Corporate Taxation, Corporate Finance and Investment: Theory and Applications in Dynamic CGE Modelling

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Abstract

Dynamic computable general equilibrium (CGE) models, such as the MONASH model developed at the Centre of Policy Studies, typically use complex dynamic investment mechanisms to generate dynamic growth paths. Even so, these models don't account for corporate finance and corporate taxation in determining investment outcomes. This paper provides final results from a work program aimed at imposing corporate finance and corporate taxation on investor choices in an inter-temporal model of investment, and then describes the process of modifying the MONASH model to take on-board this new theory. The paper will proceed in two parts.

Firstly, we briefly outline the development of an investment model that links corporate taxation and corporate finance to investment behaviour. Firms seek to maximise shareholder wealth, and firm and investor optimising-behaviour leads to a set of rate-of-return expressions that are determined by financing and investment policies, both of which, themselves, are the result of optimising decisions. Firms in this model optimise across two dimensions – they optimise the present value of shareholder distributions across time (i.e. dynamic optimisation), which they achieve by determining an optimal choice of inputs (including an investment policy) and an optimal method of financing these inputs (involving an optimal financial policy and dividend policy). The solution to the firm's optimal growth path takes account of: various company and personal income tax regimes; various capital-gains taxes regimes (including realisation-basis capital gains taxation); depreciation allowances; investment allowances; debt accumulation; transactions costs on external financing; and interest rates on debt linked to financial leverage. The resulting rate-of-return expressions highlight the complex nature of the taxation of capital income and the ways in which taxation reform can influence investment behaviour.

Secondly, we explain the implementation of this investment approach in MONASH, and report on a series of simulations. A key feature of the model in application is that firms choose their financial and dividend policies endogenously, in every period of the simulation, from eight potential alternatives. The impact of two different tax reforms are simulated using a version of MONASH-Australia aggregated to 32 industries and 33 commodities. The two experiments are: (1) a cut to the dividend-tax rate, such as that currently under debate in the US; and, (2), a cut in a realisation-basis capital gains tax. Each experiment is run twice, with recursive/backward-looking expectations and rational/forward-looking expectations imposed. The simulations show that a GE approach is necessary to gain full insight into the investment behaviour of corporations, as these tax cuts have two immediate and contradictory effects - the tax cuts reduce the taxation of capital income, increasing the rate of return, while also increasing the cost of finance and the user-cost of capital, thereby decreasing the rate of return. Furthermore, we find that anticipation effects are important in

the determination of firm behaviour and investment outcomes. A comparison of these results highlights the ways in which different capital-income tax regimes influence firm behaviour and outcomes in a dynamic setting, including the different adjustment paths that result.

Equation Section (Next)

1. Introduction and Background

This paper reports on the final stages of a project that has developed a new investment theory for a large-scale dynamic CGE model. Given the large amount of background material involved, this paper presents an overview of the theoretical side of the project, while simulation results and some additional theoretical notes (relating mostly to implementation issues) are provided as a hand-out. Readers are also referred to Winston (2004)[1] for further detail.

2. An Inter-Temporal Model of Investment with Corporate Finance

2.1. Notation

The notation used throughout this paper is given in the following table. All variables are *expected* values unless otherwise stated.

Table 1. Notation

D_t^j	Unfranked dividend
DF_t^j	Franked dividend
VN_t^j	New equity issues
m_t^j	Personal income tax rate of shareholders in industry j
u_t^j	Proportion of personal income tax rate applicable to dividend income
q_t^j	Effective dividend tax rate
g_t^j	Rate of dividend imputation available to shareholders in industry j
t_t^j	Company income tax rate
c_t^j	Accrual-equivalent capital gains tax rate
cgt_t^j	Statutory rate of capital gains tax
y_t^j	Proportion of capital gains that are taxable
e_t^j	Rate of share realisations
a_t	Dummy for capital gains indexing, 1 for real, 0 for nominal
p_t	Rate of inflation
m_t and h_t	Parameters determining the own-price elasticity of demand for output
Y_t^j	Firm/industry output
i_t	Economy-wide cash rate
h_t^j	Proportion of the personal income tax rate applicable to interest income
V_t^j	Effective rate of interest-income tax
c	Parameter setting the sensitivity financial leverage to $qtot_t^j$, $KTOT_t^j$ and B_t^j
b	Parameter determining the shape of the debt-interest function
r_t^j	Rate of interest on debt
$qtot_t^j$	Share-weighted price-index denoting the "price" of composite capital
$\Gamma_t^{n,j}$	Deductible investment allowance on capital of type n

[1] Available from the working-paper archive at www.monash.buseco.edu.au\policy.

$PC\Delta_{t,t-s}^{n,j}$	Prime-cost depreciation allowance on capital of type n, in period t, t-s periods after it was acquired
$DV\Delta_{t,t-s}^{n,j}$	Diminishing-value depreciation allowance on capital of type n, in period t, t-s periods after it was acquired
$j_t^{n,j}$	Dummy on depreciation allowances, 1 for prime-cost, 0 for diminishing-value
p_t	Producer or basic price of output
$L_t^{m,j}$	Total usage of labour of type m
w_t^m	Price of labour of type m
LN_t^j	Total usage of land
pl_t	Rental price of land
$I_t^{n,j}$	Real investment in new capital of type n
$KTOT_t^j$	Composite capital stock of industry j
$d_t^{n,j}$	Rate of physical depreciation of capital of type n
$K_t^{n,j}$	Stock of capital of type n, where n denotes effective life
q_t^n	Asset price of capital goods of type n
$Q_t^{g,j}$	Total intermediate usage of good g
P_t^g	Purchaser's price of intermediate good g
$x_t^{g,j}$	Rate of sales tax on intermediate good g
b_t^j	Term-to-maturity of debt
r_t^j	Rate of debt retirement, equals $(1/b_t^j)$
B_t^j	Total issues of bonds in period t
TB_t^j	Total bonds outstanding in period t
u_t^j	Rate of payroll tax
T_t^j	Level of the corporate income tax liability

2.2. The Objective Function – Shareholder Wealth

The firm's objective is to maximise shareholder wealth, measured as the *cum*-dividend value of the firm's equity,

$$W_0^j = \sum_{t=0}^{\infty} \left\{ \frac{\left[\frac{\left(1-c_0^j\right)\left(1-q_t^j\right)}{1-c_t^j} \right] D_t^j + \left[\frac{\left(1-c_0^j\right)\left(1-q_t^j\right)\left[1-\left(1-g_t^j\right)t_t^j\right]}{\left(1-c_t^j\right)\left(1-t_t^j\right)} \right] DF_t^j - \left(1-c_0^j\right) VN_t^j}{\prod_{s=1}^{t} \left[\frac{1+i_s\left(1-V_s^j\right)-c_s^j\left[1+a_s p_s\right]}{1-c_s^j} \right]} \right\} + c_0^j\left(1+a_0 p_0\right) V_{-1}^j . \tag{1}$$

The derivation of this equation is explained in Winston (2004), and is too lengthy to state here. Equation (1) defines the *cum*-dividend value of the firm as the sum of an infinite series of appropriately discounted after-tax dividend payments minus new-share issues.

2.5. The Constraints

2.5.1. Cash Flow Constraint

The firm's cash flow constraint, a function of its sources and uses of funds, is defined by the following two functions:

$$\begin{aligned} D_t^j + DF_t^j - VN_t^j\left(1-TRVN_t^j\right) &= m\left(Y_t^j\right)^{1-h} - \sum_{m=1}^{M} w_t^m L_t^{m,j}\left(1+u_t^j\right) - pl_t LN_t^j \\ &- \sum_{g=1}^{G} P_t^g Q_t^{g,j} - TB_{t-1}^j\left(r_t^j + r_t^j\right) + B_t^j\left(1-TRB_t^j\right) - \sum_{n=1}^{N} q_t^n I_t^{n,j} - T_t^j \end{aligned} \tag{2}$$

$$T_t^j = t_t^j\left[\begin{array}{l} m\left(Y_t^j\right)^{1-h} - pl_t LN_t^j - \sum_{m=1}^{M} w_t^m L_t^{m,j}\left(1+u_t^j\right) - \sum_{g=1}^{G} P_t^g Q_t^{g,j} - r_t^j TB_{t-1}^j - \sum_{n=1}^{N} \Gamma_t^{n,j} q_t^n I_t^{n,j} \\ -\sum_{s=1}^{N}\sum_{s=1}^{n} q_{t-s}^n I_{t-s}^{n,j} j_t^{n,j} PC\Delta_{t,t-s}^{n,j} - \sum_{s=1}^{N}\sum_{s=1}^{n}\left\{q_{t-s}^n I_{t-s}^{n,j}\left(1-j_t^{nj}\right) DV\Delta_{t,t-s}^{n,j}\prod_{h=s+1}^{t}\left(1-DV\Delta_{t,t-h}^{n,j}\right)\right\} \end{array}\right] \tag{3}$$

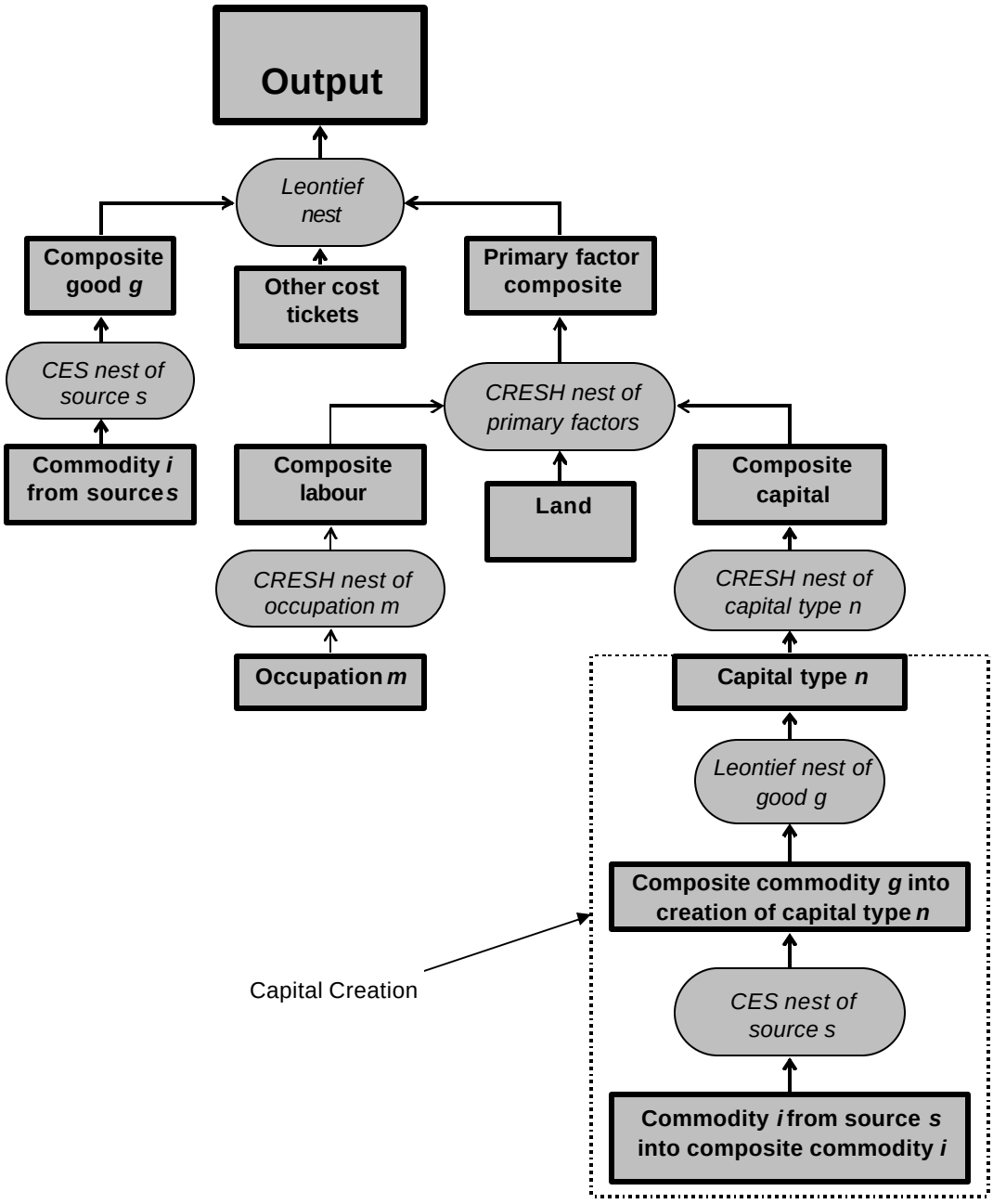
The firm's sources of funds are operating revenue and receipts from selling claims over fixed (debt) and variable (new equity) amounts of future cash flows. The firm's uses of funds are factor payments, purchases of intermediate goods, debt servicing, capital expenditures and the payment of company income tax. The company income tax liability is a function of the level of taxable income and the company income tax rate. Taxable income in period t is equal to operating revenue minus variable costs and deductions.

2.5.2. Production Technology

$$Y_t^j = f_t^j\left(K_{t-1}^{n,j}, L_t^{m,j}, LN_t^j, Q_t^{g,j}\right) \tag{4}$$

Firm output Y_t^j, is a function of three primary factor inputs and intermediate commodity usage. Producers create output according to a multi-level, nested production function;

Figure 1. Input technology, current production in the modified MONASH model



The MONASH model assumes one type of capital in each industry. The changes discussed in this paper add an additional level in the capital creation nest where the composite-input into capital creation is first combined into specific *types* of capital, differentiated by effective life, and *then* combined into a composite capital variable that enters the primary factor nest. With this greater level of detail, the model will have the ability to simulate changes in the different rates of depreciation and investment allowance available on different types of capital. Thus, industries with different capital structures will face different consequences from changes in business-taxation policy due to changes in the net-of-tax-and-allowances profitability of assets.

2.5.3. Debt Accumulation

The firm's total outstanding debt at time *t*, denoted TB_t^j, is given an accumulation relationship of the form

$$TB_t^j = B_t^j + \left(1 - r_t^j\right) TB_{t-1}^j . \tag{5}$$

where $r_t^j = 1/b_t^j$ and b_t^j is the term-to-maturity of the firm's debt. This treats the payment of principal like a geometric rate of decay in the firm's total outstanding debt. The firm's total outstanding debt in period *t* is equal to the total debt outstanding at the beginning of the previous period *plus* new bond issues in period *t minus* repayments of principal. The firm's pre-tax interest bill in any period is the current-period's interest rate multiplied by total outstanding debt,

$$\text{Interest Bill} = r_t^j TB_t^j \tag{6}$$

The interest rate on debt is assumed to be a variable rate, which allows us to treat total outstanding debt of varying maturity like a single outstanding bond.

2.5.4. Financial Leverage and the Cost of Debt

The firm's financial leverage in period *t* is

$$FLV_t^j = \frac{TB_t^j}{q_t^j K_t^j}, \tag{7}$$

where q_t^j is a weighted average of the prices of the various *n* capital types that combine to create the CRESH composite. This ratio enters a function for the rate of interest on the firm's debt[2],

$$r_{t+1}^j = \frac{\Lambda}{1 - e^{b\left\{FLV_t^j - 1\right\}}}. \tag{8}$$

The parameter Λ is set at 0.042 and b at 1.832681, giving us a function of the right qualities to model a rate of interest on debt linked to financial leverage.

2.5.5. Transaction Costs in Obtaining Finance from External Sources

Share and bond sales incur a per-dollar fee which diminishes with the size of the issue. Denoting the average transaction cost per dollar of finance for equity and debt as $TRVN_t^j$ and TRB_t^j respectively, we define these fees by

$$TRVN_t^j = \frac{1}{\ln VN_t^j + \Theta_t^j} \tag{9}$$

for new equity issues, and

$$TRB_t^j = \frac{1}{\ln B_t^j + \mathrm{X}_t^j} \tag{10}$$

[2] Thanks to Peter Dixon and Maureen Rimmer for suggesting this function.

for bond issues. Θ_t^j and X_t^j are parameters determining the maximum rate of the transaction fee. In application, we take the natural log of VN_t^j and VB_t^j *plus* 1 to rule out the chance of asking the model to calculate the natural log of 0, giving us a maximum rate of 5% on acquiring finance. The shapes of these functions imply large fixed costs in acquiring external finance, which reflects the realities of underwriting and intermediation.

2.5.6. Capital Accumulation

In any period *t*, the firm's stock of capital of type *n* will increase in size by the level of real investment in capital of type *n* in that period, and depreciate at a constant, type-specific geometric rate d^n. The size of the firm's stock of capital of type *n* in period *t* is

$$K_t^{n,j} = I_t^{n,j} + \left(1 - d_t^{n,j}\right) K_{t-1}^{n,j}. \tag{11}$$

Equation (11) says that the firm's stock of capital of type *n* at the end of period *t* is equal to the depreciated value of the previous period's capital stock of type *n* plus real investment during period *t*.

2.5.7. A Finite Demand Elasticity for the Firm's Output

Define the price of the firm's output as

$$p_t = m\left(Y_t^j\right)^{-h} \tag{12}$$

where h and m are parameters determining the own-price elasticity of demand $ED_t^j = -\frac{1}{h}$.

Multiplying output by this price function gives us a revenue function -

i.e. Given $$p_t = m\left(Y_t^j\right)^{-h}$$

it follows that $$R_t^j = p_t Y_t^j = m\left(Y_t^j\right)^{1-h} \tag{13}$$

where R_t^j denotes revenue earned by industry j in period t, and Y_t^j is a function of inputs and technology as described in section 2.5.2.

2.5.8. Franking Account Constraint (Dividend Imputation System)

Firms can pay a dollar of franked dividends for every dollar available in their franking account, and it is illegal for a firm to issue franked dividends over and above this balance. Firms are also (generally) required by law to distribute franking credits when they are available. Following Benge (1997 and 1998), we impose an inequality constraint of the form

$$\sum_{s=-\infty}^{t} \left\{ \left[\frac{1 - t_s^j}{t_s^j} \right] T_s^j - DF_s^j \right\} \geq 0. \tag{14}$$

This constraint effectively links the firm's ability to distribute franked dividend to its tax payments.

2.6. The Lagrangian Function

In order to help make the expressions more manageable, we denote the one-period modified discounting term by

$$\Phi_t^j = \frac{1+i_t\left(1-V_t^j\right)-c_t^j\left(1+a_t p_t\right)}{1-c_t^j}. \tag{15}$$

Φ_t^j is applied to period t-1's outcomes, and the cash rate i_t is the interest rate applying to deposits during period t-1 and is paid at the beginning of period t.

A Kuhn-Tucker formulation is used (because of the presence of sign constraints) and the usual first-order conditions for linear and non-linear programming are replaced by the Kuhn-Tucker conditions (discussed below).

$$\begin{aligned}
L = &\sum_{t=0}^{\infty}\left\{\prod_{s=1}^{t}\left(\Phi_s^j\right)^{-1}\left\{\left[\frac{\left(1-c_0^j\right)\left(1-q_t^j\right)}{1-c_t^j}\right]D_t^j+\left[\frac{\left(1-c_0^j\right)\left(1-q_t^j\right)\left[1-\left(1-g_t^j\right)t_t^j\right]}{\left(1-c_t^j\right)\left(1-t_t^j\right)}\right]DF_t^j-\left(1-c_0^j\right)VN_t^j\right\}\right\}\\
&+c_0^j\left(1+a_0p_0\right)V_{-1}^j\\
&+\sum_{t=0}^{\infty}\left\{l_t^{1,j}\prod_{s=1}^{t}\left(\Phi_s^j\right)^{-1}\left[\begin{array}{l} m\left[f_t^j\left\{K_{t-1}^{n,j},L_t^{m,j},LN_t^j,Q_t^{g,j}\right\}\right]^{1-h}-\sum_{m=1}^{M}w_t^m L_t^{m,j}\left(1+u_t^j\right)-plLN_t^j-\sum_{g=1}^{G}P_t^gQ_t^{g,j}\\ -TB_{t-1}^j\left(r_t^j+r_t^j\right)+B_t^j\left(1-TRB_t^j\right)+VN_t^j\left(1-TRVN_t^j\right)-\sum_{n=1}^{N}q_t^nI_t^{n,j}-T_t^j-D_t^j-DF_t^j\end{array}\right]\right\}\\
&+\sum_{t=0}^{\infty}\left\{l_t^{2,j}\prod_{s=1}^{t}\left(\Phi_s^j\right)^{-1}\left[\begin{array}{l} T_t^j-t_t^j\left[\begin{array}{l} m\left[f_t^j\left\{K_{t-1}^{n,j},L_t^{m,j},LN_t^j,Q_t^{g,j}\right\}\right]^{1-h}-plLN_t^j-\sum_{m=1}^{M}w_t^mL_t^{m,j}\left(1+u_t^j\right)\\ -\sum_{g=1}^{G}P_t^gQ_t^{g,j}-r_t^jTB_{t-1}^j-\sum_{n=1}^{N}\Gamma_t^{n,j}q_t^nI_t^{n,j}-\sum_{s=1}^{n}\sum_{s=1}^{N}q_{t-s}^nI_{t-s}^{n,j}j_t^{n,j}PC\Delta_{t,t-s}^{n,j}\\ -\sum_{s=1}^{n}\sum_{s=1}^{N}\left\{q_{t-s}^nI_{t-s}^{n,j}\left(1-j_t^{n,j}\right)DV\Delta_{t,t-s}^{n,j}\prod_{h=s+1}^{t}\left(1-DV\Delta_{t,t-h}^{n,j}\right)\right\}\end{array}\right]\end{array}\right]\right\}\\
&+\sum_{t=0}^{\infty}\left\{l_t^{3,j}\prod_{s=1}^{t}\left(\Phi_s^j\right)^{-1}\left[I_t^{n,j}+\left(1-d_t^{n,j}\right)K_{t+1}^{n,j}-K_t^{n,j}\right]\right\}\\
&+\sum_{t=0}^{\infty}\left\{l_t^{4,j}\prod_{s=1}^{t}\left(\Phi_s^j\right)^{-1}\left[TB_t^j-B_t^j-\left(1-r_t^j\right)TB_{t-1}^j\right]\right\}\\
&+\sum_{t=0}^{\infty}\left\{\prod_{s=1}^{t}\left(\Phi_s^j\right)^{-1}\left[l_t^{5,j}I_t^{n,j}+l_t^{6,j}D_t^j+l_t^{7,j}DF_t^j+l_t^{8,j}VN_t^j+l_t^{9,j}B_t^j+l_t^{10,j}\sum_{s=-\infty}^{t}\left\{\left[\frac{1-t_s^j}{t_s^j}\right]T_s^j-DF_s^j\right\}\right]\right\}
\end{aligned} \tag{16}$$

2.7.1. The Kuhn-Tucker Conditions

The Kuhn-Tucker conditions include the usual first-order conditions *plus* a set of complementary slackness conditions to impose the sign restrictions. We only provide the crucial equations for the solution to investment problem here for the purposes of this paper.

$$\frac{\partial L}{\partial L_t^{m,j}} = \frac{\left[l_t^{1,j} - l_t^{2,j} t_t^{\,j} \right] \left[MRPL_t^{m,j} - w_t^m \left(1 + u_t^j \right) \right]}{\prod_{s=1}^{t} \Phi_s^j} = 0 \tag{17}$$

$$\frac{\partial L}{\partial Q_t^{g,j}} = \frac{\left[l_t^{1,j} - l_t^{2,j} t_t^{\,j} \right] \left[MRPQ_t^{g,j} - P_t^g \right]}{\prod_{s=1}^{t} \Phi_s^j} = 0 \tag{18}$$

$$\frac{\partial L}{\partial K_t^{n,j}} = \frac{\left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^{\,j} \right) MRPK_{t+1}^{n,j} + l_{t+1}^{3,j} \left(1 - d_t^{n,j} \right)}{\prod_{s=1}^{t+1} \Phi_s^j} - \frac{l_t^{3,j}}{\prod_{s=1}^{t} \Phi_s^j} = 0 \tag{19}$$

$$\frac{\partial L}{\partial T_t^j} = \frac{-l_t^{1,j} + l_t^{2,j} + \sum_{s=t}^{\infty} l_s^{10,j} \left[\frac{1 - t_t^{\,j}}{t_t^{\,j}} \right]}{\prod_{s=1}^{t} \Phi_s^j} = 0 \tag{20}$$

$$\frac{\partial L}{\partial I_t^{n,j}} = \frac{q_t^n \left[-l_t^{1,j} + l_t^{2,j} t_t^{\,j} \left(\Gamma_t^{n,j} + \Xi_t^{n,j} \right) \right] + l_t^{3,j} + l_t^{5,j}}{\prod_{s=1}^{t} \Phi_s^j} = 0 \tag{21}$$

where

$$\Xi_t^{n,j} = j_t^{\,n,j} PC\Xi_t^{n,j} + \left(1 - j_t^{\,n,j} \right) DV\Xi_t^{n,j}, \tag{22}$$

$$PC\Xi_t^{n,j} = \left\{ PC\Delta_t^{n,j} + \sum_{s=1}^{n} \left[\frac{PC\Delta_{t+s}^{n,j}}{\prod_{h=s}^{n} \Phi_{t+h}^j} \right] \right\}, \tag{23}$$

and

$$DV\Xi_t^{n,j} = \left\{ DV\Delta_t^{n,j} + \sum_{s=1}^{n} \left[\frac{DV\Delta_{t+s}^{n,j} \left[\prod_{h=s-1}^{n-1} \left(1 - DV\Delta_{t+h}^{n,j} \right) \right]}{\prod_{z=1}^{s} \Phi_{t+z}^j} \right] \right\}. \tag{24}$$

$$\frac{\partial L}{\partial D_t^j} = \frac{\left[\frac{\left(1 - c_0^j \right) \left(1 - q_t^{\,j} \right)}{1 - c_t^j} \right] - l_t^{1,j} + l_t^{6,j}}{\prod_{s=1}^{t} \Phi_s^j} = 0 \tag{25}$$

$$\frac{\partial L}{\partial DF_t^j} = \frac{\left[\frac{\left(1 - c_0^j \right) \left(1 - q_t^{\,j} \right) \left[1 - \left(1 - g_t^j \right) t_t^{\,j} \right]}{\left(1 - c_t^j \right) \left(1 - t_t^{\,j} \right)} \right] - l_t^{1,j} + l_t^{7,j} - \sum_{s=1}^{\infty} l_s^{10,j}}{\prod_{s=1}^{t} \Phi_s^j} = 0 \tag{26}$$

$$\frac{\partial L}{\partial VN_t^j} = \frac{-\left(1-c_0^j\right) + l_t^{1,j}\left\{1-\left[\frac{\ln VN_t^j + \Theta_t^j - 1}{\left(\ln VN_t^j + \Theta_t^j\right)^2}\right]\right\} + l_t^{8,j}}{\prod_{s=1}^{t}\Phi_s^j} = 0 \tag{27}$$

$$\frac{\partial L}{\partial B_t^j} = \frac{l_t^{1,j}\Delta NBF_t^j + l_t^{9,j} - l_t^{4,j}}{\prod_{s=1}^{t}\Phi_{t+s}^j} = 0 \tag{28}$$

where $$\Delta NBF_t^j = \frac{\partial}{\partial B_t^j} B_t^j\left[1-\frac{1}{\ln B_t^j + X_t^j}\right] = 1-\frac{\left(\ln B_t^j + \mathrm{X}_t^j - 1\right)}{\left(\ln B_t^j + \mathrm{X}_t^j\right)^2} \tag{29}$$

and ΔNBF_t^j is the change in net bond-financing in period t, a function of new bond issues, net of transactions costs.

$$\frac{\partial L}{\partial TB_t^j} = \frac{-l_{t+1}^{1,j} r_{t+1}^j - \left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^j\right)\Delta IB_{t+1}^{TB_t^j j} - l_{t+1}^{4,j}\left(1-r_{t+1}^j\right)}{\prod_{s=1}^{t+1}\Phi_s^j} + \frac{l_t^{4,j}}{\prod_{s=1}^{t}\Phi_s^j} = 0 \tag{30}$$

where $$\Delta IB_{t+1}^{TB_t^j j} = \frac{\partial\left(TB_t^j r_{t+1}^j\right)}{\partial TB_t^j} = \frac{\Lambda\left[1-\frac{bTB_t^j}{q_t^j K_t^j}\right]}{1-e^{b\left\{\frac{TB_t^j}{q_t^j K_t^j}-1\right\}}} = r_{t+1}^j\left(1-bFLV_t^j\right) \tag{31}$$

$\Delta IB_{t+1}^{TB_t^j j}$ is the change in the interest bill in period t+1 due to a change in total indebtedness in period t, a function of the change in the stock of debt and the change in the interest rate due to a change financial leverage.

The complementary slackness conditions are

$$l_t^{5,j} \geq 0, \text{ and } l_t^{5,j} = 0 \text{ if } I_t^{n,j} > 0 \tag{32}$$

$$l_t^{6,j} \geq 0, \text{ and } l_t^{6,j} = 0 \text{ if } D_t^j > 0 \tag{33}$$

$$l_t^{7,j} \geq 0, \text{ and } l_t^{7,j} = 0 \text{ if } DF_t^j > 0 \tag{34}$$

$$l_t^{8,j} \geq 0, \text{ and } l_t^{8,j} = 0 \text{ if } VN_t^j > 0 \tag{35}$$

$$l_t^{9,j} \geq 0, \text{ and } l_t^{9,j} = 0 \text{ if } B_t^j > 0 \tag{36}$$

$$l_t^{10,j} \geq 0, \text{ and } l_t^{10,j} = 0 \text{ if } \left\{\left[\frac{1-t_t^j}{t_t^j}\right]T_t^j - DF_t^j\right\} \geq 0 \tag{37}$$

Equation (19) plays an important role in the solution to the investment problem. The right-hand-side of equation (19) defines the present value of a marginal unit of capital.

$$PV_t^{n,j} = \frac{\left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^{j}\right) MRPK_{t+1}^{n,j} + l_{t+1}^{3,j}\left(1-d_t^{n,j}\right)}{\Phi_{t+1}^{j}} - l_t^{3,j}, \tag{38}$$

This defines the relationships between the costs and benefits of a unit of capital. Optimising behaviour is *imposed* by assuming that this present value is zero, when (38) implies

$$\frac{\left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^{j}\right) MRPK_{t+1}^{n,j}}{\Phi_{t+1}^{j}} = l_t^{3,j} - \frac{l_{t+1}^{3,j}\left(1-d_t^{n,j}\right)}{\Phi_{t+1}^{j}}. \tag{39}$$

Equation (21) also plays a central role. Assuming the non-negativity constraint on investment, $l_t^{5,j}$, to be slack and solving for $l_t^{3,j}$ we find

$$l_t^{3,j} = q_t^n\left[l_t^{1,j} - l_t^{2,j} t_t^{j}\left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right], \tag{40}$$

the shadow price of one unit of investment. Dividing through by the asset pr ice we find

$$\Omega_t^{n,j} = \frac{l_t^{3,j}}{q_t^n} = l_t^{1,j} - l_t^{2,j} t_t^{j}\left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right). \tag{41}$$

$\Omega_t^{n,j,f}$ is a modified Lagrangian multiplier denoting the shadow price of one dollar's-worth of asset type *n*. The solution is found by determining the values of the Lagrangian multipliers at the stationary points, and from this solution we take, most importantly, the rate-of-return equations. These are discussed in some detail below.

3. Corporate Financial Policy, The Cost of Investment and the Rate of Return

3.1. Moving from the New Model to MONASH

There are two approaches to moving from the Lagrangian function to a full solution of the investment problem:

1. Equations (20), (21), (25), (26), (27), (28) and (30) can be substituted into (19) to create the foundation for a single rate-of-return equation. The resulting equation will contain a number of unknown Lagrangian multipliers attached to the slack variables (on the sign constraints) in the Lagrangian function Whether the various sign constraints are slack or binding is a numerical issue, and with full information about the values of tax rates, interest rates, imputation rates, bond rates etc. we can determine the firm's optimal financial policy and set the slack variables appropriately, in turn generating an expression for the rate-of-return on investment under the optimal financing policy. The equation that is produced under the full solution to the Kuhn-Tucker problem is large and difficult to interpret and has little intuitive appeal until the values of the slack variables are determined.

2. We can derive a set of rate-of-return functions for each potential financial policy and allow an expanded MONASH to choose between them according to a set of criteria that ensure consistency with the objective of the firm (shareholder wealth-maximisation). Thus, each rate-of-return function presumes a particular financial policy, and MONASH can choose between them by choosing the financing policy that (a) maximises the rate-of-return *given*

the level of investment or (b) provides the largest value for investment *given* a rate of return (which in equilibrium will be zero, discussed below).

The two approaches are equivalent, but the second method (i.e. separate functions for each financial policy) lends itself more readily to both intuitive interpretation and analytical clarity. The way that MONASH chooses the firm's financial policy is described in Winston (2004).

3.1.1. Reconciling the Investment Theories – Rates of Return

In MONASH year-to-year (short-run) forecast and policy (deviation) simulations, rates of return adjust to movements in usable capital stocks. Usable capital stocks are the result of a process of capital accumulation that is serviced by a lagged investment process driven by movements in the rate of return (explained below). Firms face a fixed supply of capital in the *current* period, and so disequilibria between the demand for, and supply of, capital in current production are removed by changes in rental rates and asset prices. These changes in rental rates and asset prices feed into the rate-of-return functions and drive investment to create an "optimal" capital stock in the next period according to information available today. The model also makes use of explicit capital supply functions in year-to-year simulations.

The calculation of the rate of return on capital in MONASH begins by defining the present value of a unit of physical capital[3] as

$$PV_t^j = \frac{R_t^j\left(1-TAX_{t+1}\right)+q_{t+1}\left(1-d^j\left[1-RALPH^j TAX_{t+1}\right]\right)}{1+i_{t+1}\left(1-TAX_{t+1}\right)}-q_t. \tag{42}$$

where R_t^j is the "rental price" of capital (marginal revenue product) in period t, TAX_t is an estimated tax rate on capital income, and $RALPH_t$[4] is the proportion of depreciation that is tax-deductible. Notice the similarities between this equation and the first-order condition for capital, equation (19), which we restate here, slightly rearranged,

$$PV_t^{n,j} = \frac{R_{t+1}^{n,j}\left(l_{t+1}^{1,j}-l_{t+1}^{2,j}t_{t+1}^j\right)+l_{t+1}^{3,j}\left(1-d_t^{n,j}\right)}{\left[\frac{1+i_{t+1}\left(1-V_{t+1}^j\right)-c_{t+1}^j\left(1+a_{t+1}p_{t+1}\right)}{1-c_{t+1}^j}\right]}-l_t^{3,j} \tag{43}$$

where $R_{t+1}^{n,j}=MRPK_{t+1}^{n,j}$. Dividing both sides of (42) by q_t, we find the present value of one dollar of investment – the *rate-of-return* – in the MONASH model to be

$$ROR_t^j = \frac{\left[\frac{R_{t+1}^j}{q_t}\right]\left(1-TAX_{t+1}\right)+\left[\frac{q_{t+1}}{q_t}\right]\left(1-d^j\left[1-RALPH^j TAX_{t+1}\right]\right)}{1+i_{t+1}\left(1-TAX_{t+1}\right)}-1. \tag{44}$$

Doing likewise for equation (43), we find

[3] See Dixon and Rimmer (2002).

[4] Named for John Ralph, chairman of the committee that produced Ralph (1999), a government sponsored report outlining a set of taxation policy recommendations, for the Review of Business Taxation carried out by the Australian Government in the late 1990's.

$$ROR_t^{n,j} = \frac{\left[\frac{R_{t+1}^{n,j}}{q_t^n}\right]\left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^{j}\right) + \left[\frac{q_{t+1}^n}{q_t^n}\right]\Omega_{t+1}^{n,j}\left(1-d_t^{n,j}\right)}{\left[\frac{1+i_{t+1}\left(1-V_{t+1}^{j}\right) - c_{t+1}^{j}\left(1+a_{t+1}p_{t+1}\right)}{1-c_{t+1}^{j}}\right]} - \Omega_t^{n,j}. \quad (45)$$

Comparing equations (44) and (45), firstly we can see that taxes and allowances enter the equations in different ways. In (44), rental income and interest are taxed at the estimated capital tax rate, and depreciation allowances are calculated as a proportion of the physical depreciation rate. These points highlight two important issues: equation (44) assumes that the opportunity cost of a dollar of investment funds to the firm's residual claimants is equal to one dollar; and depreciation allowances act like a subsidy on real depreciation costs.

The first point indicates that MONASH takes no account of the multiple sources of funds available to investors, and assumes that investment decisions are made after all taxation on capital income has been paid. In a sense, this approach treats the whole capital stock as if it is owned by non-corporate entities that never borrow and, although they *must* therefore finance investment with retained earnings, take no account of the way in which earnings are taxed in the calculation of the expected rate of return on investment.

The second point suggests that MONASH effectively allows deductions for *real economic* depreciation. Like virtually all real-world tax systems, the depreciation rates allowed for tax deductions are somewhat notional. Also, in reality, rates of physical depreciation will vary between uses for a given type of capital. Allowing a deduction for a true, estimated rate of decay on capital removes the important link between tax allowances and the actual physical rates of depreciation observed. Perhaps the most important argument supporting a detailed analysis of corporate finance in investment is to capture the way the tax system distorts investment decisions.

Summarising, there are four major additions to the MONASH theory provided by equation (45):

1. All issues are analysed with reference to the residual claimant on the corporations stock – the shareholder. This means that the firm's investment behaviour is now a function of the amount of capital used during the period, and the relative costs and benefits of financing the respective terminal values (i.e. the beginning and end of period values of the unit of capital) from the point of view of an agent who must pay income and capital gains taxes on cash flows;

2. The method of finance becomes a factor in the rate of return on capital;

3. Different types of capital, differentiated by effective life and tax status, are accounted for, and;

4. More of the detail of the business and personal taxation systems is included, like the addition of realisation-basis capital gains taxation and the imputation system.

3.1.2. Optimal Financial Policy and the Firm's Objective

The firm's objective is to maximise shareholder wealth, which is achieved when the marginal benefit, *MBK*, and cost, *MCK*, of one-dollar's worth of capital,

(Old MONASH) $$MBK_t^j = \frac{R_t^j\left(1-TAX_{t+1}\right)}{q_t\left[1+i_{t+1}\left(1-TAX_{t+1}\right)\right]} \tag{46}$$

(New MONASH) $$MBK_t^{n,j} = \frac{R_{t+1}^{n,j}\left(l_{t+1}^{1,j} - l_{t+1}^{2,j}t_{t+1}^{j}\right)}{q_t^n \Phi_{t+1}^j} \tag{47}$$

and

(Old MONASH) $$MCK_t^j = 1 - \frac{q_{t+1}\left(1-d^j\left[1-RALPH^j TAX_{t+1}\right]\right)}{q_t\left[1+i_{t+1}\left(1-TAX_{t+1}\right)\right]}, \tag{48}$$

(New MONASH) $$MCK_t^{n,j} = \left[l_t^{1,j} - l_t^{2,j}t_t^{j}\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right] - \frac{q_{t+1}^n\left[l_{t+1}^{1,j} - l_{t+1}^{2,j}t_{t+1}^{j}\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]\left(1-d_t^{n,j}\right)}{q_t^n \Phi_{t+1}^j} \tag{49}$$

respectively, from equations (44), (45) and (41), are equal. Thus, the first-order conditions impose optimising behaviour on the relationship described in equation (45) by ensuring that the present value of the marginal dollar's-worth of capital is zero. The optimisation problem has been structured to reflect the structure of the MONASH model. The rate of return equations and the relevant "accessory" functions[5] are taken from the model and imbedded in the MONASH equation-system to replace the existing investment and capital accumulation theory. In a sense, the construction of the model of the firm and the Kuhn-Tucker problem were disposable tools used to generate some functions describing the behaviour of investors in a MONASH-style environment with added financial complexity.

Is this "poaching" a valid use of these functions? To answer this, consider that the different financing options and deductions on capital available to the firm will determine the values of the Lagrangian multipliers $l_{t+1}^{1,j}$, $l_{t+1}^{2,j}$, $l_t^{3,j}$ and $l_{t+1}^{3,j}$. Multipliers $l_{t+1}^{1,j}$ and $l_{t+1}^{2,j}$ are the shadow price of general cash-flow and tax payments respectively, and serve to adjust gross shareholder income streams to create a measure of the after-tax value of distributions to shareholders from a marginal unit of capital. Multipliers $l_t^{3,j}$ and $l_{t+1}^{3,j}$ determine the cost of a marginal unit of capital: the *value* of physical depreciation during the period, a function of the shadow price of cash flow, asset-price inflation, physical depreciation, tax allowances and discounting factors. The l^3's are functions of the l^1's and l^2's (see equation (40)) because the cost of funds from any source used for investment is a function of the shadow price of cash flow.

Substituting equation (41) into (45), we obtain

$$ROR_t^{n,j} = \frac{\left[\frac{R_{t+1}^{n,j}}{q_t^n}\right]\left(l_{t+1}^{1,j} - l_{t+1}^{2,j}t_{t+1}^{j}\right) + \left[\frac{q_{t+1}^n}{q_t^n}\right]\left(l_{t+1}^{1,j} - l_{t+1}^{2,j}t_{t+1}^{j}\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right)\left(1-d_t^{n,j}\right)}{\left[\frac{1+i_{t+1}\left(1-V_{t+1}^j\right)-c_{t+1}^j\left(1+a_{t+1}p_{t+1}\right)}{1-c_{t+1}^j}\right]} - l_t^{1,j} + l_t^{2,j}t_t^{j}\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right). \tag{50}$$

[5] Debt accumulation relationships, financial leverage functions, depreciations allowance present-value calculations, etc. See Winston (2004).

In standard investment problems, there is implicitly one source of finance, and each dollar is valued at one dollar by the firm's "shareholders". As long as the rate of return is positive at the margin, the firm will invest. With four potential sources of finance, the criterion for investing at the margin becomes a function of four sources of finance and eight potential financial policies. There will be a distinct value for $ROR_t^{n,j}$ on a given unit of investment for each potential financial policy, and the firm will optimise (i.e. maximise shareholder wealth) by choosing the financial policy that gives the highest value of $ROR_t^{n,j}$. Facing a *set* of financing options and financial policies, the firm will invest as long as *at least one* of the financial policies generates a non-negative rate-of-return. Equilibrium will be characterised by one financial policy generating a rate-of-return of zero while all others generate negative rates-of-return. Thus, the optimal financial policy is consistent with optimal investment, optimal capital accumulation and maximum shareholder wealth. From the point of view of the firm's financial policy, a number of the variables in $ROR_t^{n,j}$ above are parameters in the following problem:

Maximise $ROR_t^{n,j}$ by choosing a financial policy (that is, choose $l_t^{1,j}$, $l_{t+1}^{1,j}$, $l_t^{2,j}$ and $l_{t+1}^{2,j}$) subject to $R_{t+1}^{n,j}$ (a function of the level of output, the input bundle and the production function – for a given unit of investment, we can take this as given), q_t^n and q_{t+1}^n (the asset price – input markets are competitive and thus the firm is a price taker), t_t^j, t_{t+1}^j, V_{t+1}^j, c_{t+1}^j, $\Gamma_t^{n,j}$, $\Xi_t^{n,j}$, $\Gamma_{t+1}^{n,j}$, $\Xi_{t+1}^{n,j}$ (these tax rates and allowances are not choice variables for the firm's managers), $d_t^{n,j}$ (a parameter) and p_{t+1} (inflation is independent of the firm's behaviour).

The actual values of these exogenous variables/parameters are irrelevant to this problem – what is important is that *we can assume them to be given regardless of the financial policy chosen by the firm.* This assumption only requires that the firm's choice of financial policy has no bearing on the values of other variables in (50), which is in fact true – the tax rates and allowances are exogenous variables, asset prices are the result of competitive markets, and the gross, before-tax rental price of the marginal unit under consideration is given by factor usage and the technical characteristics of the production function, and is independent of the firm's financial policy.

In a forward-looking environment, re-state equation (45) as

$$ROR_t^{n,j} = \left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^{\ j} \right) \Upsilon_t^{n,j} - \left(l_t^{1,j} - l_t^{2,j} t_t^{\ j} \right) \tag{51}$$

where

$$\Upsilon_t^{n,j} = \left\{ \frac{R_{t+1}^{n,j} + q_{t+1}^n \left(1 - d_t^{n,j}\right)}{q_t^n \Phi_{t+1}^j} \right\}. \tag{52}$$

All of the components of $\Upsilon_t^{n,j}$ are independent of $l_{t+1}^{1,j}$, $l_{t+1}^{2,j}$, $l_t^{1,j}$ and $l_t^{2,j}$. Therefore, the rate of return on a given unit of investment will vary according to the relative advantages of alternative financial policies for shareholders. The difference between two alternative financial policies is

$$\left(ROR_t^{n,j}\right)_a - \left(ROR_t^{n,j}\right)_b = \left[\left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^{\ j} \right)_a - \left(l_{t+1}^{1,j} - l_{t+1}^{2,j} t_{t+1}^{\ j} \right)_b \right] \Upsilon_t^{n,j} - \left[\left(l_t^{1,j} - l_t^{2,j} t_t^{\ j} \right)_a - \left(l_t^{1,j} - l_t^{2,j} t_t^{\ j} \right)_b \right] \tag{53}$$

The alternative view of the optimal financial policy says that, given a rate of return, the optimal financial policy is the one that generates the largest investment. Whether we assume that the firm is optimising (that is, the rate of return is zero at the margin) or not, cost-minimisation in financial

policy *given* the level of investment is equivalent to maximising the level of investment *given* a rate-of-return. The proof of this is trivial and is not stated here.

3.1.3. Capital Supply Functions

In year-to-year (dynamic) simulations, MONASH uses explicit capital-supply functions to help determine actual investment. However, investment is driven by *expected* rather than *actual* rates of return. In MONASH, expected rates of return $EROR_t^j$ are comprised of two parts,

$$EROR_t^j = EQEROR_t^j + DIS_t^j \tag{54}$$

where

$EQEROR_t^j$ is the equilibrium expected rate of return (i.e. that return required to sustain the period *t* rate of capital growth indefinitely), an inverse logistic function of the rate of growth in the capital stock, and

DIS_t^j is the disequilibrium in the expected rate of return – put another way, the difference between the actual rate of return and investor expectations, or a measure of investor error.

Expected rates of return, $EROR_t^j$, are generated by equation (44). The model described above enables us to derive a set of equations of the form of (45) to *replace* (44) in the determination of $EROR_t^j$.

The inverse logistic function determining $EQEROR_t^j$ is

$$EQEROR_t^j = \left[RORN_t^j + FERORJ_t^j + FEROR_t \right] + \frac{1}{C_t^j} \begin{Bmatrix} \ln\left[KGR_t^j - KGRmin_t^j \right] \\ -\ln\left[KGRmax_t^j - KGR_t^j \right] \\ -\ln\left[TREND_t^j - KGRmin_t^j \right] \\ +\ln\left[KGRmax_t^j - TREND_t^j \right] \end{Bmatrix} \tag{55}$$

where: $TREND_t^j$ is the average, observed rate of capital growth over a historical period; $RORN_t^j$ is an estimated, historically-normal rate of return, defined as the average rate of return the industry exhibited while its capital stock grew at *TREND*; KGR_t^j is the simulated rate of capital growth, where

$$KGR_t^j = \frac{K_{t+1}^j}{K_t^j} - 1; \tag{56}$$

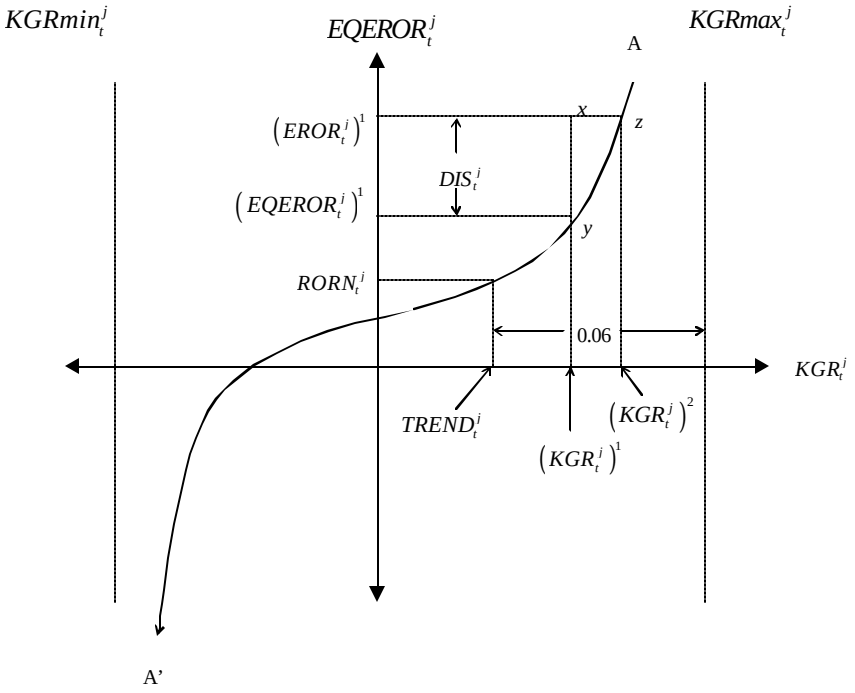
$KGRmin_t^j$ is the minimum allowable rate of growth, set at the negative of the physical depreciation rate; $KGRmax_t^j$ is the maximum allowable rate of capital growth, usually set such that

$$KGRmax_t^j = TREND_t^j + \left(0.06 \geq x \geq 0.10 \right); \tag{57}$$

C_t^j is a positive parameter, determining the responsiveness of capital growth to movements in rates of return; and $FERORJ_t^j$ and $FEROR_t$ are vector and scalar shift variables respectively, allowing for shifts in the capital supply curves.

Assuming that $FERORJ_t^j$, $FEROR_t$ and DIS_t^j are all zero, equation (55) says that for industry j to attract to attract sufficient investment funds to achieve a rate of capital growth $TREND_t^j$, it must have an expected rate of return on capital of $RORN_t^j$.

Figure 2. The Inverse Logistic Function – Capital Supply and Equilibrium Expected Rates of Return



Typically, in year t, the year t-1 capital growth data and the expected rate of return (either the simulated solution or observed data) will not give a point on the inverse logistic curve, AA'. Therefore, DIS_t^j will be non-zero. This disequilibrium will be gradually eliminated over time by reductions in DIS_t^j imposed exogenously during simulations. Thus, as DIS_t^j approaches zero from above (if $EROR_t^j > EQEROR_t^j$), or from below (if $EROR_t^j < EQEROR_t^j$), $EQEROR_t^j$ must increase or decrease, respectively. With the value of $EQEROR_t^j$ determined in this way via (54), the only free variable in (55) is the rate of capital growth, KGR_t^j. Thus, as DIS_t^j falls, KGR_t^j responds by increasing at a rate determined by the parameter values in (55) (and vice versa for an increase in DIS_t^j). Thus, capital growth rates (when they are endogenous) are determined by the elimination of disequilibria between the simulated expected rates of return and the capital supply function.

For example, firms/industries that are currently exhibiting capital growth rates *below* their historical trend values *given* a level of the expected rate of return, will have *higher* forecast capital growth rates, and vice versa. In figure 5.1, we have a simulated or observed point at *x* where the expected rate of return, $(EROR_t^j)^1$, is higher than that required to sustain the rate of capital growth, $(KGR_t^j)^1$. As DIS_t^j falls in value over time, we move up the AA' curve from point *y* toward *z*, thereby

increasing capital growth toward $\left(KGR_t^j\right)^2$. $EQEROR_t^j$ can be thought of as investors' required rate of return – the minimum return required to convince them to provide financial capital to the firm. As investors respond to the higher-than-required capital growth rate by providing more capital, they become progressively less willing to invest the marginal dollar, until the required rate of return increases to the level of the expected rate of return on offer. By determining the rate of capital growth during a period, we are then able to calculate the level of investment undertaken by the firm during that period. Rearranging (56), we have

$$KGR_t^j = \frac{K_{t+1}^j - K_t^j}{K_t^j} \tag{58}$$

noting that the numerator implies, in light of our capital stock constraint, that

$$KGR_t^j = \frac{I_t^j - d_t^n K_t^j}{K_t^j}. \tag{59}$$

With KGR_t^j determined as explained above, K_t^j known and d_t^j defined as a parameter, investment is given by

$$I_t^j = K_t^j \left(KGR_t^j + d_t^j\right). \tag{60}$$

If, instead, under an alternative closure, we know the value for investment from outside the model (or, in other words, the value of KGR_t^j is exogenous), we then free-up R_t^j and/or q_t^n to move with the expected rate of return as it adjusts to eliminate disequilibria. In figure 5.1, this would mean that, given $\left(KGR_t^j\right)^1$, the expected rate of return would fall over time from $\left(EROR_t^j\right)^1$ towards $\left(EQEROR_t^j\right)^1$ as DIS_t^j falls over time.

Equation (45) (and its required accessory equations) replace the MONASH investment theory, based on equation (44), and is used to determine the value of $EROR_t^j$, and to determine the way that $EROR_t^j$ responds to shocks. With the new specification of $EROR_t^j$, MONASH is able to directly simulate changes in business taxation and corporate finance variables.

3.2. Financial Policy and The Cost of Capital

In this section, we analyse the cost of capital at the margin under each potential financial policy. This involves determining a value for the $\Omega_t^{n,j}$'s - the shadow-price of one-dollar's worth of capital - in equation (45) for each of the eight potential financial scenarios. In each case described below, assume that period *t* is the current period (period zero), which (a) defines all the cases we analyse as involving immediate/current decisions, and (b) enables us to cancel-out some of the capital gains tax terms in the first-order conditions. The derivation is explained in some detail for the first case.

3.2.1. Financing with Retained Earnings at the Margin

From this point on, we add a financial-policy superscript to the Lagrangian multipliers ($l_t^{1,j}$, $l_t^{2,j}$, $l_{t+1}^{1,j}$, $l_{t+1}^{2,j}$, $\Omega_t^{n,j}$ and $\Omega_{t+1}^{n,j}$), so that (for example) $l_t^{1,j,f}$ now refers to the value of the general cash-flow multiplier in period *t* for industry *j* using financial policy *f*.

3.2.1.1. Financing Policy 1: Slack Franking Account Constraint (Classical System)

The first task is to determine the nature of the various inequality constraints implied by this financial policy and set the values of the slack variables appropriately. This case assumes a slack franking-account constraint, and so the firm cannot be paying the maximum amount of franked dividends - it must be true, therefore, that the firm is operating in a classical system and has no frankable earnings to distribute. If the firm was operating in a dividend imputation system and had no frankable earnings to distribute (very unlikely, as it would the firm has paid no tax), its franking account constraint would still be binding – if there are no frankable earnings to distribute, then paying none still renders the franking account constraint binding. Setting the values of the slack variables appropriately provides

$$l_t^{5,j,1} = l_t^{6,j,1} = l_t^{10,j,1} = 0, \ \ \forall t \, . \tag{61}$$

With the assumption that period t is period zero, (25) and (61) imply

$$l_t^{1,j,1} = 1 - q_t^j \, . \tag{62}$$

Equation (62) defines the shadow price of franked dividends. (62) says that (in equilibrium) a relaxation of the cash flow constraint by one dollar is worth the after-tax value of an unfranked dividend to shareholders. Expressions (20), (61) and (62) provide

$$l_t^{2,j,1} = l_t^{1,j,1} = 1 - q_t^j \, , \tag{63}$$

the shadow price of tax-related cash flow. This shows the sensitivity of the objective function to a change in tax payments by one unit in the presence of a change in unfranked dividends. Because no franking credits are distributed, there are no franking-account-related constraints on the firm's ability to pay franked dividends due to a change in tax payments, and so the shadow price of one dollar's-worth of dividends and company tax are equal. Expressions (21), (22), (61), (62), (63) imply

$$l_t^{3,j,1} = q_t^n \left(1 - q_t^j\right)\left[1 - t_t^j \left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right]. \tag{64}$$

Equation (64) defines the cost of a unit of capital as equal to its asset price, q_t^n, adjusted for the cost to shareholders of financing each of those dollars with retained unfrankable earnings, $\left(1 - q_t^j\right)$, minus the value of the depreciation and investment allowances deducted at the company tax level, $\left[1 - t_t^j \left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right]$. Dividing through by the asset price, we obtain

$$\Omega_t^{n,j,1} = \left(1 - q_t^j\right)\left[1 - t_t^j \left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right], \tag{65}$$

the cost to shareholders of purchasing one dollar's-worth of capital.

The next two cases are variations on this retained earnings example. We state the important equations only and leave discussion of their implications until we imbed them in the rate-of-return equations.

3.2.1.2. Financing Policy 2: Binding Franking Account Constraint (Dividend Imputation System)

The slack variables set to zero are

$$l_t^{5,j,2} = l_t^{6,j,2} = l_t^{7,j,2} = 0, \ \forall t . \tag{66}$$

(25) and (66) imply

$$l_t^{1,j,2} = 1 - q_t^j , \tag{67}$$

By (26), (66) and (67), we have

$$l_t^{10,j,2} = \frac{\left(1-q_t^j\right) g_t^j t_t^j}{1-t_t^j}. \tag{68}$$

the shadow price of a franking credit. If there are no franking credits (i.e. $g_t^j = 0$), as would be the case in a classical system, expression (68) is equal to zero. Along with (20), equation (68) provides

$$l_t^{2,j,2} = \left(1-q_t^j\right)\left(1-g_t^j\right). \tag{69}$$

Substituting these values into (41) provides

$$\Omega_t^{n,j,2} = \left(1-q_t^j\right)\left[1-\left(1-g_t^j\right) t_t^j \left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right]. \tag{70}$$

3.2.1.3. Financing Policy 3: Slack Franking Account Constraint (Dividend Imputation System)

The slack variables set to zero are

$$l_t^{5,j,3} = l_t^{7,j,3} = l_t^{10,j,3} = 0, \ \forall t. \tag{71}$$

Equations (26) and (71) imply

$$l_t^{1,j,3} = \frac{\left(1-q_t^j\right)\left[1-\left(1-g_t^j\right) t_t^j\right]}{1-t_t^j}. \tag{72}$$

Along with (20), (72) implies

$$l_t^{2,j,3} = l_t^{1,j,3} = \frac{\left(1-q_t^j\right)\left[1-\left(1-g_t^j\right) t_t^j\right]}{1-t_t^j}. \tag{73}$$

Equations (41), (71) and (73) provide

$$\Omega_t^{n,j,3} = \frac{\left(1-q_t^j\right)\left[1-\left(1-g_t^j\right) t_t^j\right]\left[1-t_t^j\left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right]}{1-t_t^j}. \tag{74}$$

3.2.2. Financing with New Equity Issues at the Margin

3.2.2.1. Financing Policy 4: Slack Franking Account Constraint (Classical or Dividend Imputation System)

Setting the appropriate slack variables to zero provides

$$l_t^{5,j,4} = l_t^{8,j4} = l_t^{10,j,4} = 0, \ \forall t. \tag{75}$$

$l_t^{1,j,4}$, by (27) and (75), is

$$l_t^{1,j,4} = \frac{\left(1-c_t^j\right)}{\Delta NVNF_t^j}. \tag{76}$$

where $$\Delta NVNF_t^j = \left[1-\frac{\left(\ln VN_t^j + \Theta_t^j - 1\right)}{\left(\ln VN_t^j + \Theta_t^j\right)^2}\right].$$

Equations (20), (75) and (76) imply that

$$l_t^{2,j,4} = l_t^{1,j,4} = \frac{\left(1-c_t^j\right)}{\Delta NVNF_t^j}. \tag{77}$$

Taking (41), (75) and (77), we find

$$\Omega_t^{n,j,4} = \frac{\left(1-c_t^j\right)\left[1-t_t^j\left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right]}{\Delta NVNF_t^j}. \tag{78}$$

3.2.2.2. Financing Policy 5: Binding Franking Account Constraint (Dividend Imputation System)

The appropriate slack variables to set to zero are

$$l_t^{5,j,5} = l_t^{7,j,5} = l_t^{8,j,5} = 0, \ \forall t. \tag{79}$$

Equations (27) and (79) imply

$$l_t^{1,j,5} = \frac{\left(1-c_t^j\right)}{\Delta NVNF_t^j}. \tag{80}$$

where $$\Delta NVNF_t^j = \left[1-\frac{\left(\ln VN_t^j + \Theta_t^j - 1\right)}{\left(\ln VN_t^j + \Theta_t^j\right)^2}\right].$$

These are identical to the equivalent conditions in the previous section. Expressions (26), (79) and (80) provide

$$l_t^{2,j,5} = \frac{1}{t_t^j}\left\{\frac{\left(1-c_t^j\right)}{\Delta NVNF_t^j} - \left(1-q_t^j\right)\left[1-\left(1-g_t^j\right)t_t^j\right]\right\}, \tag{81}$$

and equations (41), (79), (80) and (81) provide

$$\Omega_t^{n,j,5} = \frac{\left(1-c_t^j\right)\left[1-\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]}{\Delta NVNF_t^j} + \left(1-q_t^j\right)\left[1-\left(1-g_t^j\right)t_t^j\right]\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right). \tag{82}$$

3.2.3. Financing with Debt at the Margin

The assumptions we make about how the firm's earnings are distributed to shareholders are important for these examples. The cost of debt is partly a function of the earnings that shareholders forego in repaying debt. This is a function of foregone dividends used to repay debt when earnings are being distributed, or the capital loss from issuing equity if no dividends are being paid. Therefore, it has something to do with the cost of not paying dividends (equivalent to the cost of retained earnings) or the cost of equity issues, appropriately discounted.

Equations (28) and (30) define the inter-temporal equilibrium that will occur between the present values of the new cash flows (between funds received in period t and servicing costs) generated by the bonds when financial markets clear. Solving (30) for $l_t^{4,j}$, substituting the result into (28) and rearranging provides[6]

$$l_t^{1,j,f} = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^j\right)\left\{\frac{r_{t+z}^j l_{t+s}^{1,j,f} + \left(l_{t+s}^{1,j,f} - l_{t+s}^{2,j,f}t_{t+s}^j\right)r_{t+1}^j\left(1-bFLV_t^j\right)}{\Delta NBF_{t-1}^j\prod_{k=1}^{s}\Phi_{t+k}^j}\right\}\right\}. \tag{83}$$

The assumptions we make about the firm's dividend policy will determine the values of the various unknown l's in this equation.

3.2.3.1. Financing Policy 6: A Firm Paying only Unfranked Dividends – A Slack Franking Account Constraint (Classical System)

The slack variables we set to zero in this case are

$$l_t^{5,j,6} = l_t^{6,j6} = l_t^{9,j6} = l_t^{10,j,6} = 0, \ \forall t. \tag{84}$$

This says that the firm is investing, paying unfranked dividends, issuing debt and is not constrained by a franking account. When the firm repays debt (starting from the following period), it will need to reduce its payments of unfranked dividends. Therefore, in equation (83), the shadow price of cash flow is given by

$$l_t^{1,j,6} = 1-q_t^j, \quad \forall t, \tag{85}$$

while, by (20), (84) and (85),

$$l_t^{1,j,6} = l_t^{2,j,6} = 1-q_t^j, \quad \forall t. \tag{86}$$

As before, the equality between $l_t^{1,j,f}$ and $l_t^{2,j,f}$ is a result of the franking account constraint being slack. Substituting these results into (83), we find

[6] See Winston (2004) for background and detail.

$$l_t^{1,j,6} = l_t^{2,j,6} = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^{j}\right)\left(1-q_{t+s}^{j}\right)\left\{\frac{r_{t+s}^{j}+\left(1-t_{t+s}^{j}\right)r_{t+1}^{j}\left(1-b\,FLV_t^{j}\right)}{\Delta NBF_{t-1}^{j}\prod_{k=1}^{s}\Phi_{t+k}^{j}}\right\}\right\}. \qquad (87)$$

Assume for now that the rates of the various taxes in (87) are zero, and then

$$l_t^{1,j,6} = l_t^{2,j,6} = 1 = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^{j}\right)\left\{\frac{r_{t+s}^{j}+r_{t+1}^{j}\left(1-b\,FLV_t^{j}\right)}{\Delta NBF_{t-1}^{j}\prod_{k=1}^{s}\Phi_{t+k}^{j}}\right\}\right\}. \qquad (88)$$

This says: the shadow price of one dollar of cash flow is equal to one dollar when tax rates are zero; and the present value of the debt-repayments on a one-dollar bond issue at the margin will be equal to one dollar, because of optimising behaviour. As the firm increases its indebtedness, the financial-leverage effects drive the interest rate upwards until the equality in (88) is achieved.

The debt issue in period t increases the firm's cash flow (and therefore its value to shareholders) by the value of the bonds issued, while reducing it in periods t+1 onward by the amounts required to service the repayments. The firm will continue to increase its bond issues to finance investment until the after-tax income stream from investment is equal to the cost of investment, and the cost of investment under this financial policy increases at the margin because of leverage-related interest rate effects. Substituting (87) into (41), we find

$$\Omega_t^{n,j,6} = \left[1-t_t^{j}\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]\sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^{j}\right)\left(1-q_{t+s}^{j}\right)\left\{\frac{r_{t+s}^{j}+\left(1-t_{t+s}^{j}\right)r_{t+1}^{j}\left(1-b\,FLV_t^{j}\right)}{\Delta NBF_{t-1}^{j}\prod_{k=1}^{s}\Phi_{t+k}^{j}}\right\}\right\}. \qquad (89)$$

Expression (89) says that the cost of one dollar's-worth of capital is the present value of the infinite stream of servicing obligations minus the reduction in the number of dollars required due to tax deductions, where the value of cash flow is measured in terms of the shadow price of unfranked dividends.

Notice that the deductibility of interest is captured inside the infinite sum on the right-hand-side of (89), because this type of deduction affects the cost of a dollar of finance. The deductibility of capital expenditures and depreciation, on the other hand, enters a multiplier term on this infinite sum – this will be less than one and greater than zero for firms not paying maximum franked dividends in an imputation system – and captures the reduction in the number of dollars of finance required to purchase the asset due to tax allowances. Thus, here we see the distinction between interest and capital allowances in the firm's cost of capital – the former reduces the cost of a dollar of finance while the latter reduces the number of after-tax dollars required. This stream of payments is grossed up for transaction costs on the bond issue by the term ΔNBF_{t-1}^{j}.

The next two examples are stated briefly.

3.2.3.2. Financing Policy 7: A Firm Paying only Franked Dividends – A Binding Franking Account Constraint (Dividend Imputation System)

The appropriate slack variables to set to zero in this case are

$$l_t^{5,j,7} = l_t^{7,j,7} = l_t^{9,j,7} = 0, \ \forall t . \tag{90}$$

Equations (25) and (90) imply

$$l_t^{1,j,7} = 1 - q_t^j, \ \ \forall t , \tag{91}$$

while equations (26), (90) and (91) provide

$$l_t^{10,j,7} = \frac{\left(1-q_t^j\right) g_t^j t_t^j}{1-t_t^j}, \ \ \forall t . \tag{92}$$

Along with (20), this gives us

$$l_t^{2,j,7} = \left(1-q_t^j\right)\left(1-g_t^j\right), \ \ \forall t . \tag{93}$$

Substituting these results into equation (83), we obtain

$$l_t^{1,j,7} = \left(1-q_t^j\right) = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^j\right)\left(1-q_{t+s}^j\right)\left\{\frac{r_{t+z}^j + \left[1-\left(1-g_{t+s}^j\right)t_{t+s}^j\right] r_{t+1}^j\left(1-b\,FLV_t^j\right)}{\Delta NBF_{t-1}^j \prod_{k=1}^{s}\Phi_{t+k}^j}\right\}\right\}. \tag{94}$$

Equation (94) tells us about the present value of the debt-servicing obligations stemming from a bond issue. As implied by (91), (93) and (94), $l_t^{2,j,7}$ can be defined by

$$l_t^{2,j,7} = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^j\right)\left(1-q_{t+s}^j\right)\left\{\frac{r_{t+z}^j + \left[1-\left(1-g_{t+s}^j\right)t_{t+s}^j\right] r_{t+1}^j\left(1-b\,FLV_t^j\right)}{\Delta NBF_{t-1}^j \prod_{k=1}^{s}\Phi_{t+k}^j}\right\}\right\}\left(1-g_t^j\right). \tag{95}$$

Substituting into (41) we have

$$\Omega_t^{n,j,7} =$$

$$\sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^j\right)\left(1-q_{t+s}^j\right)\left\{\frac{r_{t+z}^j + \left[1-\left(1-g_{t+s}^j\right)t_{t+s}^j\right] r_{t+1}^j\left(1-b\,FLV_t^j\right)}{\Delta NBF_{t-1}^j \prod_{k=1}^{s}\Phi_{t+k}^j}\right\}\right\}\left[1-\left(1-g_t^j\right)t_t^j\left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right] \tag{96}$$

3.2.3.3. Financing Policy 8: A Firm Paying NO Dividends – A Slack Franking Account Constraint (Classical or Dividend Imputation System)

Setting the appropriate slack variables to zero, we have

$$l_t^{5,j8} = l_t^{8,j8} = l_t^{9,j8} = l_t^{10,j8} = 0, \ \forall t \, . \tag{97}$$

Along with equations (27) and (20), this implies

$$l_t^{1,j,8} = l_t^{2,j,8} = 1 - c_t^j, \quad \forall t \, . \tag{98}$$

Substituting (98) into (30) we obtain

$$l_t^{1,j,8} = l_t^{2,j,8} = \left(1 - c_t^j\right) = \sum_{s=1}^{\infty} \left\{ \prod_{z=1}^{s} \left(1 - r_{t+z}^j\right)\left(1 - c_{t+s}^j\right) \left\{ \frac{r_{t+s}^j + \left(1 - t_{t+s}^j\right) r_{t+1}^j \left(1 - b\, FLV_t^j\right)}{\Delta NBF_{t-1}^j \prod_{k=1}^{s} \Phi_{t+k}^j} \right\} \right\}. \tag{99}$$

Substituting these results into (41) we find

$$\Omega_t^{n,j,8} = \sum_{s=1}^{\infty} \left\{ \prod_{z=1}^{s} \left(1 - r_{t+z}^j\right)\left(1 - c_{t+s}^j\right) \left\{ \frac{r_{t+s}^j + \left(1 - t_{t+s}^j\right) r_{t+1}^j \left(1 - b\, FLV_t^j\right)}{\Delta NBF_{t-1}^j \prod_{k=1}^{s} \Phi_{t+k}^j} \right\} \right\} \left[1 - t_t^j \left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right] \tag{100}$$

3.3. The Rate of Return on Capital

In this section, we provide a brief description of the derivation of rate-of-return functions.

3.3.1. Aggregating Rentals

Rental data is available at the industry level but not for individual capital goods. The model developed above assumes that the individual rentals applying to each capital good are known.

Our ability to aggregate rentals rests on having accurate information about investment shares and the ways in which these change over time. Accurate information about investment shares is available from the MONASH database. On the second issue, the technical composition of an industry's capital stock is constrained primarily by the technological constraints inherent in the specific process to which it is applied, and it is likely that price-substitution possibilities between inputs to capital creation are small, particularly in the short-run. The cost-minimisation occurring in the capital nests implies a technologically efficient aggregate capital stock, meaning the shares of each capital good in the capital stock are at their optimal values given the use to which the capital is being put, the technological parameters of production and the relative prices of the different capital goods. Therefore, substitution elasticities between the capital goods in the CRESH nest are likely to be low, and sometimes zero. As the change in these shares approaches zero, the potential error arising from using aggregate rental data also approaches zero. Modifying (45) to obtain a firm or industry level rate-of-return function that utilises aggregate rental data *and* detailed cost of capital data, we find

$$ROR_t^{j,f} = \frac{\left(l_{t+1}^{1,j,f} - l_{t+1}^{2,j,f} t_{t+1}^{j}\right) R_{t+1}^{j}}{q_t^n \Phi_{t+1}^{j}} - \sum_{n=1}^{N} ISH_t^{n,j} \left\{ \Omega_t^{n,j,f} - \frac{q_{t+1}^n \Omega_{t+1}^{n,j,f} \left(1 - d_t^{n,j}\right)}{q_t^n \Phi_{t+1}^{j}} \right\} \quad (101)$$

where
$$R_{t+1}^{j} = \sum_{n=1}^{N} ISH_t^{n,j} R_{t+1}^{n,j} \quad (102)$$

and
$$q_t = \sum_{n=1}^{N} ISH_t^{n,j} q_t^n . \quad (103)$$

A weighted-average asset price is used to calculate an effective rental per dollar from firm or industry level rental data, again using investment shares as weights. From this point on, (101) provides the basis for the model's rate-of-return equations.

3.3.2. Expected Rates of Return with Forward-Looking Expectations

The rates of return under forward-looking expectations, with actual rates of return implicit, are listed below. They are found by substituting the various values of $l_t^{1,j,f}$, $l_t^{2,j,f}$ and $\Omega_t^{n,j,f}$ for the $f = 1$ to 8 financial policies into (104).

3.3.2.1. Financing Policy 1: Retained Unfrankable Earnings with a Slack Franking Account Constraint (Classical System)

$$EROR\left(FL\right)_t^{j,1} = \frac{R_{t+1}^{j}}{q_t \Phi_{t+1}^{j}} \left(1 - q_{t+1}^{j}\right)\left(1 - t_{t+1}^{j}\right) - \sum_{n=1}^{N} ISH_t^{n,j} \left\{ \begin{array}{l} \left(1 - q_t^{j}\right)\left[1 - t_t^{j}\left(\Gamma_t^{n,j} + \Xi_t^{n,j}\right)\right] \\ - \dfrac{q_{t+1}^n}{q_t^n \Phi_{t+1}^{j}} \left(1 - q_{t+1}^{j}\right)\left[1 - t_{t+1}^{j}\left(\Gamma_{t+1}^{n,j} + \Xi_{t+1}^{n,j}\right)\right]\left(1 - d_t^{n,j}\right) \end{array} \right\} \quad (105)$$

The rate of return on investment- the net-present-value of one dollar's-worth of investment – is always a function of the relationship between the income received (the after-tax, present value of the rental on a dollar's-worth of capital) and the costs borne (i.e. the cost of capital used up in producing this income, a function of asset prices, tax allowances, physical depreciation rates, and financing costs).

The shareholders in this firm don't benefit from dividend-imputation, and so rental income is subject to double-taxation. The gross income stream from one dollar of capital at the margin, $\left(R_{t+1}^{j}/q_t\right)$, is taxed twice, once at the company level and once at the personal level, at rates expected to exist in period t+1, $\left(1 - q_{t+1}^{j}\right)\left(1 - t_{t+1}^{j}\right)$. This payment occurs in a future period and is therefore discounted appropriately by Φ_{t+1}^{j}.

The cost of capital is some function of the asset's cost at the time of purchase and its residual value after one period's use. A firm has two things to consider – how many dollars are required to purchase an asset, and the shadow price of each of those dollars in light of the corporate tax and financial system. These equations define the rate of return (the return on one-dollar's-worth of capital) and so the starting point for an answer to the "how-many dollars" question is *one dollar* at the time of purchase, minus some amount at the end of the period adjusted for asset price inflation/deflation and physical depreciation. The cost of one dollar of finance is partly a function of the shadow price of the source, and in this case it is retained unfrankable earnings, hence the

various coefficients containing $\left(1-q_t^{\ j}\right)$ or $\left(1-q_{t+1}^{\ j}\right)$. Depreciation and investment allowances reduce the number of after-tax dollars that are required to purchase an asset, and in the absence of dividend imputation, these allowances will have non-zero value to shareholders – thus, the term $\left[1-t_t^{\ j}\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]$ appears in (105).

Capital gains taxation is always relevant to shareholders, even when the firm is not sourcing finance from equity issues or retaining all dividends all of the time. The capital gains tax terms in the discount rate (see equation (1)) serve two purposes; in the numerator of the discount rate, the CGT term reduces the size of the discount rate to *add* back in the value of initial value of the firm for capital gains tax calculations. This is also the place where CGT base-indexing occurs – if real gains are taxed, the base must be inflated to remove the effects of inflation for calculating capital gains. The CGT term in the denominator of the discount rate takes account of the fact that the value of the firm to shareholders at any point in time is given by the realisable value of their shares. The *realisable* value of shares is their after-CGT value. This term adjusts the entire expression for the capital gains tax that would be payable on the shares if they were realised. In effect, the combination of these CGT terms imposes the capital gains tax on the model as if it was a discounting factor.

3.3.2.2. Financing Policy 2: Retained Unfrankable Earnings with Binding Franking Account Constraint (Dividend Imputation System)

$$EROR(FL)_t^{j,2}=\frac{R_{t+1}^j}{q_t\Phi_{t+1}^j}\left(1-q_{t+1}^{\ j}\right)\left[1-\left(1-g_{\ t+1}^{\ j}\right)t_{\ t+1}^{\ j}\right]$$
$$-\sum_{n=1}^{N}ISH_t^{n,j}\left\{\left(1-q_t^{\ j}\right)\left[1-\left(1-g_t^{\ j}\right)t_{\ t}^{\ j}\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]-\frac{q_{t+1}^n}{q_t^n\Phi_{t+1}^j}\left(1-q_{t+1}^{\ j}\right)\left[1-\left(1-g_{\ t+1}^{\ j}\right)t_{\ t+1}^{\ j}\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]\left(1-d_t^{\ n,j}\right)\right\} \quad (106)$$

The differences between this equation and (105) stem from the effects of dividend-imputation in the presence of a binding franking account constraint. Firstly, the value of tax-deductions approaches zero as the rate of dividend imputation approaches one. This has been discussed in some detail earlier. The crucial factor in this is the payment of maximum franking-credits, imposed by the binding franking account constraint. Secondly, rental income is taxed at more concessional rates as the rate of dividend imputation approaches one, causing the company-tax term $\left[1-\left(1-g_{\ t+1}^{\ j}\right)t_{\ t+1}^{\ j}\right]$ to disappear.

There are some important taxation issues highlighted by a comparison of equations (105) and (106). The taxation of rental income under this financial policy occurs according to a tax coefficient that includes an imputation rate. A question arises - given that the firm is paying the maximum amount of franked dividends, why would a change in investment influence the payment of franked dividends, as suggested by the presence of the imputation rate in the coefficient? The answer is that it doesn't – it changes the amount of unfranked dividends being paid at the margin. Because we are assuming an imputation system, unfranked dividends are paid out of *untaxed* company income, and thus the rate company taxation must disappear regardless of which type of dividend is financing investment. This, actually, is the goal of the imputation system – to subject all capital income from investment to the same rates of taxation. This is easier to understand if we remember that the variable R_{t+1}^j is more closely related to EBT_{t+1}^j than it is to dividends – assuming full imputation, the relationship between R_{t+1}^j, D_{t+1}^j and $DF_{t+1}^{\ j}$ is given by

$$\left(R_{t+1}^{j}-TD_{t}^{j}\right)=\frac{DF_{t+1}^{j}}{1-t_{t+1}^{j}} \qquad \text{and} \qquad TD_{t}^{j}=D_{t+1}^{j} \tag{107}$$

Therefore, we could replace the income term in (106) by substituting for (107), finding

$$MBK_{t}^{j}=R_{t+1}^{j}\frac{\left(1-q_{t+1}^{j}\right)}{q_{t}\Phi_{t+1}^{j}}=\left\{D_{t+1}^{j}+\frac{DF_{t+1}^{j}}{1-t_{t+1}^{j}}\right\}\frac{\left(1-q_{t+1}^{j}\right)}{q_{t}\Phi_{t+1}^{j}} \tag{108}$$

This *would* be double counting, however, because our rate-of-return functions capture the value of tax deductions in the *cost* of capital. Therefore, we can ignore tax deductions in capital income and treat all of this income as if it is franked (i.e. that is hasn't been quarantined from taxation due to deductions), so that (107) can be simplified to

$$R_{t+1}^{j}=\frac{DF_{t+1}^{j}}{1-t_{t+1}^{j}}. \tag{109}$$

Thus, the rental variable is equivalent to the grossed-up value of the franked dividends paid by the firm, regardless of whether the income received is actually a franked or unfranked dividend. This is why there is no grossing-up term explicit in the tax coefficient on R_{t+1}^{j}. As we reduce the rate of imputation, the nature of the tax system approaches a classical system, and (108) becomes

$$MBK_{t}^{j}=\frac{R_{t+1}^{j}}{q_{t}\Phi_{t+1}^{j}}\left(1-q_{t+1}^{j}\right)\left(1-t_{t+1}^{j}\right)=\frac{D_{t+1}^{j}}{q_{t}\Phi_{t+1}^{j}}\left(1-q_{t+1}^{j}\right), \tag{110}$$

reflecting that

$$R_{t+1}^{j}\left(1-t_{t+1}^{j}\right)=D_{t+1}^{j} \tag{111}$$

in a classical system (ignoring deductions). This is seen in the rate-of-return function for financial policy 1 above.

3.3.2.3. Financing Policy 3: Retained Frankable Earnings with a Slack Franking Account Constraint (Dividend Imputation System)

$$\begin{aligned}EROR\left(FL\right)_{t}^{j,3}=&\frac{R_{t+1}^{j}}{q_{t}\Phi_{t+1}^{j}}\left(1-q_{t+1}^{j}\right)\left[1-\left(1-g_{t+1}^{j}\right)t_{t+1}^{j}\right]\\&-\sum_{n=1}^{N}ISH_{t}^{n,j}\left\{\begin{array}{l}\dfrac{\left(1-q_{t}^{j}\right)\left[1-\left(1-g_{t}^{j}\right)t_{t}^{j}\right]\left[1-t_{t}^{j}\left(\Gamma_{t}^{n,j}+\Xi_{t}^{n,j}\right)\right]}{1-t_{t}^{j}}\\-\dfrac{q_{t+1}^{n}}{q_{t}^{n}\Phi_{t+1}^{j}}\dfrac{\left(1-q_{t+1}^{j}\right)\left[1-\left(1-g_{t+1}^{j}\right)t_{t+1}^{j}\right]\left[1-t_{t+1}^{j}\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]}{1-t_{t+1}^{j}}\left(1-d_{t}^{n,j}\right)\end{array}\right\}\end{aligned} \tag{112}$$

For this financing policy, rental income is taxed in the same way as under financial policy 2 because of the imputation system – whether the dividends being paid are franked or not, under an imputation system they have the same value to shareholders.

The cost of a dollar of finance – a retained franked dividend of one dollar – depends on the rate of imputation. As it approaches one, the term $\left(1-\left(1-g_t^{\,j}\right)t_t^{\,j}\right)$ also approaches one, and so the cost of a dollar of finance is equal to the fully-grossed up value of a franked dividend worth one dollar.

The cost of a dollar of finance in this example is partly a function of a grossing-up term $1/\left(1-t_t^{\,j}\right)$. This appears here, and not in the equivalent place in equations (105) or (106) for financial policies 1 or 2, because the *source of finance* in this case is a dividend carrying a tax credit. Why, then, does this grossing-up term *not* appear in the tax coefficient on rental income? The answer, as discussed in different context above, is that we are measuring the value of rental payments and the costs of finance from slightly different perspectives. In the case of rental income, the variable R_{t+1}^{j} is the *before-company-tax* value of capital income, while in the case of finance costs, we measure retained earnings from the perspective of *after-company-tax* income (dividends). When the focus is finance costs and the firm is retaining post-company-tax frankable earnings, the grossing-up term appears in the denominator of the tax coefficient for retained frankable earnings. On the other hand, when the focus is on rental income, measured by a pre-company-tax variable R_{t+1}^{j}, the company-tax term enters as a multiplier.

The firm is not paying the maximum amount of franking credits, and so a change in its tax bill due to capital allowances has a non-zero effect on shareholder wealth. Hence the term that nets-out deductions, $\left[1-t_t^{\,j}\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]$, is independent of the imputation rate. In the previous example, which assumed the firm was paying the maximum amount of franked dividends, dividend imputation rendered tax deductions worthless to shareholders.

3.3.2.4. Financing Policy 4: A New Equity Issue with a Slack Franking Account Constraint (Classical or Dividend Imputation System)

$$EROR\left(FL\right)_t^{\,j,4}=\frac{R_{t+1}^{j}}{q_t^{n}\Phi_{t+1}^{j}}\frac{\left(1-c_{t+1}^{j}\right)\left(1-t_{t+1}^{\,j}\right)}{\Delta NVNF_{t+1}^{\,j}}$$
$$-\sum_{n=1}^{N}ISH_t^{n,j}\left\{\frac{\left(1-c_t^{\,j}\right)\left[1-t_t^{\,j}\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]}{\Delta NVNF_t^{\,j}}-\frac{q_{t+1}^{n}\left(1-c_{t+1}^{j}\right)\left[1-t_{t+1}^{\,j}\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]}{q_t^{n}\Phi_{t+1}^{j}\Delta NVNF_{t+1}^{\,j}}\left(1-d_t^{n,j}\right)\right\} \quad (113)$$

where $\Delta NVNF_t^{\,j}$ is the change in net invest-able equity finance after transactions costs,

$$\Delta NVNF_t^{\,j}=\frac{\partial VN_t^{j}\left[1-TRVN_t^{\,j}\right]}{\partial VN_t^{j}}=1-\frac{\left(\ln VN_t^{\,j}+\Theta_t^{j}-1\right)}{\left(\ln VN_t^{\,j}+\Theta_t^{j}\right)^{2}}$$

and

$$TRVN_t^{\,j}=\frac{1}{\ln VN_t^{\,j}+\Theta_t^{j}}$$

is the transaction cost on new equity issues.

Under this financial policy, firms are paying no dividends at all (in an imputation *or* classical system). If the firm is paying no dividends, it must be passing on shareholder income only as capital gains. Rental income is therefore taxed first at the company tax rate and then at the capital-gains tax rate. This amount is grossed-up by $\Delta NVNF_t^{\,j}$, the net change in cash flow due to the share issue, partly a function of the change in the total transaction costs on all shares due to the sale of the

marginal dollar of equity. The rental reported here is the rental on one-dollar's-worth of capital, and the shadow price of this amount is a function of its source – capital gains. Because the firm is retaining all of its earnings, an extra dollar of retainable earnings reduces its need to acquire it from an external source – in this case, an equity issue, which would involve a transactions cost. It is against this opportunity cost that the value of a dollar of cash flow is measured in this example.

The cost of the capital that produces this income stream is partly a function of the cost of a dollar of equity finance at the margin, which is an increasing function of the size of the share issue, *but* with a negative second derivative (due to the fall in the average transaction cost at the margin). To acquire one-dollar's-worth of finance to spend on a capital good, the firm needs to issue $\Delta NVNF_t^j$ dollars of shares, and each of these dollars costs existing shareholders $\left(1-c_t^j\right)$.

The franking account constraint in this example is slack. Shareholders will benefit from any deductions available on an asset, because no tax credits are transferred to shareholders and, thus, tax savings at the company-tax level have value.

Equation (113) is equivalent to equation (105) for financial policy 1 with the dividend-tax terms, $\left(1-q_t^{\,j}\right)$, replaced by the capital-gains-tax and transaction cost terms $\left(1-c_t^j\right)\Big/\Delta NVNF_t^{\,j}$. If transaction costs were zero and the rates of dividend and capital-gains tax were the same, these equations would be identical. The equivalency is driven mainly by the slack franking account, which removes imputation issues form the analysis.

3.3.2.5. Financing Policy 5: A New Equity Issue with a Binding Franking Account Constraint (Dividend Imputation System)

$$EROR\left(FL\right)_t^{j,5}=\frac{R_{t+1}^j}{q_t^n\Phi_{t+1}^j}\left(1-q_{t+1}^{\,j}\right)\left[1-\left(1-g_{t+1}^{\,j}\right)t_{t+1}^{\,j}\right]$$
$$-\sum_{n=1}^{N}ISH_t^{n,j}\left\{\begin{array}{l}\dfrac{\left(1-c_t^j\right)\left[1-\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]}{\Delta NVNF_t^j}+\left(1-q_t^{\,j}\right)\left[1-\left(1-g_t^{\,j}\right)t_t^{\,j}\right]\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\\ -\dfrac{q_{t+1}^n\left(1-d_t^{\,n,j}\right)}{q_t^n\Phi_{t+1}^j}\left[\begin{array}{l}\dfrac{\left(1-c_{t+1}^j\right)\left[1-\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]}{\Delta NVNF_{t+1}^j}\\ +\left(1-q_{t+1}^{\,j}\right)\left[1-\left(1-g_{t+1}^{\,j}\right)t_{t+1}^{\,j}\right]\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\end{array}\right]\end{array}\right\} \qquad (114)$$

where $$\Delta NVNF_t^{\,j}=\frac{\partial VN_t^j\left[1-TRVN_t^j\right]}{\partial VN_t^j}=1-\frac{\left(\ln VN_t^j+\Theta_t^j-1\right)}{\left(\ln VN_t^{\,j}+\Theta_t^j\right)^2}$$

and $$TRVN_t^j=\frac{1}{\ln VN_t^j+\Theta_t^j}.$$

This is perhaps the most difficult financial policy to intuitively grasp. The effect on shareholder wealth under this policy is a relatively complicated function of the interplay between the dividend imputation system and the realisation-basis taxation of capital gains.

The taxation of rental income is identical to that for financial policies 2 and 3. This firm is issuing equity at the margin with a binding constraint on the franking account, implying that the firm is paying the maximum amount of franked dividends. Permutations of this policy could include the

firm retaining all unfrankable earnings infra-marginally, or paying the maximum amount of them – empirical evidence suggests that firms are likely to have retained all unfrankable earnings under this policy, although it is not crucial to the solution. The tax coefficient on rental income in this case is

$$l_{t+1}^{1,j,5} - l_{t+1}^{2,j,5} t_{t+1}^{j} = \frac{\left(1-c_{t+1}^{j}\right)}{\Delta NVNF_{t+1}^{j}} - \left\{ \begin{array}{l} \dfrac{\left(1-c_{t+1}^{j}\right)}{\Delta NVNF_{t+1}^{j}} \\ -\left\{ \dfrac{\left(1-q_{t+1}^{j}\right)\left[1-\left(1-g_{t+1}^{j}\right)t_{t+1}^{j}\right]}{\left(1-t_{t+1}^{j}\right)} - \dfrac{\left(1-c_{t+1}^{j}\right)}{\Delta NVNF_{t+1}^{j}} \right\} \left[\dfrac{1-t_{t+1}^{j}}{t_{t+1}^{j}} \right] \end{array} \right\} t_{t+1}^{j}. \qquad (115)$$

$$= \left(1-q_{t+1}^{j}\right)\left[1-\left(1-g_{t+1}^{j}\right)t_{t+1}^{j}\right]$$

This is the tax coefficient applicable to an imputation system when franked dividends are being paid (compare it with equations (106) and (112) for financial policies 2 and 3 respectively), and shows that the benefits of investment are actually received as franked dividends. The first instance of $\left(1-c_{t+1}^{j}\right)\Big/\Delta NVNF_{t+1}^{j}$ on the right hand-side captures the shadow price (or in this case, benefit) of a dollar of capital gains to shareholders generated by investment, before deductions and company tax are accounted for. Tax is subtracted from this amount by the term inside the large brackets, itself comprised of two parts: (a) the reduction in cash flow caused by tax payments minus (b) the increase in the franking account balance it creates. The net result is that shareholders receive a distribution as a franked dividend.

For capital expenditures, the story is more complex. When a firm issues one dollar of equity while paying all tax-credits to shareholders, the net effect on shareholder wealth is a function of the shadow price of new equity, $\left(1-c_{t}^{j}\right)\Big/\Delta NVNF_{t}^{j}$, and the effect of deductions on the firm's tax liabilities and, thus, ability to pay franked dividends. Capital expenditures cause changes in tax payments due to the availability of capital allowances, and changes in tax payments influence the value of the firm's equity to shareholders by changing the franked/unfranked dividend payments mix and the net value of the firm's cash flows. To illustrate the point, assume that the there are no transactions costs on an equity issue and a full imputation system, and so equation (114) becomes

$$EROR\left(FL\right)_{t}^{j,5} = \frac{R_{t+1}^{j}}{q_{t}^{n}\Phi_{t+1}^{j}}\left(1-q_{t+1}^{j}\right)$$

$$-\sum_{n=1}^{N} ISH_{t}^{n,j} \left\{ \begin{array}{l} \left(1-c_{t}^{j}\right)\left[1-\left(\Gamma_{t}^{n,j}+\Xi_{t}^{n,j}\right)\right]+\left(1-q_{t}^{j}\right)\left(\Gamma_{t}^{n,j}+\Xi_{t}^{n,j}\right) \\ -\dfrac{q_{t+1}^{n}\left(1-d_{t}^{n,j}\right)}{q_{t}^{n}\Phi_{t+1}^{j}}\left[\left(1-c_{t+1}^{j}\right)\left[1-\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]+\left(1-q_{t+1}^{j}\right)\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right] \end{array} \right\}. \qquad (116)$$

Making an additional assumption that the rates of dividend- and capital-gains tax are identical, this simplifies further to

$$EROR\left(FL\right)_{t}^{j,5} = RORact_{t}^{j,5} = \frac{R_{t+1}^{j}}{q_{t}^{n}\Phi_{t+1}^{j}}\left(1-c_{t+1}^{j}\right) - \sum_{n=1}^{N} ISH_{t}^{n,j}\left\{\left(1-c_{t}^{j}\right)-\left(1-c_{t+1}^{j}\right)\frac{q_{t+1}^{n}\left(1-d_{t}^{n,j}\right)}{q_{t}^{n}\Phi_{t+1}^{j}}\right\}. \qquad (117)$$

At first glance, setting the rate of dividend imputation at 1 should make any deductions worthless to shareholders. This is clearly not true in equation (116), but certainly is true of equation (117). Given that the only difference between (116) and (117) is that the rates of dividend tax and capital-gains tax are identical in the latter, it is apparent that the difference between the two comes from

this divergence. The reason for this is subtle, and due to two factors: firstly, the deductions reduce the number of after-CGT dollars that need to be raised from the equity issue to finance investment, reducing the cost of investment; and, secondly, deductions reduce the ability of the firm to distribute franked dividends by an amount calculated against the dividend tax rate. Both of these amounts are calculated with reference to the tax rate against which the deduction itself is calculated – the company income tax, but some of the company tax terms cancel out in the derivation.

3.3.2.6. Financing Policy 6: A Debt Issue while Paying ONLY Unfranked Dividends and a Slack Franking Account Constraint (Classical System)

$$\begin{aligned} EROR(FL)_t^{j,6} &= \frac{R_{t+1}^j}{q_t^n \Phi_{t+1}^j}\left(1-q_{t+1}^j\right)\left(1-t_{t+1}^j\right) \\ &-\sum_{n=1}^{N} ISH_t^{n,j}\left\{\left[1-t_t^j\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]PVDS_t^{j,6}-\frac{q_{t+1}^n\left[1-t_{t+1}^j\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]PVDS_{t+1}^{j,6}\left(1-d_t^{n,j}\right)}{q_t^n\Phi_{t+1}^j}\right\} \end{aligned} \tag{118}$$

where $PVDS_t^{j,f}$ denotes the present value of debt servicing arising from the bonds issued in period t, given by

$$PVDS_t^{j,6} = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^j\right)\left(1-q_{t+s}^j\right)\left\{\frac{r_{t+s}^j+\left(1-t_{t+s}^j\right)r_{t+1}^j\left(1-bFLV_t^j\right)}{\Delta NBF_t^j\prod_{k=1}^{s}\Phi_{t+k}^j}\right\}\right\} \tag{119}$$

where

$$\Delta NBF_t^j = 1-\frac{\left(\ln B_t^j+\mathrm{X}_t^j-1\right)}{\left(\ln B_t^j+\mathrm{X}_t^j\right)^2} \tag{120}$$

is the change in net bond financing after transaction costs. If q_t^j, b_t^j, t_t^j, c_t^j and i_t^j are constant, we can simplify this to obtain

$$PVDS_t^{j,6} = \left\{\frac{\Phi_t^j\left(1-q_t^j\right)}{\Phi_t^j-1+r_t^j}\right\}\left[\frac{r_t^j+\left(1-t_t^j\right)r_{t+1}^j\left(1-bFLV_t^j\right)}{\Delta NBF_t^j}\right]. \tag{121}$$

This financial policy is the first rate of return equation we analyse that involves debt issues. A bond issue has three important effects – it changes the firm's tax bill via the deductibility of interest, it pushes the cost component of investment finance out into future periods, and the firm's financial leverage and bond-rate changes.

Rental income is taxed in an identical way to financial policy 1. This firm is issuing a bond to finance investment at the margin, but the rentals it generates are still paid as classical, double-taxed dividends. This highlights the central role of dividend policy in the firm's rate of return – it is important to know both how the investment is financed and how the proceeds are distributed.

On capital expenditures, it is useful to take the coefficients on asset purchases from financial policy 1 and compare them with those in equation (118). In equation (105), the shadow price of finance is the after-dividend-and-company-tax value of a dollar of unfrankable earnings, and deductions carry full tax benefits for the shareholder because of the absence of dividend imputation. This is also true of equation (118). The two expressions differ *only* because of the effect of the bond issue – it

imposes an "inflator" on the values discussed above (via the bond interest rate), and moves all of the cash flow consequences out into future periods. The shadow price of a dollar of debt finance in the presence of classical dividend payments is the present value (note the different discount rates in (105) and (118)) of the reduction in dividend payments over the periods in which the debt and interest must be serviced. In effect, which of (105) and (118) is optimal will depend on the relationship between the levels and tax treatments of interest on debt and the rate of interest in the investor's discount rate. This highlights the importance of the interplay between the rates at which different sides of the finance market discount cash flows. This is true for every debt-related rate of return equation and its equivalent in terms of dividend policy - (105) and (118); (106) and (122), and (113) and (126). Readers are referred to a comparison of these equations in light of the preceding discussion for an insight into the last two rate-of-return equations.

3.3.2.7. Financing Policy 7: A Debt Issue while Paying ONLY Franked Dividends with a Binding Franking Account Constraint (Dividend Imputation System)

$$EROR\left(FL\right)_t^{j,7} =$$

$$\frac{R_{t+1}^j}{q_t \Phi_{t+1}^j}\left(1-q_{t+1}^j\right)\left[1-\left(1-g_{t+1}^j\right)t_{t+1}^j\right]-\sum_{n=1}^{N} ISH_t^{n,j}\left\{\begin{array}{l}\left[1-\left(1-g_t^j\right)t_t^j\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]PVDS_t^{j,7} \\ -\dfrac{q_{t+1}^n\left[1-\left(1-g_{t+1}^j\right)t_{t+1}^j\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]PVDS_{t+1}^{j,7}\left(1-d_t^{n,j}\right)}{q_t^n \Phi_{t+1}^j}\end{array}\right\} \tag{122}$$

where

$$PVDS_t^{j,7} = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^j\right)\left(1-q_{t+s}^j\right)\left\{\frac{r_{t+z}^j+\left[1-\left(1-g_{t+s}^j\right)t_{t+s}^j\right]r_{t+1}^j\left(1-b\,FLV_t^j\right)}{\Delta NBF_t^j\prod_{k=1}^{s}\Phi_{t+k}^j}\right\}\right\} \tag{123}$$

and

$$\Delta NBF_t^j = 1-\frac{\left(\ln B_t^j + \mathrm{X}_t^j - 1\right)}{\left(\ln B_t^j + \mathrm{X}_t^j\right)^2}. \tag{124}$$

With constant values for q_t^j, b_t^j, t_t^j, c_t^j and i_t^j,

$$PVDS_t^{j,7} = \left\{\frac{\Phi_t^j\left(1-q_t^j\right)}{\Phi_t^j - 1 + r_t^j}\right\}\left[\frac{r_t^j+\left[1-\left(1-g_t^j\right)t_t^j\right]r_{t+1}^j\left(1-b\,FLV_t^j\right)}{\Delta NBF_t^j}\right] \tag{125}$$

3.3.2.8. Financing Policy 8: A debt Issue with NO Dividends Payments and a Slack Franking Account Constraint (Classical or Dividend Imputation System)

$$\begin{aligned} EROR\left(FL\right)_t^{j,8} &= \frac{R_{t+1}^j}{q_t \Phi_{t+1}^j}\left(1-c_{t+1}^j\right)\left(1-t_{t+1}^j\right) \\ &-\sum_{n=1}^{N} ISH_t^{n,j}\left\{\left[1-t_t^j\left(\Gamma_t^{n,j}+\Xi_t^{n,j}\right)\right]PVDS_t^{j,8}-\frac{q_{t+1}^n\left[1-t_{t+1}^j\left(\Gamma_{t+1}^{n,j}+\Xi_{t+1}^{n,j}\right)\right]PVDS_{t+1}^{j,8}\left(1-d_t^{n,j}\right)}{q_t^n \Phi_{t+1}^j}\right\} \end{aligned} \tag{126}$$

where
$$PVDS_t^{j,8} = \sum_{s=1}^{\infty}\left\{\prod_{z=1}^{s}\left(1-r_{t+z}^{j}\right)\left(1-c_{t+s}^{j}\right)\left\{\frac{r_{t+s}^{j}+\left(1-t_{t+s}^{j}\right)r_{t+1}^{j}\left(1-b\,FLV_t^{j}\right)}{\Delta NBF_t^{\,j}\prod_{k=1}^{s}\Phi_{t+k}^{j}}\right\}\right\} \tag{127}$$

and
$$\Delta NBF_t^{\,j} = 1-\frac{\left(\ln B_t^j + \mathrm{X}_t^j - 1\right)}{\left(\ln B_t^j + \mathrm{X}_t^j\right)^2}. \tag{128}$$

Finally, with constant values of q_t^j, b_t^j, t_t^j, c_t^j and i_t^j,

$$PVDS_t^{j,8} = \left\{\frac{\Phi_t^j\left(1-c_t^j\right)}{\Phi_t^j - 1 + r_t^j}\right\}\left[\frac{r_t^j+\left(1-t_t^j\right)r_{t+1}^j\left(1-b\,FLV_t^j\right)}{\Delta NBF_t^{\,j}}\right]. \tag{129}$$

3.3.3. Expected Rates of Return with Recursive/Static Expectations

Expected rates-of-return under recursive/static expectations are denoted by $EROR(SE)_t^{j,f}$. In recursive/static mode, agents in MONASH expect that capital rentals and asset prices will grow over time at the rate of inflation, and that tax rates and tax allowances are constant. Thus, they expect that

$$q_{t+1}^{n,j} = \left(1+p_t\right)q_t^{n,j} \tag{130}$$

and

$$R_{t+1}^{n,j} = \left(1+p_t\right)R_t^{n,j}. \tag{131}$$

All the tax rates and allowances in these expressions are set constant, and the investor's expectation of future asset prices and capital rentals is imposed according to (130) and (131).

Likewise, if their expectations are truly static, investors will assume that the current rates of interest and inflation will continue unchanged. Under static expectations, the coefficient Φ_t^j is applicable to all future periods, i.e. so that we could replace a term like

$$\prod_{s=1}^{t}\Phi_s^j = \prod_{s=1}^{t}\left[\frac{1+i_s\left(1-V_s^j\right)-c_s^j\left[1+a_s p_s\right]}{1-c_s^j}\right]$$

with

$$\left[\Phi_1^j\right]^t = \left[\frac{1+i_1\left(1-V_1^j\right)-c_1^j\left[1+a_1 p_1\right]}{1-c_1^j}\right]^t.$$

Implicit in this approach is that investors predict the future path of the rate of return on capital based on information about the past and present. With these assumptions in place, the model cannot analyse the results of changes in future variables on the investor's current behaviour. In other words, investors are always "surprised" by changes in taxation policy, and have no ability to "prepare" for policy shocks. An advantage of using static expectations is that it highlights the role

of the *levels* of tax and allowance rates, rather than the interplay between their current values and future values when changes are expected. Other than for that, the main benefit of using backward-looking-expectations versions of the rate-of-return equations is their relative simplicity and transparency. Deriving these expressions from the forward-looking versions above is trivial and not stated here.

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