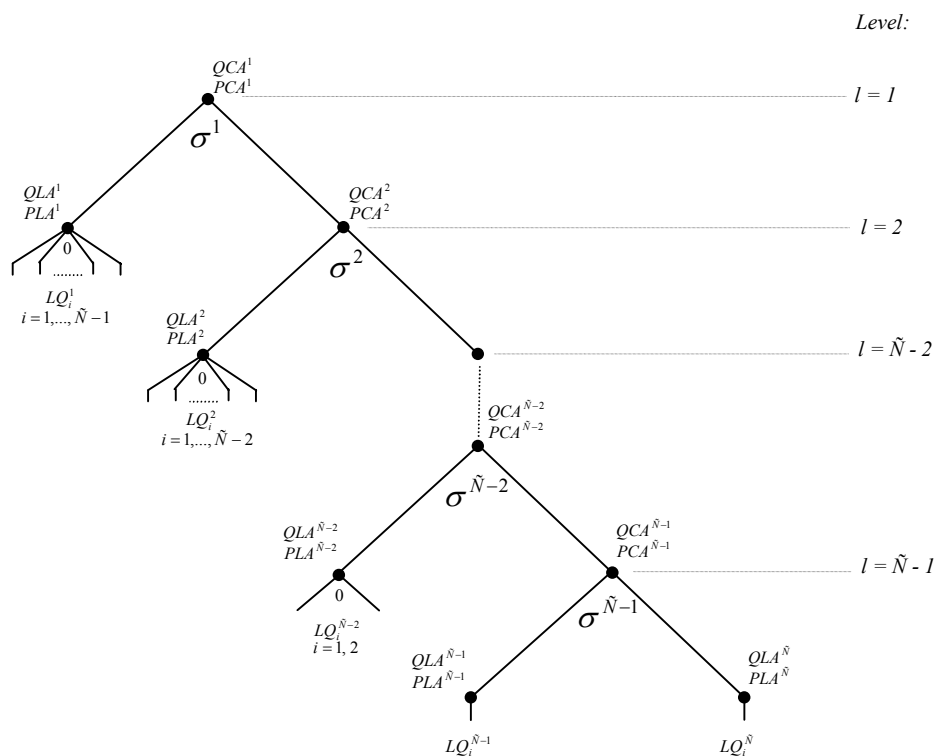


Table 1a. Initial uncompensated price and income elasticities of final demand

	Food	Alcohol	Fuel	Clothing	Transport	Services	Oth non durables	Expenditure
Food	-0.544	0.085	-0.046	-0.056	0.043	0.064	-0.094	0.547
Alcohol	0.418	-1.623	0.732	0.192	-0.111	-0.073	-0.156	0.621
Fuel	-0.198	0.584	-0.496	-0.058	-0.433	-0.066	0.098	0.569
Clothing	-0.314	0.107	-0.076	-0.618	-0.161	-0.369	0.529	0.903
Transport	-0.370	-0.124	-0.313	-0.189	-0.814	-0.217	0.173	1.855
Services	-0.247	-0.121	-0.153	-0.414	-0.322	-0.765	0.216	1.806
Oth non durables	-0.413	-0.115	0.058	0.528	0.481	0.358	-1.727	0.830
Share	0.345	0.067	0.084	0.102	0.178	0.117	0.103	

Table 1b. Initial compensated price of final demand

	Food	Alcohol	Fuel	Clothing	Transport	Services	Oth non durables
Food	-0.354	0.122	0	0	0.141	0.128	-0.037
Alcohol	0.634	-1.582	0.785	0.255	0	0	-0.092
Fuel	0	0.622	-0.448	0	-0.331	0	0.157
Clothing	0	0.167	0	-0.526	0	-0.264	0.623
Transport	0.275	0	-0.157	0	-0.483	0	0.365
Services	0.381	0	0	-0.230	0	-0.554	0.403
Oth non durables	-0.124	-0.059	0.128	0.612	0.629	0.455	-1.641

Table 2a. Sensitivity analysis on income effects to the own price compensated elasticity of the hidden good

	Income elasticities	$\tilde{\varepsilon}_{hi,hi}^h = -1.75$		$\tilde{\varepsilon}_{hi,hi}^h = -1$		$\tilde{\varepsilon}_{hi,hi}^h = -0.25$	
		$\dot{R} = 10\%$	$\dot{R} = 50\%$	$\dot{R} = 10\%$	$\dot{R} = 50\%$	$\dot{R} = 10\%$	$\dot{R} = 50\%$
Food	0.547	5.47	27.35	5.48	27.64	5.51	28.05
Alcohol	0.621	6.21	31.05	6.04	27.31	5.42	17.36
Fuel	0.569	5.69	28.45	5.76	29.92	6.05	34.80
Clothing	0.903	9.03	45.15	9.05	45.55	9.38	51.49
Transport	1.855	18.55	92.75	18.55	92.75	18.71	95.66
Services	1.806	18.06	90.30	18.06	90.30	18.23	93.52
Oth non durables	0.830	8.30	41.51	8.30	41.38	7.56	27.94

Table 2b. Calibration of price effects: substitution elasticities and latent allocations

Level	1	2	3	4	5	6	7	8
Substitution	33.169	48.765	20.056	4.761	4.460	1.432	0.029	
Food	0.705	0.070	0.060	0.165				
Alcohol	0.665	0.335						
Fuel	0.693		0.055			0.014	0.238	
Clothing	0.513	0.121		0.121	0.244			
Transport		0.300	0.115	0.113	0.082			0.390
Services	0.026	0.293	0.105	0.073	0.219	0.284		
Oth. non durables	0.552	0.143	0.305					
Hidden good	1							

Table 3a. Impacts on UK markets of the food experiment (Differences in percentages from the base)

Specifications	Production		Final consumption		Imports		Final consumer price	
	Base	Flexible	Base	Flexible	Base	Flexible	Base	Flexible
Food	-13.30	-14.03	3.38	1.83	130.4	128.04	-4.268	-4.293
Alcohol	3.40	2.70	-0.24	-1.56	-47.65	-48.31	-0.797	-0.807
Fuel	0.70	0.85	-0.71	0.10	-19.20	-19.10	-0.327	-0.321
Clothing	-2.40	2.66	-0.91	0.61	84.94	86.86	-0.121	-0.124
Transport	-0.40	0.42	-0.89	0.11	2.10	2.83	-0.140	-0.143
Services	0.12	0.14	-0.62	-0.27	-58.67	-58.67	-0.415	-0.426
Other non durables	0.99	1.07	-0.83	0.96	4.62	4.84	-0.201	-0.206
Hidden good	-	0.00	-	0.00	-	-	-	11.547

Table 3b. Impacts on UK macro-economic variables of the food experiment

Specifications	Base	Flexible
Income (Differences in percentages from the base)	-1.03	-1.02
Welfare (billion dollars)	1158	1180

Table 4a. Impacts on UK markets of the trade experiment (Differences in percentages from the base)

Specifications	Production		Final consumption		Imports		Final consumer price	
	Base	Flexible	Base	Flexible	Base	Flexible	Base	Flexible
Food	-13.31	-13.75	2.36	1.44	127.44	126.04	-4.49	-4.50
Alcohol	2.34	1.82	-0.26	-1.25	-42.53	-43.06	-1.98	-1.98
Fuel	1.08	1.21	-1.69	-0.89	-19.41	-19.32	-0.55	-0.54
Clothing	1.63	1.64	1.51	1.48	96.57	96.54	-3.69	-3.69
Transport	0.58	0.59	-2.02	-1.12	0.74	1.41	-0.22	-0.22
Services	0.23	0.29	-1.52	-0.61	-59.22	-59.18	-0.72	-0.73
Other non durables	1.29	1.31	-1.35	-0.73	6.25	6.31	-0.90	-0.90
Hidden good	-	0.00	-	0.00	-	0.00	-	12.16

Table 4b. Impacts on UK macro-economic variables of the trade experiment

Specifications	Base	Flexible
Observable income (Differences in percentages from the base)	-2.23	-2.22
Welfare (million dollars)	204	230

Table 5a. Impacts on UK markets of the income experiment (Differences in percentages from the base)

Specifications	Production		Final consumption		Imports		Final consumer price	
	Base	Flexible	Base	Flexible	Base	Flexible	Base	Flexible
Food	18.44	11.55	29.69	15.65	106.287	91.23	-0.38	-0.59
Alcohol	19.84	14.49	29.12	18.88	-33.00	-38.14	-0.70	-0.79
Fuel	7.48	5.82	28.15	15.86	-12.57	-14.48	0.05	-0.28
Clothing	8.72	8.81	28.47	26.00	123.37	120.23	-0.20	-0.22
Transport	3.12	3.56	28.56	53.19	24.34	42.38	-0.28	-0.29
Services	5.58	7.02	29.70	52.48	-56.96	-55.82	-1.15	-1.21
Other non durables	7.33	7.12	28.8	23.97	12.76	11.43	-0.46	-0.50
Hidden good		0		0				11.57

Table 5b. Impacts on UK macro-economic variables of the income experiment

Specifications	Base	Flexible
Observable income (Differences in percentages from the base)	28.21	28.10
Welfare (million dollars)	49752	49836

Table 6a. Impacts on UK markets of the full experiment (Differences in percentages from the base)

Specifications	Production		Final consumption		Imports		Final consumer price	
	Base	Flexible	Base	Flexible	Base	Flexible	Base	Flexible
Food	3.35	-3.27	31.70	17.78	169.00	148.25	-4.83	-5.01
Alcohol	22.09	15.94	28.78	16.93	-28.97	-35.20	-2.67	-2.74
Fuel	8.58	7.00	26.01	14.51	-13.49	-15.24	-0.53	-0.83
Clothing	10.33	10.40	30.40	28.21	134.66	131.88	-3.88	-3.89
Transport	3.76	4.21	25.97	51.42	20.56	39.11	-0.50	-0.51
Services	5.83	7.32	27.75	51.49	-58.33	-57.14	-1.88	-1.92
Other non durables	8.75	8.49	27.07	22.80	12.78	11.59	-1.36	-1.39
Hidden good								

Table 6b. Impacts on UK macro-economic variables of the full experiment

Specifications	Base	Flexible
Observable income (Differences in percentages from the base)	25.34	25.46
Welfare (million dollars)	50008	50099