

Chapter 12.A

Primary Factor Shares

***Thomas Hertel, Marinos Tsigas and
Badri Narayanan G.***

12.A.1 Overview and Motivation

In versions 1 - 3 of the GTAP Data Base¹, there was only one type of natural resource endowment identified, namely agricultural land. Because this factor was only used in agricultural production, it was also, by default, a sector-specific factor. The presence of a specific, or fixed, factor serves to dampen the supply response in the sector in question. When demand increases, part of that increase is translated into increased land rents. Indeed, the only way to increase agricultural output is to substitute other factors of production for land. This type of situation stands in sharp contrast to some manufacturing activities, say the production of wearing apparel, where supply can be increased indefinitely — provided the demand is there — by simply building new factories and hiring more workers. The only thing that forces the supply curve for such a sector to slope upward is the fact that as additional workers are bid away from other activities, wages will rise. This is a general equilibrium supply constraint.

Of course the presence of non-producible natural resource inputs arises in other sectors as well. Coal, oil, natural gas, minerals, fisheries and forestry are all examples of sectors where partial equilibrium supply response is constrained by the underlying natural resource input. As the range of GTAP applications has broadened, the supply response of these sectors has been called into question. In recent work conducting projections with GTAP, we have introduced specific factors in these sectors in order to restrain the responsiveness of supply to increases in the economy-wide capital stock (Dimaranan, 1996). Others conducting simulations of trade liberalization in resource-abundant economies have commented about the very large increases in production and export of extractive products following trade liberalization. It became clear that steps needed to be taken here as well.

Having identified the need to constrain supply response in the other natural resource sectors, it remains to determine how the relevant cost share was to be obtained. In the case of agriculture, a wide range of studies exist which estimate these cost shares — often as part of an econometric study of the sectoral cost function. However, such studies are not common in other natural resource sectors. Thus we decided to take a more indirect approach to the problem. We

¹Editor's Note: This chapter documents changes in the treatment of primary factor shares in agricultural and natural resource- based industries that were implemented in the GTAP 4 Data Base (McDougall, et al., 1998). We continue to use the same assumptions in the GTAP 8 Data Base.

begin with the desired outcome, i.e. what is the target, partial equilibrium supply elasticity? We then compute the natural resource cost share which is implied by that target elasticity.

In partial equilibrium, with fixed prices for all mobile factors of production (non-natural resource inputs) and a fixed quantity of the natural resource input, supply response equals the negative of the own-Allen-Uzawa elasticities of substitution (AUES) for the natural resource input, so that:

$$(1) \quad q = -\eta_s p$$

where: q and p stand for the percentage change in quantity and price of the sectoral output respectively, and η_s , the supply elasticity, is equal to the negative of the AUES:

$$(2) \quad \eta_s = -\sigma_{RR}$$

The sectoral supply response is simply determined by the industry's ability to substitute away from the fixed factor. If $\sigma_{RR} = 0$, then no amount of price increase will be able to induce a supply response. But how does this Allen elasticity relate to the parameters in GTAP? This relationship is a function of the cost shares and elasticities of substitution in the nested CES production function. Specifically:

$$(3) \quad \sigma_{RR} = -(\sigma_{VA}[\theta_R^{-1} - \theta_{VA}^{-1}] + \sigma_T[\theta_{VA}^{-1} - \theta_O^{-1}])$$

where σ_{VA} corresponds to the elasticity of substitution among components of value-added and σ_T corresponds to the elasticity of substitution between value-added and intermediates. The latter is zero in the standard GTAP Model, so that the second term in parentheses falls out. The parameter θ_R corresponds to the cost share of the natural resource input. θ_{VA} refers to the cost share of value added, and the last parameter, θ_O , is the cost share of all inputs — or output — which is simply equal to one. This is a special case of the formula derived by Keller (1976, Theorem 2) for an arbitrary tree of origin-shifted CES functions.¹

Assuming σ_{VA} and σ_T are given by the GTAP Data Base, we can solve equations (2) and (3) for the natural resource share implied by a given supply elasticity. This is the general approach taken here. Of course, it may be that the required cost share exceeds the share of capital in the original data base — or even the share of all value-added! In order to eliminate such outcomes, we have taken two steps. First of all, rather than just taking the resource share out of capital (which is highly volatile and may even be negative in a bad year), we take it out of all value-added. Secondly, the elasticity of substitution in value-added is reduced from its value in the GTAP 3 Data Base. In the GTAP 4 Data Base we have set this value equal to 0.2. When implemented, this procedure results in less than one-third of value-added going to the natural resource input in most cases. Of course there are still a few instances where a very low share of value-added in the source data results in a much higher share of value-added being assigned to natural resource inputs. Future

¹ Refer to the appendix of this chapter.

refinements of this approach to calibrating supply response should seek to take into account regional variation in the targeted elasticity.

12.A.2 *Non-Agricultural, Natural Resource-Based Sectors*

Forestry: The first question which arises when one turns to specific sectors is: how large is the partial equilibrium supply response? In the case of forestry, we draw on econometric estimates as reported in column one of table 1 in Catimel (1996). The estimated supply elasticities are for the Canadian softwood timber industry and they range from 1.6 to 5.8, depending on the region of Canada and the nature of the product supply (e.g., logs vs. pulpwood). Discarding the high and low estimates and averaging the remaining elasticities gives a mean supply response of 2.5. This is the targeted elasticity in version 4 of GTAP, and it is imposed in every region of the data base. Therefore, equations (2) and (3) are solved for a cost share of natural resources which satisfies them in each region.

Fisheries: The case of fisheries is more difficult due to the fact that the response of supply to price varies depending on the management regime in place. Campbell (1996) explores this issue in considerable detail for the case of tuna. In the case of an “open access” fishery, anyone seeking to come and harvest the stock of fish can do so. Open access leads to essentially an infinite discount rate, over-fishing, and hence a very low supply response to price (0.29). On the other hand, some fisheries are managed by an industry group. When such a group operates as a single, profit-maximizing entity, supply is more responsive to price. In this type of “private management” regime, reductions in the discount rate will increase the responsiveness of supply. In the case of a zero discount rate and private management, Campbell computes a supply elasticity for tuna of 0.76. These represent the two extremes for the tuna industry. For purposes of GTAP 4, we average the two and obtain an elasticity of supply equal to 0.525. Someone studying a question in which fisheries supply response is a crucial parameter could specify a distribution of supply elasticities, based on the Campbell study (e.g., triangular distribution with minimum value 0.29 and maximum value 0.72), thereby generating a distribution of possible outcomes (see Arndt and Pearson, 1996, for more details on how to conduct such systematic sensitivity analysis in GTAP).

Fossil fuels and mining: Whalley and Wigle (1991) spend considerable time studying the supply response of the fossil fuels sector in their attempt to assess the implications of taxation designed to reduce global carbon dioxide emissions. Because of the importance of this parameter to their study, they conduct a systematic sensitivity analysis for which they set the minimum value to 0.1 and the maximum to 1.5. Their central case is 0.5. This is the value which we choose as a target for version 4 of the GTAP Data Base.

Unfortunately, a similar study was not available for the other mining sectors. However, in the case of many minerals, extensive deposits exist around the world and the owners are ready to bring some of these mines back on line (or temporarily shut some down) when prices rise (or fall). This suggests a higher supply response. We have adopted the value of 2.5 here, based on experience in Australia (Robert McDougall, personal communication).

12.A.3 *Agriculture*

Having gone to the trouble to target supply response in the non-agricultural, natural resource-based sectors, we found ourselves asking whether this might not also be a good idea in the case of agriculture. Of course the situation is somewhat different here, since we have additional information in the form of independent estimates of primary factor shares obtained from a survey of the literature (see table 12.A.1). Since these are deemed more reliable than the estimated elasticities of substitution, we adjust the latter in order to achieve the targeted supply response. Furthermore, we only target the global supply elasticity for agriculture as a whole.

What degree of supply response is appropriate for agriculture? Despite the extensive literature on this topic, the agricultural economics profession is still far from agreement. Some argue that supply response is very low, even in the long run, while others argue that once full adjustment of labor and capital has occurred, the scope for sizable responses to price changes is considerable. Given the propensity of governments to intervene in agricultural markets, the degree of supply response is much more than an academic issue. Sizable long run supply elasticities translate into high costs for government price support schemes and distorted world markets. In this study, we target a global supply response for agriculture of 1.19, based on Peterson's (1988) cross-section, international study. This is larger than what some of the supply pessimists might like, but smaller than what the supply optimists would argue for. When applied to equations (2) and (3), using average shares for world agriculture, this yields a value of $\sigma_{VA} = 0.23$. This is substituted in the parameter file for the previous value which was larger (0.56 in versions 1 - 3). This smaller elasticity of substitution in production results in a smaller agricultural supply response in models based on the GTAP 4 Data Base.

There are two areas of agricultural supply response where broad agreement exists. We accommodate both of them in GTAP. First of all, supply response is relatively smaller in less industrialized economies, where agriculture is less commercialized. This feature comes through in GTAP, via the overall share of value-added in costs. In Sub-Saharan Africa, where purchased inputs are far less important, the share of value-added, and hence land, is much higher than in the United States. Consider the case of rice production. The global cost share of land in this sector is about 22 percent. However, in the US and Australia, this figure is only 10 percent, whereas in India it is 28 percent and the Indonesian rice sector has a cost share of land equal to 43 percent. Based on equations (1) - (3), supply response in the US and Australia will be much higher than in India and Indonesia, given a common elasticity of substitution in production.

A second point of general agreement is that supply response at the level of individual crops is much higher than for agriculture as a whole. This makes sense, since agricultural land (and farmers as well) can shift from one crop to another much more readily than they can shift into another sector, such as wearing apparel. Since agricultural land is sector specific in the GTAP Model, those who choose to disaggregate agricultural subsectors will find that this proposition also holds. Namely, the supply response of individual agricultural activities is larger than for the sector as a whole. Land is no longer a fixed factor for individual crops as it may be bid away from competing crop or grazing activities. The degree of land mobility across subsectors is determined by the elasticity of transformation (ETRAE in GTAP notation), as discussed in the GTAP book (Hertel, 1997).

The shares of primary factors in agricultural value-added for the 87 regions of the GTAP 7 Data Base given in table 12.A.3 were generated from the factor splits obtained from the literature

and summarized in table 12.A.1. The factor shares data in table 12.A.1 were assigned to the 244 standard countries in the GTAP 8 Data Base (see chapter 3) by mapping the regions to the standard countries. The factor shares for the 129 GTAP regions in GTAP 8 were then obtained by aggregating the factor shares for the 244 countries using GDP weights.

There have been some changes made to this data in version 7 and carried forward in version 8, based on the recent literature that computes factor shares for agricultural sectors, summarized by Shinsuke Uchida, for a potential update in GTAP Data Base. The major concerns with the old figures are that they are obsolete and suffer from methodological limitations.

Table 1: Changes in Factor Shares

COUNTRY	Labor Share		Capital Share	
	GTAP 6 DB	GTAP 7 & 8 DB	GTAP 6 DB	GTAP 7 & 8 DB
Nepal	38	57	18	14
Canada	39	41	44	43
Peru	47	60	25	5
South Africa	40	34	45	52
Zimbabwe	60	41	25	28

Following are the justifications for these changes:

1. Nepal: Earlier figure was based on a study on India, while this is based on a 1996 study on Nepal published (Abdulai and Regmi, 2000); so, the update is better in terms of being recent, regional focus and methodology. Factor shares were estimated by the household utility maximization approach, which seeks to maximize two persons' (male and female) joint household production function with a budget constraint by choosing the consumption of home produced goods, market goods, allocation of time between market work, own-farm work, home production, and leisure, as well as the inputs of hired labor into own-farm production². This model has some advantages, compared to the separated production model from household, in the sense that non-separability between production and consumption decisions makes it possible to take non-waged family labors into account. In this model, wages of family labor can be recovered as a shadow value even without the family labor market. The fact that this is a cross-sectional survey type of analysis with only 280 household observations could render estimated factor shares less consistent.³ But another statistical problem – endogeneity – was dealt with by using instrument variables. This result can be more appropriate as a benchmark of the Nepalese agricultural factor shares than the alternative estimates in the GTAP 6 Data Base (i.e. the Indian factor shares shown

² See also Jacoby (1993) to learn more about this agricultural household model. He estimated factor shares in Peru as mentioned below.

³ Ideal samples must be panel nature, but it is hard to collect such data.

in Table 18.C.1 in Hertel and Tsigas, 2002), considering several agricultural heterogeneities between the countries. Also, labor shares would be more reliable with this methodology as family labor is accounted for. As a result, the Nepalese agriculture is shown to be relatively labor intensive in the 1996-97 season, in which family labor has a greater impact (28%).

2. Canada: The update is based on Echevarria (1997). These estimates are based on the TFP (Total Factor Productivity) procedure, estimating the cost shares of the primary inputs for Canadian agriculture in 1971 - 93. This author uses the value-added production function. It can be a restricted version of the profit maximization, since maximization of value-added depends only on the primary factors. Constant return to scale and Hicks-neutral technical change were assumed for the value-added function. Value-added in Canadian agriculture and values of land, labor and capital were used as output and inputs, respectively. Values of labor were carefully imputed by including unpaid workers to reduce biases. As a result, a weighted average of estimates at province levels is coincidentally equivalent to the one in GTAP 6 Data Base. This would be a firm evidence of the trend of the primary factor shares in the Canadian agriculture until the 1990s. Our earlier estimate was based on an econometric exercise for a single year.
3. South Africa: Factor shares in South African agricultural value-added are obtained by TFP measurement process, which is based on the profit maximization assumption. The factor shares are derived as the cost share of inputs from first derivatives of zero profit max condition with respect to time. Details of theory can be seen in Thirtle et. al. (2000). This approach does not require econometric process, so the endogeneity problem does not arise; instead, it requires data on changes in prices and quantities of outputs and inputs. Thus, reliability of this estimation would mainly depend on accuracy of data on periodic changes⁴. Their data on labor account for only full-time employment, which would result in downward bias to the labor share. However, it seems still a better estimate than the figure in GTAP 6 Data Base (Masters, 1994), since the previous one has the same deficit in addition to an unclear estimation procedure with only single year data. With national aggregate data in 1947-91, Thirtle et. al. (2000) conclude that South African agriculture would be relatively capital intensive (52%).
4. Zimbabwe: The methodology adopted by Thirtle et. al. (1993) is analogous to the one for South Africa except that the Cobb Douglas (C-D) production function was replaced with the translog function, so it is less restricted. Average factor shares during 1970-89 were estimated as the cost shares of inputs. The labor share (41%) is much lower than that measured from Masters (1994) in the GTAP (60%), but there is no clear distinction between these studies; both data were collected from the national government sources. It can be said that such difference in shares is due to conducted methodologies, since the propensity of the results - higher labor share for the GTAP measurement and higher capital share for the TFP approach - is common to estimates for Zimbabwe, as well as South Africa. Hence, as in the case of the South African factor shares, the new estimates were incorporated for Zimbabwe in GTAP 7 Data Base.

⁴ Note that this implies that inputs with bigger changes are more likely to have higher factor shares. This could yield a problem when changes in each input are very different.

5. Peru: This is based on a Household study on Peru published in Jacoby (1993). The methodology is similar to that adopted by Abdulai and Regumi (2002)⁵. Jacoby extracted 1034 household observations from the Peruvian Living Standards Survey administered by the World Bank during the 1985-86 period. All of them are farmers in the Sierra area, highlands region. They are not national representative, since it is a sampled survey and also Peru has two other ecologically different lands, the eastern jungle and coastal areas. However, it can be used as a benchmark because Peruvian agriculture is based on the subsistence farmers, most of which are concentrated in the Sierra. Very low capital share is justified by the dominance of labor-intensive subsistence farming, requiring a major labor share.

12.A.4 *Summary*

In summary, these changes substantially affect the performance of the natural resource-based sectors in applied GE models drawing on the GTAP 7 and 8 Data Base releases. With less elastic supply, we expect larger price changes and smaller quantity changes for natural resource based products. These changes can also have macro-economic implications in economies which are heavily dependent on natural resources. Consider for example, the adjustment of an oil-exporting economy to a cut in its import tariffs. In order to pay for the increased imports, more oil must be exported. However, if supply of that product is less responsive, then the process of reestablishing external balance will be quite different. We will expect more of the adjustment to occur in the form of higher resource rents, and less in the form of higher exports and depressed product prices.

⁵ One distinction from the other listed studies is that hours of work were used as labor variables.

Table 12.A.1 Shares of Primary Factors in Agricultural Value-Added (Survey of Literature)

Region	Land	Labor	Capital	Reference Document
Australia	0.21	0.51	0.28	ABS I-O table for labor, Ball (1996) for others
New Zealand	0.14	0.62	0.24	NZIER I-O table
Japan	0.18	0.51	0.31	Kuroda (1995) for labor, Ball (1996) for others
Korea	0.51	0.40	0.09	Ban (1996)
Philippines	0.41	0.55	0.04	Crisostomo and Barker (1979)
Southeast Asia	0.51	0.42	0.07	OECD (1993) - GREEN Model
China	0.29	0.59	0.12	Martin (1993)
Taiwan	0.38	0.54	0.08	Lee and Chen (1979)
India	0.44	0.38	0.18	Chadha, et. al. (1997)
Nepal	0.29	0.57	0.14	Abdulai and Regmi (2000)
Canada	0.17	0.39	0.44	Echevarria (1997)
United States	0.28	0.38	0.34	Ball (1998) for labor, Ball (1996) for others
Brazil	0.16	0.24	0.60	Brandao et. al. (1994)
Latin America	0.35	0.60	0.05	Jacoby (1993)
United Kingdom	0.14	0.68	0.18	Heinrichsmeyer et. al. (1998) for labor, Ball (1996) for others
Germany	0.15	0.68	0.17	Heinrichsmeyer et. al. (1988) for labor, Ball (1996) for others
Denmark	0.13	0.40	0.47	DIAFE
European Union	0.11	0.68	0.21	Heinrichsmeyer et. al. (1988) for labor, Ball (1996) for others
European FTA	0.27	0.35	0.38	OECD (1993) - GREEN Model
Central Europe	0.35	0.53	0.12	OECD (1993) - GREEN Model
Former Soviet Union	0.28	0.60	0.12	OECD (1993) - GREEN Model
Middle East	0.11	0.57	0.32	Haley (1991)
South Africa (CU)	0.14	0.34	0.52	Thirtle et. al (2000)
Southern Africa	0.31	0.41	0.28	Thirtle et. al (1993)
Sub-Saharan Africa	0.12	0.72	0.16	Haley (1991)

References

- Abdulai, A. and Regmi, P.P., (2000) "Estimating Labor Supply of Farm Households under Nonseparability: Empirical Evidence from Nepal", *Agricultural Economics*, 22: 309-20.
- Arndt, C. and K.R. Pearson. 1996. "Implementing Systematic Sensitivity Analysis Using GEMPACK," GTAP Technical Paper #3, Center for Global Trade Analysis, Purdue University: <http://www.agecon.purdue.edu/gtap/techpapr/index.htm>.
- Ball, V.E. 1988. "Modeling Supply Response in a Multiproduct Framework," *American Journal of Agricultural Economics*, 70(4):813-25.
- Ball, V.E. 1996. unpublished worksheets, personal communication, April 30.
- Ban, Sung Hwan. 1979. "Agricultural Growth in Korea, 1918-1971," Chapter 4 in Y. Hayami, V.W. Ruttan, and H.M. Southworth, *Agricultural Growth in Japan, Korea, and the Philippines*, East-West Center.
- Brandão, A.S.P., T.W. Hertel, A. Campos. 1994. "The Implications of International Agricultural Trade Liberalization for Brazilian Agriculture," in I. Goldin, O. Knudsen, and A.S. Brandao, (eds.), *Modeling Economy-Wide Reforms Development Centre of the Organization for Economic Co-Operation and Development*. Paris: Chapter 4, pp 193-222.
- Campbell, H.F. 1996. "Prospects for an International Tuna Resource Owner's Cartel," *Marine Policy*, vol. 20, no.5, pp. 419-427.
- Catimel, Jerome. 1996. "Econometric Estimation of Canadian Softwood Timber Supply Curves: An Exploratory Analysis," Research Note No. 41, Canadian Forest Service, April.
- Chadha, Rajesh, Sanjib Pohit, Alan V. Deardorff and Robert M. Stern. 1998. *The Impact of Trade and Domestic Policy Reforms in India: A CGE Modeling Approach*, The University of Michigan Press, Ann Arbor.
- Crisostomo, C. and R. Barker. 1979. "Agricultural Growth in the Philippines, 1948-1971," Chapter 5 in Y. Hayami, V.W. Ruttan, and H.M. Southworth, *Agricultural Growth in Japan, Korea, and the Philippines*, East-West Center.
- Dimaranan, B. 1996. "Growth Projection Using GTAP," presentation at the Advanced Short Course in Global Trade Analysis, Purdue University, 2-8 June.
- Echevarria, C. (1997) "A Three-Factor Agricultural Production Function: The Case of Canada", *International Economic Journal*, 12(3): 63-75.
- Haley, S.L. 1991. "Capital Accumulation and the Growth of Aggregate Agricultural Production," *Agricultural Economics*, 6:129-157.

- Henrichsmeyer, W. and A. Ostermeyer-Schlöder. 1988. "Productivity Growth and Factor Adjustment in EC Agriculture," *European Review of Agricultural Economics*, 15:137-54.
- Hertel, T.W. (Editor). 1997. *Global Trade Analysis: Modeling and Applications*, Cambridge University Press: New York, NY.
- Hertel, T. and Tsigas, M., (2002) "Primary Factor Shares", Section 18C in Dimaranan, B.V. and McDougall, R.A., (2002) *Global Trade, Assistance, and Production: The GTAP 5 Data Base*, Center for Global Trade Analysis, Purdue University.
- Higgs, P. J. 1986. *Adaptation and Survival in Australian Agriculture*, Oxford University Press.
- Jacoby, H.G., (1993) "Shadow Wages and Peasant Family Labor Supply: An Econometric Application to the Peruvian Sierra", *The Review of Economic Studies*, 60(4): 903-21.
- Wouter J. Keller. A nested CES-type utility function and its demand and price-index functions. *European Economic Review*, 7:175–186, 1976.
- Kuroda, Y. 1995. "Labor Productivity Measurement in Japanese Agriculture, 1956-90," *Agricultural Economics*, 12:55-68.
- Lee, Teng-hui and Yueh-eh Chen. 1979. "Agricultural Growth in Taiwan, 1911-1972," Chapter 3 in Y. Hayami, V.W. Ruttan, and H.M. Southworth, *Agricultural Growth in Japan, Korea, and the Philippines*, East-West Center.
- Martin, W. 1993. "Modeling the Post-Reform Chinese Economy," *Journal of Policy Modeling* 15(5&6):545-579.
- Masters, W. 1994. *Government and Agriculture in Zimbabwe*. Westport: Praeger.
- McDougall, R.A., A. Elbehri, and T.P. Truong. (Editors) 1998. *Global Trade, Assistance and Protection: The GTAP 4 Data Base*. Center for Global Trade Analysis, Purdue University.
- Narayanan, S. and E. Kizito. 1992. "Multifactor Productivity for Canadian Agriculture: Update to 1990 with Analysis," Working Paper 1/93, Agriculture Canada, Policy Branch, Farm Economic Analysis Division, Ottawa, September.
- Organization for Economic Co-Operation and Development. 1993. "GREEN: The User Manual," Resource Allocation Division, Economics Department, OECD, Paris, France, 82 pages, May.
- Peterson, W. 1988. "International Supply Response," *Agricultural Economics*, vol 2, pp. 365-374.
- Thirtle, C.; Atkins, J.; Bottomley, P.; Gonese, N. and Govereh, J., (1993) "The Productivity of Commercial Agriculture in Zimbabwe, 1970 – 89", *Journal of International Development*, 5(2):193-205.

- Thirtle, C.; van Zyl, J. and Vink, N. (2000) *South African Agriculture at the Crossroads: An Empirical Analysis of Efficiency, Technology and Productivity*, Macmillan Press LTD, London, UK.
- Whalley, J. and R. Wigle. 1991. "The International Incidence of Carbon Taxes," in *Global Warming: Economic Policy Responses*, R. Dornbusch and J.M. Poterba, eds. MIT Press: Cambridge, Mass.

Appendix

Elasticity of Supply in GTAP

Robert A. McDougall

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In the GTAP 7 documentation (Hertel, Tsigas, and Narayanan [1]), a formula for the elasticity of supply is derived from two formulae,

$$\eta_S = -\sigma_{RR},$$

where η_S denotes the elasticity of supply and σ_{RR} , the Allen-Uzawa own-price elasticity of substitution, and

$$\sigma_{RR} = -\left(\sigma_{VA} \left[\theta_R^{-1} - \theta_{VA}^{-1}\right] + \sigma_T \left[\theta_{VA}^{-1} - \theta_O^{-1}\right]\right),$$

where σ_{VA} denotes the elasticity of substitution between components of value added, σ_T , the elasticity of substitution between value added and intermediates, θ_R , the share of natural resources in total input cost, θ_{VA} , the share of value added in total input cost, and θ_O , the share of all inputs together in total input cost, that is, one. The latter formula is a special case of the formula derived by Keller [2, Theorem 2] for an arbitrary tree of origin-shifted CES functions. In the GTAP standard data base, the elasticity σ_T is equal to zero, so the formula simplifies to

$$\sigma_{RR} = -\sigma_{VA} \left(\theta_R^{-1} - \theta_{VA}^{-1}\right).$$

combining the two formulae, we have

$$\eta_S = \sigma_{VA} \left(\theta_R^{-1} - \theta_{VA}^{-1}\right).$$

Alternatively, we have a direct derivation of the elasticity of supply, for the particular production tree used in GTAP, in which, with standard parameter settings, output is a Leontief aggregate of intermediate inputs and value added, and value added is a CES aggregate of primary factors; with the additional assumption that one primary factor, natural resources, is fixed, and all other inputs are mobile. Let P_i and Q_i denote the price and quantity of i . Let p_i and q_i denote the corresponding relative differentials,

$$p_i = \frac{d P_i}{P_i}, \quad q_i = \frac{d Q_i}{Q_i}.$$

Let S_{ij} the share of j in the cost of i ,

$$S_{ij} = \frac{P_j Q_j}{P_i Q_i},$$

where i and j may take the values R for natural resources, F for primary factors, and O for output. For some industry in some region, let η_S denote the elasticity of supply, and σ_F the elasticity of

substitution between primary factors. Assume that prices of inputs other than natural resources are constant. Then, from the GTAP equation system, we extract the subsystem

$$\begin{aligned} q_R &= q_F - \sigma_F(p_R - p_F), \\ p_F &= S_{FR}p_R, \\ q_F &= q_O, \\ p_O &= S_{OR}p_R, \end{aligned}$$

in which p_O and q_R are given, and from which p_R , p_F , q_F , and q_O may be found. Solving, we obtain the supply equation

$$q_O = q_R + \sigma_F \frac{1 - S_{FR}}{S_{OR}} p_O.$$

Then the elasticity of supply is the coefficient of p_O in that equation, that is,

$$\eta_S = \sigma_F \frac{1 - S_{FR}}{S_{OR}}.$$

Relating notation, σ_F in the second derivation matches σ_{VA} in the first, S_{OR} in the second matches θ_R in the first, and S_{FR} matches θ_R/θ_{VA} in the first. So the two derivations agree.

References

- [1] Thomas Hertel, Marinos Tsigas, and Badri Narayanan G. *Global Trade, Assistance, and Protection: The GTAP 7 Data Base*, chapter 12.A. Purdue University, 2004.
- [2] Wouter J. Keller. A nested CES-type utility function and its demand and price-index functions. *European Economic Review*, 7:175–186, 1976.