



***Calibration of a Land Cover Supply Function Using Transition
Probabilities***

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1. Introduction

An important question that arises frequently in the economic analysis of environmental and energy policies is, how does the supply of land across various uses change in response to policies? The GTAP modeling framework of Hertel (1997) addresses this question by determining the supply of land across different uses through a Constant Elasticity of Transformation (CET) supply function. In the standard GTAP model, the only type of land explicitly modeled is agricultural land, and this is distributed across uses with a one-level, CET function. However, in the GTAP-AEZ framework, data are available on a wider range of activities, including forestry (Sohnen et al., forthcoming). Given the recent availability of data on land cover (Ramankutty et al., 2007) and harvested cropland (Monfreda et al., forthcoming) the land supply decision is more naturally divided into the allocation of land cover across forestry, grazing and crops, followed by the allocation of harvested area across cropping activities.

In this Research Memorandum, we focus on the former problem, namely the allocation of land cover between these three competing commercial uses. Naturally, the quality of the land cover responses produced by GTAP-based simulations is contingent on the value of the CET parameter. And the value of this parameter is likely to depend on the length of run for the analysis in question. This research memorandum describes the empirically based calibration strategy used to determine the value for the CET parameter, based on recent research in the United States.

The following section reviews the theory of the CET parameter and how it is relevant for modeling land supply. Sections 3 and 4 describe the calibration methodology and data, while the final section discusses the calibration results.

2. CET Supply Function

Before getting into how the CET parameter is determined, it is worth understanding what needs to go into a land supply function and the role of the CET functional form here. In a production context, we need a supply function that allows us determine multiple outputs from a set of inputs, or a single input. This latter case is the case of land supply, where there is a single input that supplies land specific to different uses. In the GTAP-BIO model, the multiple outputs that are

produced would be the three types of land cover – forest, pasture, and cropland. Represented algebraically, this function would be as follows:

$$Q_X = f(Q_{Y_i}) \quad (1)$$

Q_X is the quantity of a single input x (total land), while Q_{Y_i} refers to the vector of land used in a specific use i ($i: \{1, 2, \dots, N\}$). We conceptualize the land supply by use problem as one of revenue maximization, in which the owner allocates land to different uses in order to maximize the total value of rents. For this to be guaranteed, the input requirement function f must be strictly convex in Y_i . The CET form adopted in equation 2, gives us the necessary convexity (Powell and Gruen, 1968).

$$Q_X = \alpha \left[\sum_i \beta_i Q_{Y_i}^\rho \right]^{1/\rho} \quad (2)$$

The exponent ρ is used to determine the elasticity of transformation, and they are algebraically related through equation 3:

$$\sigma = \frac{1}{1-\rho} < 0 \quad (3)$$

A property of the CET form is that ρ must be strictly greater than one, which implies that the transformation elasticity itself must be negative¹.

3. Methodology

The key concept in the methodology used is the own return elasticities of land quantity, for each use ($\varepsilon_{j,t+5}$). This is the estimated own return elasticity of land use j , determined as the percentage difference of the estimated perturbed land use quantity and the benchmark land use quantity for a given five year period.

However, two factors must be considered when studying land use changes in response to a policy. The first is that land uses respond to the market. If the net return per unit of land in a given use – i.e. the profitability of that use – increases, then the quantity of land supplied for that use will increase. The second factor to consider is that there are secular transitions in land use over time, which are independent of changes in the profitability of a particular land use.

The method used for the estimation of own return elasticities of land quantity takes into account the two factors described above. The first is the matrix of land use transition probabilities² for a

¹ For more details on the CET, please see Shumway and Powell (1984).

² This is the probability that a unit of land will remain in its current use or move to an alternative use.

five year period and the second is the set of own return elasticities of transition probability. Following Lubowski (2002) these two sets of data can then be used to simulate land use transitions over a baseline time path where there are no changes to use returns, and then a time path of land use transitions where the transition probabilities in each period changed due to a one percent increase in returns to each use. With a time series of land use acreages in a base case, and a time series of land use acreages with the own returns to a use increasing by a percent in each period, the percent difference between land quantities by use in each path will yield the requisite own return elasticities of land quantity.

Now the relationship between the own return elasticities of land use the CET must be determined. Let us return to the changes in land quantities and prices. The revenue generated by aggregate land endowment (R_X) is the product of an average land price per unit and total land quantity. R_X must be equal to the sum of the revenues generated by the individual uses of land (R_{Yi}). These revenues are in turn products of the prices commanded by land uses j (P_j) and the land quantities (Q_{Yj}). The revenue share for a given use, θ_j , is now $P_j Q_{Yj}/P_X Q_X$, where Q_X is total land supply.

Now, the compensated (input constant) price elasticities (ε_{ij}) can be written in terms of revenue shares and the CET elasticity, such that:

$$\varepsilon_{ij} = \theta_j * \sigma \quad (4)$$

Homogeneity of supply implies that:

$$\varepsilon_{ii} = - \sum_{j \neq i} \varepsilon_{ij} \quad (5)$$

From equations 4 and 5, it now possible to determine a relationship between the own price elasticity and the σ :

$$\begin{aligned} \varepsilon_{ii} &= - \sum_{j \neq i} \varepsilon_{ij} \\ &= - \sum_{j \neq i} (\theta_j * \sigma) \\ &= - \sigma \sum_{j \neq i} \theta_j \\ &= - \sigma (1 - \theta_i) \\ \Rightarrow \sigma &= - \frac{\varepsilon_{ii}}{(1 - \theta_i)} \end{aligned} \quad (6)$$

Equation 3 thus provides an estimate of the CET parameter value associated with use j in the five year period considered. The revenue share weighted average of the CET values for each use gives us the average CET value (Equation 7) over all the uses.

$$\bar{\sigma}_{t+5} = - \sum_i \theta_i * \left(\frac{\varepsilon_{ii,t+5}}{1 - \theta_i} \right) \quad (7)$$

4. Data

There are two sets of data for the calibration. The first set is the revenue shares are from the GTAP Land Use Database by Lee, Hertel, Rose, and Avetisyan (*forthcoming*), and are 0.7489, 0.0975, and 0.1023 for the revenue shares of cropland, pasture, and forest in the United States, respectively. The second set is the baseline and perturbed time paths of land quantities by use over a 100 year period. As mentioned earlier, these projections are based on matrices of land use transition probabilities and own return elasticities of transition probabilities. The transition probabilities matrix is from Lubowski (2002), while the own return elasticities of probabilities are estimated following a methodology similar to Lubowski et al (2006), but with a critical innovation.

The methodological innovation of Lubowski (2002) is that instead of using national level data for the USA, the estimates are based on plot level data from USDA (2000). This approach tracks the amount of land of each quality in each use at each point in time, and ultimately an elasticity estimate that is more appropriate for a longer time frame. Thus, at each five-year step, the overall amount of land transitioning between all the uses changes based on the new amounts of land of different qualities allocated to each of the different uses, in both the baseline time path, as well as the time path where own returns to use are perturbed by one percent in every period. Figure 1 illustrates the differences in land use acreages over time, between the baseline path and the path where own-returns for each use are perturbed by one percent.

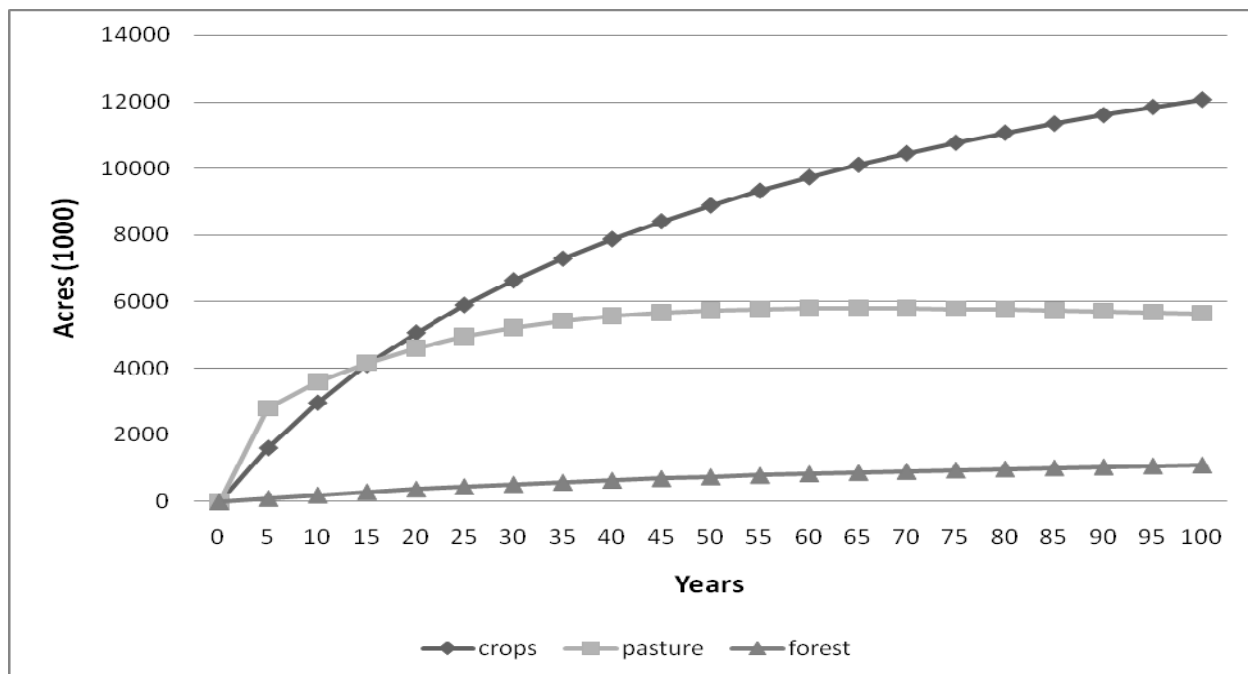


Figure 1: Estimated Land Use Quantities at t : Difference Between Perturbed and Baseline Paths

Source: Authors' Simulations

These land use quantity time paths are determined for crops, forest, and pasture, which are the same three categories of land cover in the GTAP Land Use Database. The average CET parameter in a given period is thus based on elasticities calculated from projected changes in acreages by use, followed by taking their revenue share weighted averaged across the three land cover uses in the GTAP land data.

5. Calibration Results

As was seen in Figure 1, the land use response to increases in own-returns rises throughout the timeframe. Accordingly, the own-return elasticities also rise over this period (Figure 2), while the calibrated CET parameter falls (Figure 3). The revenue share-weighted CET parameter decreases by at a decreasing rate every five-year period, indicative of an asymptotic relationship in the long run. However, in the short to medium run, this can be approximated by a linear relationship. Figure 4 illustrates how the estimated CET values compare to the CET values that would have been produced by a linear interpolation using the five-year, ten-year, and fifteen-year CET values generated by the calibration method. We can use this linear interpolation to estimate the average CET parameter associated with any desired length of run. This parameter is used in the GTAP model to govern the mobility of land between cropland cover, pastureland and accessible forest. In the US, the latter category is assumed to correspond to the privately held land sampled in the National Resources Inventory utilized by Lubowski (2002) in his econometric estimation.

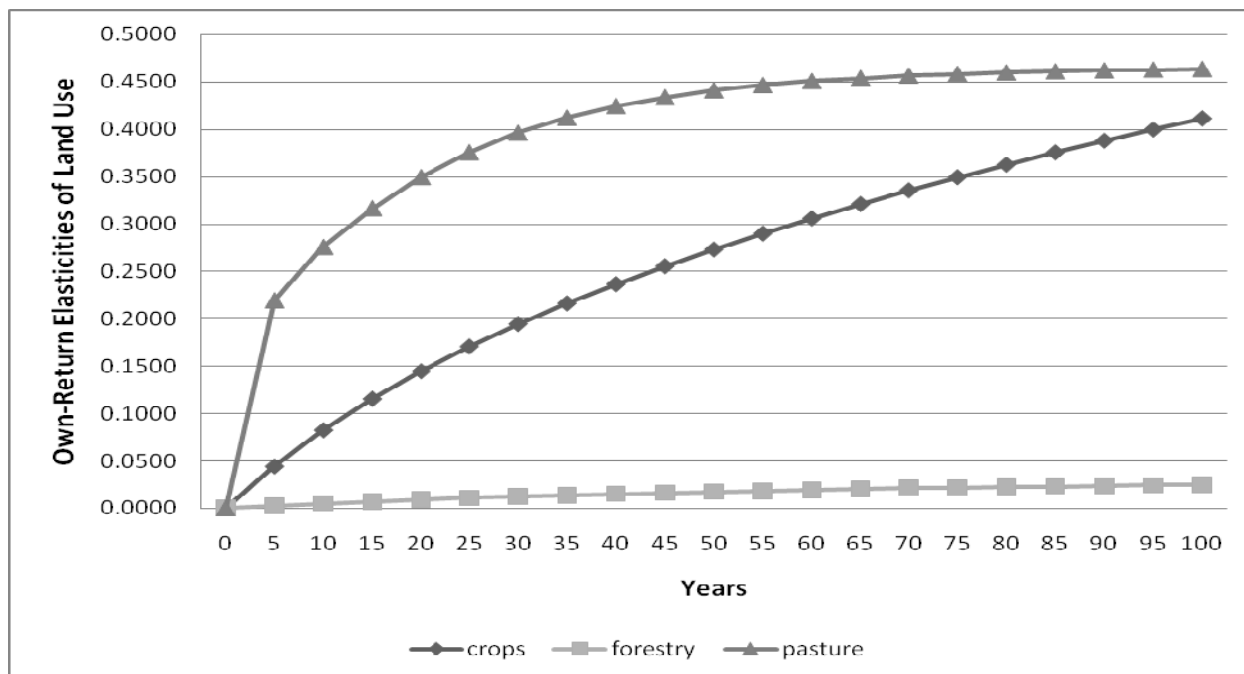


Figure 2: Own-Return Elasticities of Land Use at t for Use i
Source: Authors' Simulations

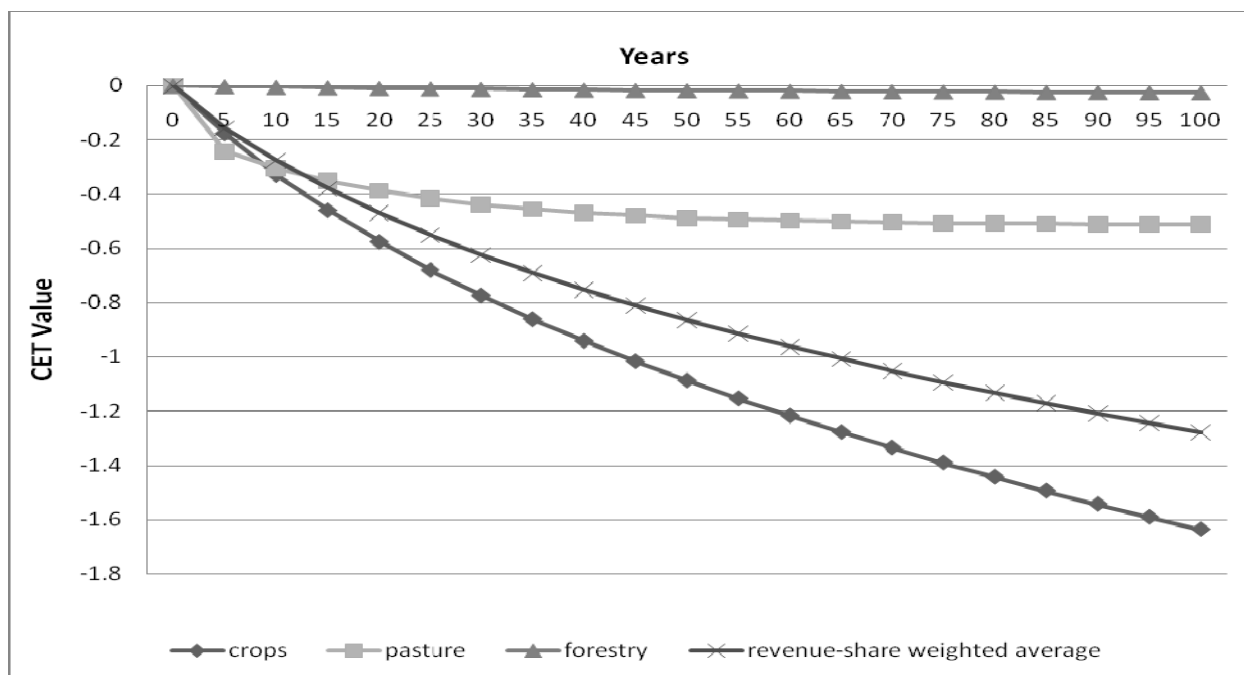


Figure 3: CET Calibration Estimates by Land Use at time t , for $t=5$ to $t=100$
Source: Authors' Simulations

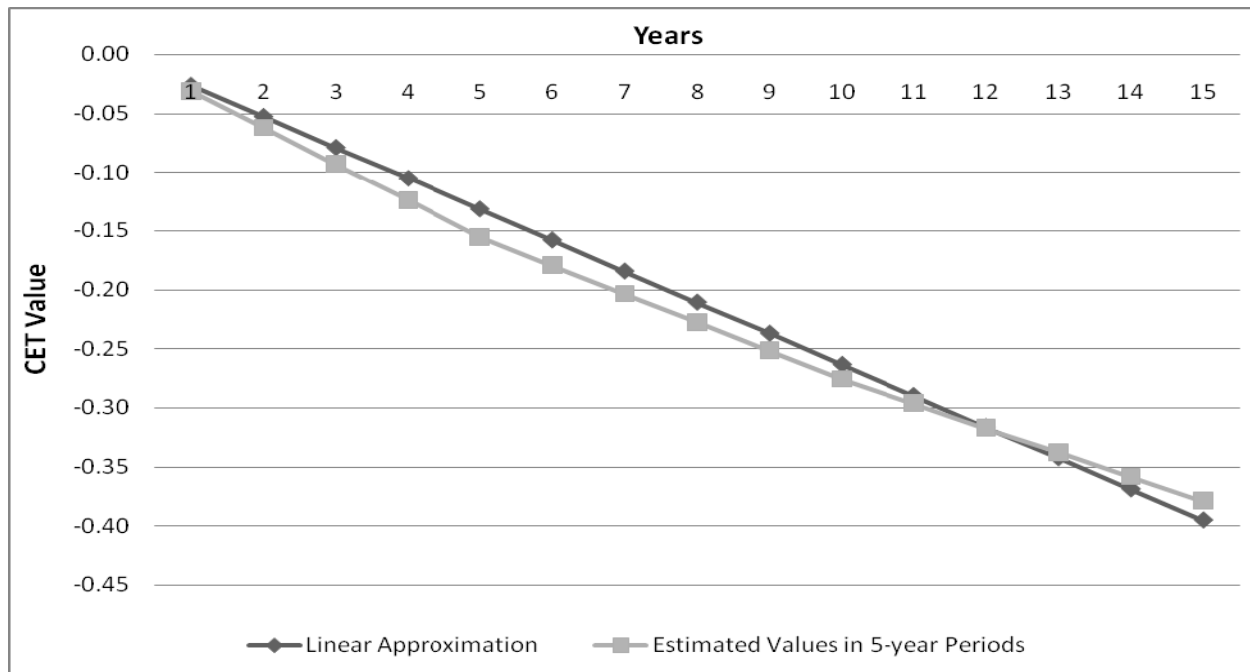


Figure 4: Comparison of Estimated CET Values with Linearly Interpolated Approximations of CET at time t
Source: Authors' Simulations

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