Elasticity of Supply in GTAP

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2013–05–20

In the GTAP 7 documentation (Hertel, Tsigas, and Narayanan [1]), a formula for the elasticity of supply is derived from two formulae,

$$\eta_S = -\sigma_{RR},$$

where η_S denotes the elasticity of supply and σ_{RR}, the Allen-Uzawa own-price elasticity of substitution, and

$$\sigma_{RR} = -\left(\sigma_{VA}\left[\theta_R^{-1} - \theta_{VA}^{-1}\right] + \sigma_T\left[\theta_{VA}^{-1} - \theta_O^{-1}\right]\right),$$

where σ_{VA} denotes the elasticity of substitution between components of value added, σ_T, the elasticity of substitution between value added and intermediates, θ_R, the share of natural resources in total input cost, θ_{VA}, the share of value added in total input cost, and θ_O, the share of all inputs together in total input cost, that is, one. The latter formula is a special case of the formula derived by Keller [2, Theorem 2] for an arbitrary tree of origin-shifted CES functions. In the GTAP standard data base, the elasticity σ_T is equal to zero, so the formula simplifies to

$$\sigma_{RR} = -\sigma_{VA}\left(\theta_R^{-1} - \theta_{VA}^{-1}\right).$$

combining the two formulae, we have

$$\eta_S = \sigma_{VA}\left(\theta_R^{-1} - \theta_{VA}^{-1}\right).$$

Alternatively, we have a direct derivation of the elasticity of supply, for the particular production tree used in GTAP, in which, with standard parameter settings, output is a Leontief aggregate of intermediate inputs and value added, and value added is a CES aggregate of primary factors; with the additional assumption that one primary factor, natural resources, is fixed, and all other inputs are mobile. Let P_i and Q_i denote the price and quantity of i. Let p_i and q_i denote the corresponding relative differentials,

$$p_i = \frac{d\,P_i}{P_i}, \quad q_i = \frac{d\,Q_i}{Q_i}.$$

Let S_{ij} the share of j in the cost of i,

$$S_{ij} = \frac{P_j Q_j}{P_i Q_i},$$

where i and j may take the values R for natural resources, F for primary factors, and O for output. For some industry in some region, let η_S denote the elasticity of supply, and σ_F the elasticity of

substitution between primary factors. Assume that prices of inputs other than natural resources are constant. Then, from the GTAP equation system, we extract the subsystem

$$\begin{aligned} q_R &= q_F - \sigma_F(p_R - p_F), \\ p_F &= S_{FR} p_R, \\ q_F &= q_O, \\ p_O &= S_{OR} p_R, \end{aligned}$$

in which p_O and q_R are given, and from which p_R, p_F, q_F, and q_O may be found. Solving, we obtain the supply equation

$$q_O = q_R + \sigma_F \frac{1 - S_{FR}}{S_{OR}} p_O.$$

Then the elasticity of supply is the coefficient of p_O in that equation, that is,

$$\eta_S = \sigma_F \frac{1 - S_{FR}}{S_{OR}}.$$

Relating notation, σ_F in the second derivation matches σ_{VA} in the first, S_{OR} in the second matches θ_R in the first, and S_{FR} matches θ_R/θ_{VA} in the first. So the two derivations agree.

References

[1] Thomas Hertel, Marinos Tsigas, and Badri Narayanan G. *Global Trade, Assistance, and Protection: The GTAP 7 Data Base*, chapter 12.A. Purdue University, 2004.

[2] Wouter J. Keller. A nested CES-type utility function and its demand and price-index functions. *European Economic Review*, 7:175–186, 1976.