

Analyzing Country Strategies for Structural
Transformation, Poverty Eradication and Shared Prosperity:
Method and Application to Sub-Saharan Africa

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Abstract

In order to eradicate extreme poverty and raising the income share of the bottom 40 percent, African countries need to design policies that use international trade to generate jobs and inclusive growth. This paper develops an approach to medium- and long-run analysis that integrates elements of product-space (PS) analysis with CGE modeling. In this pilot study, the approach is applied to a stylized Sub-Saharan country and used to analyze the impact of alternative export market conditions and government policies on growth, sector structure, poverty, and shared prosperity.

The analysis uses a recursive-dynamic CGE model, augmented to incorporate the following PS features: (a) proximity between sectors influences loss in labor effectiveness from reallocation; (b) proximity-weighted learning-by-doing, driven by accumulated production, influences activity TFP; and (c) penetration of export markets may lead to quality upgrading and higher export prices.

Due to the addition of PS features, the database is expanded to include a matrix of global proximities between sectors based on export data for tradable goods sectors and value-added data for other sectors; (b) unit export price data by sector and distance from unit price frontier for top-quality exporters; and (c) econometric estimates of the determination of country-level deviations from average unit export prices. In simulations, selected goods sectors are defined as promising on the basis of scope for labor absorption without productivity losses, and for raising productivity, and export revenue. It is anticipated that policy discrimination (via taxes, subsidies and/or public investment) in favor of promising sectors may have significant positive effects (in terms of growth, poverty reduction, and reduced inequality). Policy discrimination in favor of sectors that lack these characteristics has opposite effects. Such results may be negated by adverse world price changes for promising sectors.

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1. INTRODUCTION

Many countries in Sub-Saharan Africa (SSA) are striving to identify strategies for structural transformation that bring about rapid growth, often with the aim of achieving middle-income status. This paper develops an innovative approach to the analysis of such strategies and applies it to a stylized pilot country. The approach augments a Computable General Equilibrium (CGE) model by incorporating indicators from product-space (PS) analysis that influence the extent and consequences of changes in sector structure. In this case, the CGE model is an augmented version of MAMS (Maquette for MDG Simulations), a CGE model for country-level strategy analysis. The model is used to simulate the effects of changes in policies and world markets on macro, sector, and welfare indicators. More specifically, this study provides a forward-looking perspective with the aim of identifying strategies for structural transformation that, during the next two decades, promise to further reduce poverty, enhance shared prosperity, and accelerate growth in production, incomes, and employment.

We proceed as follows. Section 2 [draft] describes the model and its disaggregation, with a focus on the PS-related extensions while Section 3 overviews economic and social developments for low-income SSA countries during the period 1990-2010. Section 4 [in progress] covers the base run and a set of scenarios that focus on policy changes related to structural transformation and aspects of the external environment that condition the effects of these policy changes. The concluding Section 5 [in progress] extracts the implications of the preceding analysis for country development strategy and discusses a future research agenda.

Appendices [partly completed] provide additional background and details: Appendix A [draft] reviews key concepts of product-space analysis; B [draft] derives the first-order conditions for producer profit maximization in MAMS in a PS context, including their derivation; C-D presents additional data related to the model database and the simulations, respectively.

2. METHOD: BRINGING PRODUCT-SPACE INTO A CGE MODEL

Given its wide coverage of policies and indicators of interest to long-run strategy analysis in low-income countries (including endogenous productivity growth and a built-in poverty module), we use MAMS.¹ The version of MAMS used for this study, here referred to as SSA MAMS, is extended to incorporate mechanisms and parameters from PS analysis. In this

¹ For more on MAMS, see Lofgren et al. (2013) and Bourguignon et al. (2008).

section, we provide a non-technical overview of MAMS, including its disaggregation, as well as technical detail on the PS-related extensions

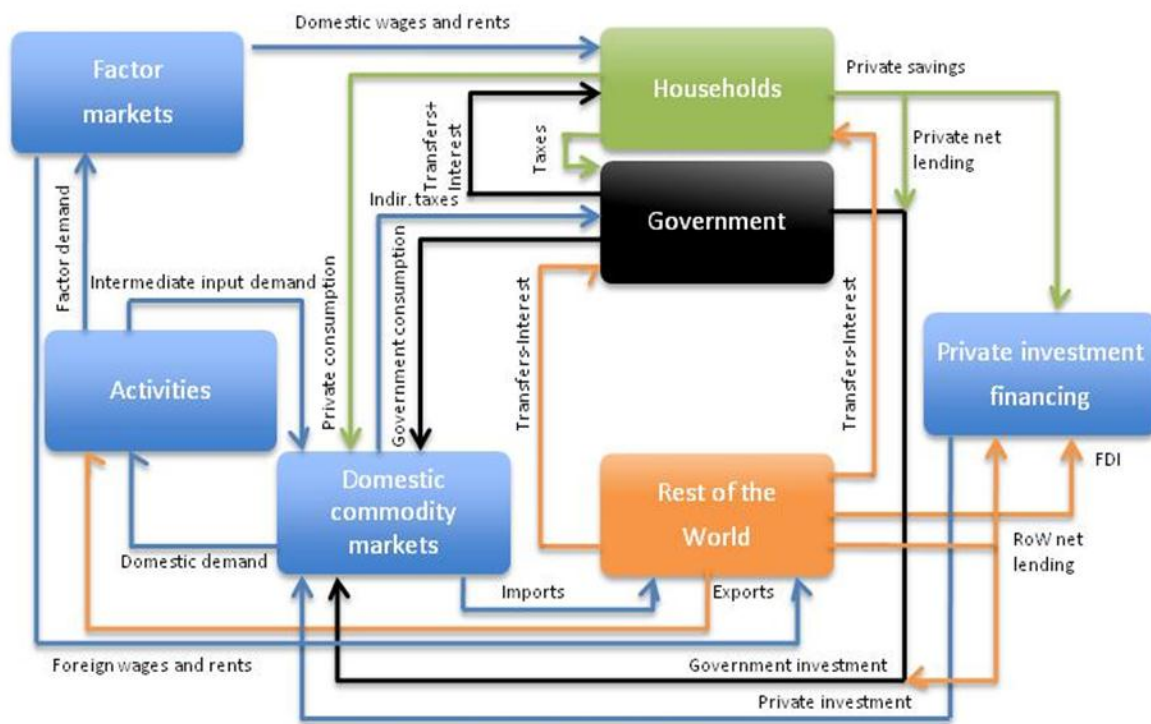
Overview of MAMS structure

This section provides a non-technical summary of MAMS, covering its two standard modules – core and MDG – as well as extensions to PS analysis.

Core module

As indicated by Figure 2.1, which will serve as the reference point for our model overview, the major building blocks of MAMS are activities (the entities that carry out production), commodities (activity outputs or, exceptionally, imports without domestic production; linked to markets), factors (also linked to markets), institutions (households, the government, and the rest of the world). In the Figure, private investment financing also has its own box given the multiple links that exist between private investment demands and their financing. In any MAMS application (and database), most blocks are disaggregated – the disaggregation in the SSA MAMS application is shown in Table 2.1.

Figure 2.1. Overview of MAMS



<< Table 2.1. Disaggregation of SSA MAMS >>

In any single year, SSA MAMS has the structure summarized in the above figure. Activities produce, selling their output at home or abroad, and using their revenues to cover their costs (of intermediate inputs, factor hiring and taxes). Their decisions to pursue particular activities with certain levels of factor use are driven by profit maximization. The shares exported and sold domestically depend on the relative prices of their output in world and domestic markets.

SSA MAMS includes three types of institutions: households, government, and the rest of the world. Households earn incomes from factors, transfers and interest from the government (with the interest due to loans from the households to the government), and transfers from the rest of the world, net of interest on household foreign debt, if any. These are used for direct taxes, savings, and consumption. The savings share depends on per-capita incomes. Their consumption decisions change in response to income and price changes. By construction (and as required by the household budget constraints), the consumption value of the households

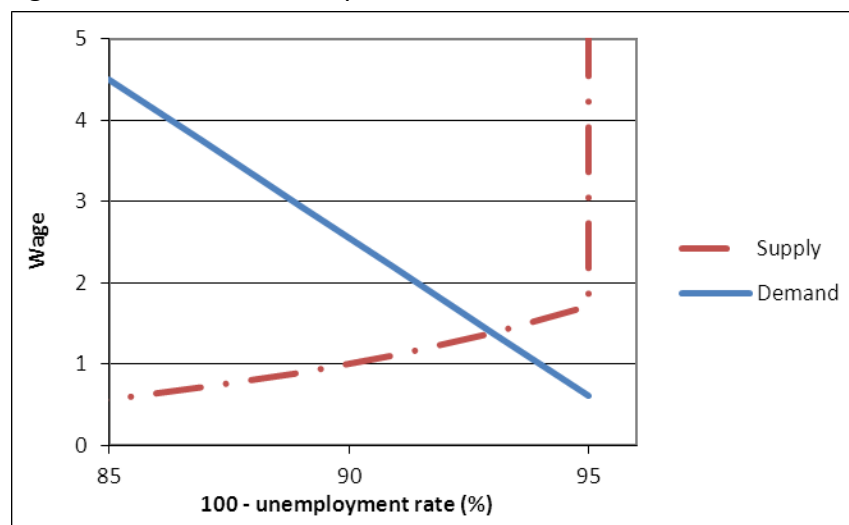
equals their income net of direct taxes and savings. The government gets its receipts from taxes and transfers from abroad; it uses these for consumption, transfers to households, and investments (providing the capital stocks required for producers of government services), drawing on domestic and foreign borrowing for supplementary investment funding.

To remain within its budget constraint, it either adjusts some part(s) of its spending on the basis of available receipts or mobilizes additional receipts of one type or more in order to finance its spending plans -- in the base scenario of the current analysis, the government adjusts part of its current (consumption) spending in order to stay within its budget constraint. (The rules for the government budget in non-base simulation are discussed in the simulation section.) The rest of the world (which appears in the balance of payments) sends foreign currency to the model country in the form of transfers to its government and households (for the government net of interest payments on its foreign debt), FDI, loans, and export payments. The model country uses these inflows to finance its imports. The balance of payments clears (inflows and outflows are equalized) via adjustments in the real exchange rate (the ratio between the international and domestic price levels), influencing export and import quantities and values in foreign currency. Private investment financing is provided from domestic private savings (net of lending to the government) and foreign direct investment (FDI). For the base scenario, private investment is an exogenous share of GDP.

In domestic commodity markets, flexible prices ensure balance between demands for domestic output from domestic demanders and supplies to the domestic market from domestic suppliers. The part of domestic demands that is for imports faces exogenous world prices – the model country is viewed as small in world markets without any impact on the import and export prices that it faces. Domestic demanders decide on import and domestic shares in their demands on the basis of the relative prices of commodities from these two sources. Similarly, domestic suppliers (the activities) decide on the shares for exports and domestic supplies on the basis of the relative prices received in these two markets.

Factor markets reach balance between demands and supplies via wage (or rent) adjustments. Across all factors, the factor demand curves are downward-sloping reflecting the responses of production activities to changes in factor wages. On the supply side of the labor markets, unemployment is endogenous – for each labor type, the model includes a wage (or supply) curve that is upward-sloping until full employment is reached, at which point it becomes vertical (see Figure 2.2; its supply curve assumes a minimum unemployment rate of 5%). For non-labor factors, the supply curves and total employment are vertical in any single year.

Figure 2.2. Labor market specification



The above discussion refers to the functioning of model economy in a single year. In MAMS, growth over time is endogenous. The economy grows due to accumulation of capital (determined by investment and depreciation), labor (determined by share of the population in labor force age, the labor force participation rate, and the educational system), other factors (on the basis of exogenous growth data), as well as because of improvements in total factor productivity (TFP). Apart from an exogenous component, TFP of any production activity potentially depends on the levels of government capital stocks, economic openness and, as will be discussed below, a learning-by-doing component that is related to the product-space features of the model.

MDG and education module

MAMS generates a set of HD (MDG and education) indicators. Foster-Green-Thorbecke poverty indicators, Gini, and indicators related to shared prosperity (defined on the basis of the incomes of the bottom 40 percent) are computed on the basis of a simple micro simulation approach in which each model household is mapped to a subset of the observations in a household survey. In each simulation, the changes in per-capita consumption for each model household are passed on to the mapped survey observations, assuming that the observed initial distribution within each household group is unchanged. At the same time, the mapped survey households are reweighted on the basis of simulated changes in the population shares of each model household.

A different treatment is used for other MDG and education indicators: MDGs 4 (under-five mortality), 7w (improved water access), 7s (improved sanitation access); net intake rate to first grade of primary, promotion rate to next grade (one rate for each of the three education cycles), and continuation into next cycle (the share of those who are promoted from the last grade who continue to next cycle; for primary and secondary). For these indicators, a two-level formulation is used. At the bottom level, an intermediate variable specific to each indicator is expressed as a CE function of a set of determinants, summarized in Table 3.1 and extracted from the literature on determinants of these outcomes.

Table 3.1. Determinants of selected MDG outcomes

MDG	Service delivery	Household consumption per capita	Wage incentives	Public infra-structure	Other MDGs
2. Primary education	x	x	x	x	4
4. Under-five mortality	x	x		x	7w, 7s
7w. Access to safe water	x	x		x	
7s. Access to basic sanitation	x	x		x	

Note: Service delivery refers to services in the relevant area (primary education, health, water, or sanitation).

At the top level, each MDG and education indicator is a logistic function of its intermediate variable. The parameters of these two-level functions are calibrated to replicate data on projected progress for the top-level indicators under the *base* scenario, with upper or lower limits imposed on the basis of logic or cross-country experience (e.g. the net intake rate has an upper limit of 100 percent; the under-five mortality rates cannot fall below some low positive value). As shown in Table 3.1, the determinants include the supply of services by government or private sector (measured per student for each cycle of education and per-capita for health), the stocks of different types of relevant government infrastructure (energy and other), real household consumption per-capita (an indicator of the ability of households to make purchases in support of stronger MDG and education outcomes), and other MDG indicators (to reflect the fact that progress for one MDG may have a positive impact on other MDGs). A wage incentive indicator is a determinant specific to education; it is expressed as the ratio between the wages for labor at the next higher and current levels of education for the student in question (an indicator of payoff from continued education). MDG 2 – the objective of making sure that each

child completes primary education -- is measured by the net (on-time) primary completion rate expressed as a period measure, i.e. the share of the population that enters the first primary grade in the relevant year that would graduate on time if the rates of net intake and promotion in the year in question were to prevail throughout their primary cycle (a total of six years).² In MAMS, primary education is an integral part of a broader education module that tracks base-year stocks of students and new entrants through the different grades of the three cycles (primary, secondary, and tertiary). In each year, students will successfully complete their grade, repeat it, or drop out of their level. A share of labor-force-age students exiting from the school system enters the labor force in the segment that matches their educational background. Similarly, a share of the non-school population (perhaps drop outs at an earlier age) that each year reaches labor force age enters the labor force, typically in its least educated segment.

Database

The database for SSA MAMS consists of an extensive data set for the simulation base-year – a social accounting matrix (SAM);³ stocks for production factors (including different types of labor and capital), population, and school enrollment; indicators for selected MDGs and the educational system – as well as a set of elasticities (for production, consumption, trade, and

² More specifically, the net completion rate is defined as net completion rate expressed as a period measure, i.e. the share of the population that enters the first primary grade in the current year that would graduate on time given the current rates of net entry (neting1) and promotion (prom), i.e. the measure uses current rates assuming that the rate of promotion of the current year prevails throughout their primary cycle (the current year and five more years). Mathematically, $NPCR_t = NIR_t \cdot (PR_t)^y$ where $NPCR$ = net primary completion rate; NIR = net intake rate ($0 \leq NIR \leq 1$); PR = promotion rate ($0 \leq PR \leq 1$); and y = number of grades in the primary cycle. As a simplification, MAMS assumes a uniform PR for all primary grades. Compared to alternative indicators, the gross completion rate or the gross or net enrollment rates, the net completion rate provides a better measure of the performance of primary education in terms of ensuring completion of the full cycle for each child. Gross completion rates (the total number of students completing primary education, irrespective of age, as a share of the total population in the theoretical graduation age) may reach above 100 percent at the same time as far from all children manage to complete the full primary cycle; enrollment rates do not provide information about completion of the cycle.

³ A SAM is a square matrix with identical row and column accounts, providing a comprehensive representation of payments flows in the economy of a geographical unit (typically a country) during a period of time (typically one year). Cell entries represent payments from its column account to its row account. In a SAM without errors, row and column totals are equal. SAMs appear with widely varying degrees of disaggregation. The payments flows are expressed in current local currency or some transformation thereof – in Table 2.??, the value of each cell has been transformed into percent of GDP at market prices in the same year. For more on SAMs, see for example Reinert and Roland-Holst (1997) and Round (2003). The detailed MAMS SAM for this application is available on request.

human development relationships), and projections into the future (for growth in GDP at factor cost and the evolution of disaggregated MDG and education indicators and determinants), to which the MAMS baseline simulation is calibrated. (See Appendix 1 for more details.)

The basic accounting structure and much of the underlying data of MAMS, like other CGE models, are derived from a Social Accounting Matrix (SAM) for the base year of the model, in this case 2010; more specifically, the SAM is used to define base-year values for the bulk of the model parameters.⁴ Most features of a SAM for MAMS are familiar from SAMs used for other models. However, a MAMS SAM has some unconventional features related to the explicit treatment of financial flows. The disaggregation of the SSA SAM coincides with that of the rest of the model database. As shown in Table 2.1, the database of SSA MAMS includes 24 sectors (activities and commodities) – 3 in agriculture, 5 in industry, and 16 in services – with each activity producing a single commodity for which it is the only domestic producer. The factors are split into labor (three types, disaggregated by education), capital (one private and one for each government service sector), and natural resources (4 types: agricultural land, forestry land, fishing resources, and extractive industry resources). The institutions are split into households (further divided into 7 groups differentiated on the basis of their sources of income: specialized in unskilled labor, semi-skilled labor, skilled labor, capital, land, remittances, or diversified), government, and the rest of world, each of which has current and capital accounts. A set of auxiliary accounts includes different tax types as well as domestic and foreign interest payments.

MAMS extensions for PS analysis

MAMS is extended to incorporate key insights from PS analysis regarding factors that influence the ease or difficulty with which sector structure can evolve over time. (Appendix A defines the product-space indicators to which we refer in the main body of the text.) The aim is to enable MAMS to replicate historical changes in sector structure and appropriately respond to changes

⁴ A SAM is a square matrix in which each account is represented by a row and a column. It provides a comprehensive picture of the economic transactions of an economy during a time period, almost invariably one year. Each cell shows the payment from the account of its column to the account of its row. Thus, the incomes of an account appear along its row and its expenditures along its column. For each account in the SAM, total revenue (row total) should be equal to total expenditure (column total).

in structural policies and in the external environment. Specifically, the following mechanisms are built into the model:

(a) Proximity-based labor reallocation.⁵ The *proximity* indicator ($0 \leq \varphi_{a,a',t} \leq 1$) reflects the central insight of PS analysis: competitive production in one activity requires capabilities that are more or less close to the capabilities associated with other activities; it is relatively easy (difficult) to expand an activity if other activities with similar capabilities represent a large (small) share of internationally competitive production. In order to capture the implications of this insight in MAMS, we will distinguish between effective and physical labor employment. In each period, physical labor quantities are associated with activities of origin, a' , on the basis of previous employment. Effective employment in the activity of destination, a , falls short of the physical quantity that is reallocated from a' to a whenever factor reallocation occurs between two activities a' and a for which $\varphi_{a',a,t} < 1$, i.e. (with the factor and time subscripts suppressed)

$$QF_a = \sum_{a'} \varphi_{a',a}^{scal} \cdot QFR_{a',a} \text{ where } QF_a = \text{effective quantity employed in destination activity } a;$$

$\varphi_{a',a}^{scal}$ = scaled *proximity* between the activities a' ($0 < \varphi_{a',a}^{scal} \leq 1$); and $QFR_{a',a}$ = physical quantity of f reallocated from a' to a .⁶ The magnitude of the scaling of the *proximity* parameter is determined as part of the validation procedure (discussed below). This formulation is embedded in the first-order conditions for profit-maximizing producer decisions and

⁵ For private capital, new investment is allocated across activities so that the marginal value product (MVP) is equal across activities (potentially adjusted for exogenous relative rent differentials). In order to ensure that installed capital is not reallocated – in general reallocation is not feasible – the model includes the constraint that

$$QF_{a,t} \geq (1 - depr) QF_{a,t-1} \text{ where } QF = \text{private capital employment; } depr = \text{depreciation rate for private capital;}$$

and a and t refer to production activities and time, respectively. If this constraint is binding for an activity a in t and unless rents are distorted by an exogenous term, the MVP of capital in this activity is smaller than the economy-wide average; if so, this activity will not receive new investment.

⁶ Note that, for the special case with $a' = a$ (factor is “reallocated” to the same sector), $\varphi_{a',a} = 1$. The scaled *proximity* parameter may be derived from the original *proximity* parameter using the following formula:

$$\varphi_{a',a}^{scal} = \varphi_{a',a} + \varphi^{scal} \cdot (1 - \varphi_{a',a}), \text{ where } \varphi^{scal} = \text{factor-specific scaling parameter, the value of which should be set}$$

to ensure that $\varphi_{a',a}^{scal}$ is within the prescribed range. Note that, $\lim_{\varphi^{scal} \rightarrow 1} (\varphi_{a',a} + \varphi^{scal} \cdot (1 - \varphi_{a',a})) = 1$. The value of

φ^{scal} is selected as part of the validation procedure. It may be the case that raw proximities are too low, as a result of which simulated factor reallocation falls short of what is observed in practice; if so a value should be selected such that $0 < \varphi^{scal} < 1$.

economywide constraints for factor markets.⁷ The assignment of physical labor quantities by activity of origin is based on activity employment shares in a weighted average of recent years.⁸ *Ceteris paribus*, as a result, activities prefer (and are willing to pay a higher wage to) labor belonging to activities that are close (with the top priority being attached to labor that already belong to the destination activity). Reallocation between very distant activities is discouraged as the loss in effective quantity employed would be stronger.

Given that factor demands from goods and service activities are part of the same broader factor (especially labor) markets, it is necessary to extend the *proximity* measures to cover sectors (activities and related commodities) that are not covered by Comtrade, i.e. a subset of the goods sectors and all service sectors. In general, the non-Comtrade sectors tend to have low export/output ratios, both in SSA and elsewhere. If a sector does not export or exports very little, export data (if available from some other source) do not provide a good measure of *proximity* (in capabilities). Given their availability and the fact that they conceptually seem more appropriate, we turn to value-added data to generate RCA and *proximity* indicators for non-Comtrade sectors, thereby ensuring an exhaustive activity coverage. Thus, we view non-Comtrade sectors as constituting an important of the sectoral structure, both in their own right and by making different paths of structural transformations more or less difficult for Comtrade sectors. In fact, this treatment addresses one of the main criticisms of PS analysis: it is likely misleading to analyze strategies for structural transformation without considering the role of sectors not covered by non-Comtrade.

(b) Learning by doing. [Description of treatment – a proximity-weighted learning by doing process influencing TFP by activity – to be added here.]

The two augmentations of the CGE model from the PS literature differ in their effects on the ease of transforming production structure. Efficiency losses in the context of labor reallocation make structural change more difficult. On the other hand, productivity changes due to learning by doing may reinforce structural change in favor of one or more related activities once the

⁷ This *proximity*-based factor reallocation treatment is inspired by Dantzig's transportation model, in which the total cost of transporting a set of items from supply to demand depots is minimized subject to demand and supply constraints (Dantzig 1963, pp. 35-42). In our version, the unit transportation costs are replaced by effective employment losses driven by PS *proximity* measures.

⁸ If only the shares of the last year are used, then the effective employment loss would only be imposed in a single year. By using a weighted average of a series of recent years, it is possible to gradually reduce effective employment losses over time, mimicking a process of learning by doing.

process is under way.⁹ The model is designed to capture any influence of policy makers on these processes via investments, subsidies and taxes. Such policies may have a positive impact on GDP growth and other indicators if they promote (or reduce discrimination against) activities that contribute to stronger growth in production and incomes.

3. COUNTRY BACKGROUND: ECONOMIC STRUCTURE AND RECENT PERFORMANCE

4. SIMULATIONS

5. CONCLUSIONS

⁹ Changes in export prices represent an additional mechanism in favor of structural transformation: as a country gains experience as a successful exporter, it is likely that it will be able to increase the prices it charges for a given export relative to the average world market price; the strength of this mechanism is likely to vary across sectors, being stronger for manufactures than agricultural products (ZZ reference??). For evidence of this phenomenon, see Hausmann, Hwang, and Rodrik (2007, pp. 13-14).

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APPENDIX A. KEY CONCEPTS IN PRODUCT-SPACE ANALYSIS

PS analysis offers a data-driven evaluation of the feasibility and desirability of alternative sectoral transformation options for a country, considering its initial structure and the global record. Drawing on data for exports and, in our application, VA, it generates both standard trade indicators and a set of new indicators that are related to the *proximity* between different commodities, the income levels that are associated with the countries that export individual commodities, and the export basket of individual countries. In this section, we will review these concepts, including the extensions and adjustments that we introduce for the purposes of this study.

PS analysis was pioneered in Hausmann, Hwang, and Rodrik (2007), Hausmann and Klinger (2006), and Hidalgo et al. (2007). This analysis is based on a country's current capabilities (or initial conditions) and on international evidence that arises from patterns of comparative advantage and path dependence in the basket of goods exports of a country, implicitly underpinned by similar path dependence in the production basket of the country. PS analysis provides information about the income and diversification potential of different sector strategies, considering links between sectors. The key data source for the PS analysis at its typical, most disaggregated level, is Comtrade's global trade records (the SITC Rev. 2 Classification at the 4-digit level, covering 784 goods and 130 countries from 1980 to 2010); this information is complemented by country-level data on GDP per capita.¹⁰

One significant shortcoming of PS analysis is that it has typically been limited to the products (goods) covered by Comtrade, viewing these in isolation from other products.¹¹ Moreover, for products that are non-traded, export data cannot be used to assess the capability of a country. Furthermore, *RCA* indicators are influenced by policy-induced distortions – although, given the large number of countries involved, the resulting distortion of *proximity*-based indicators (which appear in all core PS concepts) may be quite limited – and they are not ideal proxies for capability (a concept that is distinct from comparative advantage).

Since its inception, PS analysis has been applied to a wide range of countries, including numerous studies by the World Bank that have enriched its dialogue with clients. (See the reference section for a selection of studies.)

¹⁰ The disaggregation of PS analysis is determined by the dataset that is used. For even more finely disaggregated analysis, the Comtrade HS database may be used.

¹¹ In a recent pioneering analysis, Mishra, Lundstrom, and Anand (2011) address the role of service export sophistication in economic growth using the tools of PS analysis.

Empirically, PS analysis involves the construction of set of export-related indicators, showing both the historical record of a country and indicative projections into the future. Some of these – including the Herfindahl index (HI), and revealed comparative advantage (*RCA*) are standard.¹² Other indicators, derived from network theory and validated empirically, have more recently been pioneered in the context of PS analysis (Hausmann and Klinger 2006; Hidalgo and Hausmann 2009; Hidalgo et al. 2007).

More specifically, the standard indicators that are relevant in the current model may be defined as follows, with the indices r (or r'), c (or c' or c''), and t referring to countries (“regions”), commodities (often referred to as products, typically limited to goods), and years, respectively.¹³

- *HI* is a time- and country-specific measure of export concentration by country, and time period:

$$HI_{r,t} = \sum_c \left(\frac{E_{r,c,t}}{\sum_{c'} E_{r,c',t}} \right)^2$$

where $E_{r,c,t}$ is the value of exports for country r of commodity c in time t .

- *RCA* shows the degree of comparative advantage by country, commodity, and time. A country has an *RCA* in a commodity c if the following indicator has a value above unity:

¹² Indicators of the technological content of exports are also frequently included in analyses of structural transformation as it is indicative of the sophistication of a country in a given commodity category. Technologically sophisticated commodities tend to be associated with a high *PRODY*. For example, Lall (2000) classifies commodities into primary, resource-based, low-, medium- or high-tech; see also Lall, Weiss, and Zhang (2006).

¹³ In PS analysis, the indices c and i are typically used for country and product; in MAMS and many other CGE models, c is used for commodities (goods or services). In order to avoid confusion and to keep notation consistent throughout this study, we switch to r for countries (or “regions”; this follows the example of GTAP) and use c for commodities. It should also be noted that MAMS makes a distinction between commodities c (outputs) and activities a (producing outputs); in this application, there is a one-to-one mapping between the two. Whether the MAMS code refers to a or c depends on context.

$$RCA_{r,c,t} = \frac{\frac{E_{r,c,t}}{\sum_{c'} E_{r,c',t}}}{\frac{\sum_{r'} \sum_{c'} E_{r',c',t}}{\sum_{r'} E_{r',c,t}}} = \frac{\left[\begin{array}{c} \text{shr of commodity} \\ \text{in country exports} \end{array} \right]}{\left[\begin{array}{c} \text{shr of commodity} \\ \text{in world exports} \end{array} \right]} = \frac{\frac{E_{r,c,t}}{\sum_{r'} E_{r',c,t}}}{\frac{\sum_{r'} \sum_{c'} E_{r',c',t}}{\sum_{r'} \sum_{c'} E_{r',c',t}}} = \frac{\left[\begin{array}{c} \text{shr of country in} \\ \text{exports of commodity} \end{array} \right]}{\left[\begin{array}{c} \text{shr of country in} \\ \text{world exports} \end{array} \right]}$$

In the following, *RCA* refers to the above definition, i.e. a continuous variable with a lower limit of zero. The corresponding binary 0-1 variable is referred to as *RCA01* ($RCA01_{r,c,t} = 0$ if $RCA_{r,c,t} < 1$; $RCA01_{r,c,t} = 1$ if $RCA_{r,c,t} \geq 1$). [zz. A second continuous variant, *RCA01cnt*, is used in MAMS: $RCA01cnt_{r,c,t} = RCA_{r,c,t}$ if $RCA_{r,c,t} \leq 1$; $RCA01cnt_{r,c,t} = 1$ if $RCA_{r,c,t} > 1$). zz] On the basis of the evolution of their *RCAs*, exported goods may be classified as classic, emerging, disappearing, or marginal. The classics may be understood as the traditional exports of a country, i.e., commodities in which the country always has had a comparative advantage. The emerging champions are commodities in which the country did not have a comparative advantage in the past but developed it in recent years. The time periods ‘past’ and ‘present’ can be specified by the analyst. The disappearing commodities are those in which the country had a comparative advantage in the past but does not have it anymore, and the marginal commodities are those in which the country never has had a comparative advantage.

Indicators that more recently have been pioneered by PS analysis include:

- *PRODY_{c,t}* indicates the potential income level of a commodity *c* in *t*. It is defined on the basis of GDP per capita (*GDPPC*) of all exporting countries, with the weight of each country defined by how important *c* is in its exports (measure by the share of *c* in the total export value of country *r*).¹⁴

¹⁴ The *PRODY* concept was developed by Hausmann, Hwang and Rodrik (2007). According to Schott (2008), *PRODY* may overestimate the income potential of complex manufactured commodities such as hi tech electronics if they are exported both by relatively poor countries like China and rich Western countries. However, Schott (2008) has also noted that for simpler commodities exported by most developing countries, especially low income ones, the *PRODY* is a reasonable representation of the income potential of the commodities exported.

$$PRODY_{c,t} = \sum_r \left[\frac{\left(\frac{E_{r,c,t}}{\sum_{c'} E_{r,c',t}} \right)}{\sum_{r'} \left(\frac{E_{r',c,t}}{\sum_{c'} E_{r',c',t}} \right)} \cdot GDPPC_{r,t} \right]$$

Some evidence suggests that developing countries that start to produce and export a commodity over time may be able to raise the prices that they receive as quality improves, reducing the price gap vis-à-vis developed country exporters of the same commodity (Hausmann, Hwang, and Rodrik 2007, pp. 13-14).

- *EXPY* shows the potential income level of the export basket of a country. It is defined using *PRODY* with each commodity c weighted by its share in the exports the country:

$$EXPY_{r,t} = \sum_c \frac{E_{r,c,t}}{\sum_{c'} E_{r,c',t}} \cdot PRODY_{c,t}$$

Econometric cross-country time-series indicates that *EXPY* is a strong and robust predictor of subsequent economic growth (Hausmann, Hwang, and Rodrik 2007, p. 3).

- The *proximity* between two commodities, c and c' , in time t , $\varphi_{c,c',t}$ ($0 \leq \varphi_{c,c',t} \leq 1$) is the key building-block for all network indicators in the PS analysis. It is derived from data on probabilities of having $RCA \geq 1$ ($rca01 = 1$) simultaneously for c and c' :

$$\varphi_{c,c',t} = \min \left\{ P(rca01_{c,t} | rca01_{c',t}), P(rca01_{c',t} | rca01_{c,t}) \right\}$$

where P (the conditional probability) is computed using all countries r in year t , and where

$$rca01_{r,c,t} = \begin{cases} 1 & \text{if } RCA_{r,c,t} > 1 \\ 0 & \text{if } RCA_{r,c,t} \leq 1 \end{cases}$$

- The *density* of a commodity c in which a country does *not* have an *RCA*, scaled to vary from 0 to 1, can be seen as a measure of the probability (or capability) of developing an $RCA \geq 1$ in commodity c in the future. For each commodity, it is the ratio between (a)

the sum of all proximities between that particular commodity and all commodities in which the country has an $RCA \geq 1$; and (b) the sum of all proximities of the commodity (irrespective of whether or not the country has an RCA in the other commodity):

$$density_{r,c,t|rca01_{r,c,t}=0} = \frac{\sum_{c'|c \neq c'} \varphi_{c,c',t} \cdot rca01_{r,c',t}}{\sum_{c'|c \neq c'} \varphi_{c,c',t}}$$

In MAMS, we changed the above definition of *density*, replacing $rca01$ by an indicator we termed symmetric revealed capability ($RC02$), derived from an indicator of revealed capability (RC):

$$RC02_{c,i,t} = 1 + \frac{RC_{c,i,t} - 1}{RC_{c,i,t} + 1}$$

and

$$RC_{c,i,t} = \frac{\frac{E_{c,i,t}}{GDP_{c,t}}}{\frac{\sum_c E_{c,i,t}}{\sum_c GDP_{c,t}}} = \frac{\left[\begin{array}{l} \text{country export-GDP} \\ \text{ratio for good } i \end{array} \right]}{\left[\begin{array}{l} \text{world export-GDP} \\ \text{ratio for good } i \end{array} \right]}$$

where $RC02_{c,i,t}$ = symmetric revealed capability ($0 \leq RC02_{c,i,t} < 2$), $RC_{c,i,t}$ = revealed capability ($0 \leq RC_{c,i,t} < \infty$). The density definition is as follows:

$$density02_{r,c,t} = \frac{\sum_{c'} \varphi_{c,c',t} \cdot RC02_{r,c',t}}{\sum_{c'|c \neq c'} \varphi_{c,c',t}}$$

Both cover all commodities (irrespective of capability level). In the implementation, the numerator is in the definition of RC is in real terms (at base-year prices) while the denominator is not updated (i.e., the original world ratio is kept in place). Compared to the standard density concept ours is (a) more sensitive to changes over time; this may be particularly apparent when the number of commodities is smaller than in typical PS analysis, which is the case when *density* is used in a CGE context; (b) includes the export performance of the commodity itself; (c) covers all commodities (irrespective of capability level), and (d) is less prone to generate a seeming unrealistic dampening impact on the measure of capability from a declining share in the export

basket for other commodities whenever one commodity expands rapidly – no such effect would be present if other commodities maintain their own export-GDP ratios.

- Open forest (*OF*) may be seen as the option value of a country's unexploited export opportunities. It is defined on the basis of proximities to and potential incomes of all the commodities in which the country could potentially develop an *RCA*. Mathematically, it may be expressed as follows:

$$OF_{r,t} = \sum_c \left\{ \sum_{c'} \left[\frac{\varphi_{c,c',t}}{\sum_{c''} \varphi_{c'',c',t}} (1 - rca01_{r,c',t}) \cdot rca01_{r,c,t} \cdot PRODY_{c',t} \right] \right\}$$

$$= \sum_{c|rca01_{r,c,t}=1} \left\{ \sum_{c'|rca01_{r,c',t}=0} PRODY_{c',t} \left[\frac{\varphi_{c,c',t}}{\sum_{c''} \varphi_{c'',c',t}} \right] \right\}$$

The second formulation of the definition states that, for an individual commodity *c* in which a country has an $RCA > 1$ ($RCA01 = 1$), *OF* may be defined as the potential income of every other commodity *c'* for which $RCA < 1$ ($RCA01 = 0$), weighted by the share of *c'* in the total proximities of *c*.

In addition, in MAMS, a proximity-weighted learning by doing process influences TFP by activity. The PS indicators in MAMS follow the disaggregation of the MAMS database and cover both Comtrade and non-Comtrade commodities. For the former, the Comtrade database is used; for the latter, we use 2007 data in the GTAP 8 database. The non-Comtrade commodities consist of services and a small subset of goods (including utilities and construction). Globally and for SSA, the shares of exports in total output for these commodities are very small compared to the shares for Comtrade commodities. Given this, we used GTAP VA data for the non-Comtrade PS indicators. Technically, it is straightforward to define *RCA* and *proximity* indicators using export data for one commodity subset and VA data for another commodity subset. The economic interpretation is that, for relatively *non-tradable* commodities (covered by GTAP data), the degree of comparative (or competitive) advantage for country is measured by how important a commodity is in the VA of a country compared to their importance in global VA. At the same time, for relatively tradable commodities (covered by Comtrade), the degree of comparative advantage for country is measured in a standard manner by the how important a commodity is in the exports of a country compared to their importance in global exports. In effect, this

formulation makes it possible to consider in an integrated manner the existing patterns in terms of the development of capabilities in service sectors in parallel with goods sectors.^{15 16}

Among the above indicators, *proximity* is most important to MAMS and its database as it influences both labor reallocation and learning by doing. Drawing on *PRODY* and additional econometric analysis, MAMS also incorporates insights about the likely evolution of export prices as a country expands exports for goods associated with countries at a higher level of GDP per capita. MAMS reports of simulation results cover a wider range of indicators: *EXPY*, *HI*, *RCA*, *RCA01*, *RCA01CNT*, *DENSITY*, and *DENSITYCNT*, and *OF*).

¹⁵ Note that the purpose is to assess the capability of a country to become an efficient producer in a given sector, either for exports or, when trade is less important, as a supplier of inputs or labor to sectors that are close in product space. In such an assessment, it is clearly not possible to use export data for sector without exports. However, for sectors with exports accounting for only a minor part of output (for example construction), it would not be meaningful to assess capability with reference to exports, hence the preference for using VA data.

¹⁶ To add a further consideration, for some export-focused indicators, like *HI*, export data (globally very limited) will be used also for GTAP commodities.

APPENDIX B: PRODUCER PROFIT MAXIMIZATION IN A PRODUCT-SPACE SETTING

This appendix analyzes the profit-maximization problem of the producer of an activity a in a setting where product space *proximity* influences the profitability of the reallocation of factors between activities (sectors). A simplified version such a profit-maximization may be stated as follows:¹⁷

$$\text{maximize } p_a \cdot Q_a - \sum_f \sum_{a'} w_{f,a'} \cdot QFR_{f,a',a}$$

subject to

$$Q_a = \alpha_a \left[\sum_f \delta_{f,a} \cdot f p_{f,a}^{-\rho} \cdot QF_{f,a}^{-\rho} \right]^{-1/\rho} ; QF_{f,a} = \sum_{a'} \varphi_{f,a',a}^{scal} \cdot QFR_{f,a',a} ; \text{ and } QFR_{f,a',a} \geq 0$$

where p_a = output price; Q_a = output quantity; $w_{f,a'}$ = market wage per physical unit of factor f linked to in source activity a' ; $QFR_{f,a',a}$ = the physical quantity of f reallocated from source activity a' to the current activity a . α_a , $\delta_{f,a}$, ρ , and $f p_{f,a}$ = CES function parameters for efficiency, factor shares, exponent, and factor-specific productivity, respectively; $QF_{f,a}$ = effective employed quantity of factor f in activity a ; $\varphi_{f,a',a}^{scal}$ = scaled *proximity* between any source activity a' and the current destination activity a with respect to factor f ($0 < \varphi_{f,a',a}^{scal} \leq 1$, with a value of 1 for the special case where $a = a'$ also covered by QFR , i.e. “reallocation of a factor to its original activity with identical physical and effective factor quantities”).¹⁸ For factors without this factor reallocation mechanism, the *proximity* parameter is 1 for all relevant a - a' mappings.

The Lagrangian, L , for this problem may be stated as follows:

¹⁷ To simplify, this mathematical statement suppresses intermediate input use, domain controls, the time index, and the activity subscript for the exponent ρ .

¹⁸ As noted in footnote of Appendix A, in the MAMS database, there is a one-to-one mapping between commodities, c , and activities, a . Given this, PS indicators may refer to either, depending on what is most convenient in any specific context. Since Appendix B refers to activities and not to their outputs, we here use a .

$$L = p_a \cdot Q_a - \sum_f \sum_{a'} w_{f,a'} \cdot QFR_{f,a',a} + V_a \left[\alpha_a \left[\sum_f \delta_{f,a} \cdot \bar{p}_{f,a}^{-\rho} \cdot QF_{f,a}^{-\rho} \right]^{-1/\rho} - Q_a \right] \\ + \sum_f r_{f,a} \left[\sum_{a'} \varphi_{f,a',a}^{scal} \cdot QFR_{f,a',a} - QF_{f,a} \right]$$

where V_a = the shadow price of output; $r_{f,a}$ = the efficiency-unit shadow rent of factor f in activity a . The first-order conditions may be rendered as:

1. $\frac{\partial L}{\partial Q_a} = p_a - V_a = 0$
2. $\frac{\partial L}{\partial QF_{f,a}} = (-1/\rho) V_a \cdot \alpha_a \left[\sum_{f'} \delta_{f',a} \cdot \bar{p}_{f',a}^{-\rho} \cdot QF_{f',a}^{-\rho} \right]^{-1/\rho-1} [(-\rho) \cdot \delta_{f,a} \cdot \bar{p}_{f,a}^{-\rho} \cdot QF_{f,a}^{-\rho-1}] - r_{f,a} = 0$
3. $\frac{\partial L}{\partial QFR_{f,a',a}} = -w_{f,a'} + r_{f,a} \cdot \varphi_{f,a',a}^{scal} \leq 0;$
4. $QFR_{f,a',a} \geq 0;$
5. $QFR_{f,a',a} (-w_{f,a'} + r_{f,a} \cdot \varphi_{f,a',a}^{scal}) = 0$
6. $\frac{\partial L}{\partial V_a} = \alpha_a \left[\sum_f \delta_{f,a} \cdot \bar{p}_{f,a}^{-\rho} \cdot QF_{f,a}^{-\rho} \right]^{-1/\rho} - Q_a = 0$
7. $\frac{\partial L}{\partial r_{f,a}} = \sum_{a'} \varphi_{f,a',a}^{scal} \cdot QFR_{f,a',a} - QF_{f,a} = 0$

Using 1 to substitute for V in 2, noting that $(-1/\rho) \cdot (-\rho) = 1$, and rearranging the remaining equations permits us to summarize the first-order conditions for producer profit maximization as follows (with the above equation numbers in paranthesis):

$$\begin{aligned}
8 \text{ (1-2). } & p_a \cdot \alpha_a \left[\sum_f \delta_{f,a} \cdot \bar{p}_{f,a}^{-\rho} \cdot QF_{f,a}^{-\rho} \right]^{-1/\rho-1} \left[\delta_{f,a} \cdot \bar{p}_{f,a}^{-\rho} \cdot QF_{f,a}^{-\rho-1} \right] = r_{f,a} \\
9 \text{ (3). } & w_{f,a'} \geq r_{f,a} \cdot \varphi_{f,a',a}^{scal} \\
10 \text{ (4). } & QFR_{f,a',a} \geq 0 \\
11 \text{ (5). } & QFR_{f,a',a} (r_{f,a} \cdot \varphi_{f,a',a}^{scal} - w_{f,a'}) = 0 \\
12 \text{ (6). } & Q_a = \alpha_a \left[\sum_f \delta_{f,a} \cdot \bar{p}_{f,a}^{-\rho} \cdot QF_{f,a}^{-\rho} \right]^{-1/\rho} \\
13 \text{ (7). } & QF_{f,a} = \sum_{a'} \varphi_{f,a',a}^{scal} \cdot QFR_{f,a',a}
\end{aligned}$$

The last set of equations may be interpreted as follows:

8. The marginal value product of factor f in a (RHS) equals the shadow wage of f in a (LHS); both RHS and LHS are expressed in efficiency units.
9. The market wage of f in a' is larger than or equal to the shadow rent of f in a' scaled by the relevant *proximity* indicator.
10. The physical quantity of f reallocated from any activity a' to a is non-negative.
11. If a positive quantity of f is reallocated from a' to a , then the market wage of f in source activity a' has to be equal to the shadow rent of f in destination activity a scaled by the *proximity* indicator. (If the source wage were larger/smaller, then a marginal decrease/increase in the reallocated quantity would raise profits.) In effect, this means that, the lower the scaled *proximity*, the higher the effective-unit destination sector shadow wage that is required for reallocation to take place.
12. Output level for activity a is a function of effective-unit factor quantities.
13. Available quantity of f in a in efficiency units is the sum of quantities in physical units received from different activities a' multiplied by scaled proximity.

APPENDIX C. MODEL DATABASE.

APPENDIX D. ADDITIONAL SIMULATION RESULTS

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(b) *Density*. In PS analysis, *density* is typically based on the standard Belassa 0-1 RCA indicator and limited to sectors with an RCA < 1. It is used as an indicator of the probability (or capability) of developing an RCA > 1. Several other RCA indicators have been proposed (Dalum et al., 1998; Lafay, 1992). For the purpose of the MAMS analysis, we needed a *density* indicator that reflects the capability of a country of high-productivity production in any sector covered by the model, i.e. it should also cover services and, among Comtrade sectors, not exclude sectors with strong export performance. Moreover, the indicator should be sensitive to changes over time also in a setting with a relatively small number of sectors. Against this backdrop, we define *density* as follows for Comtrade sectors:

$$density_{r,c,t} = \frac{\sum_{c'} \varphi_{c,c',t} \cdot rasym02_{r,c',t}}{\sum_{c'} \varphi_{c,c',t}}$$

where

$$rasym02_{r,c,t} = 1 + \frac{ra_{c,t} - 1}{ra_{c,t} + 1}; \quad 0 < rasym02_{r,c,t} < 2$$

and

$$ra_{r,c,t} = \frac{\frac{E_{r,c,t}}{GDP_{r,t}}}{\frac{\sum_{r'} E_{r',c,t}}{\sum_{r'} GDP_{r',t}}} = \frac{\left[\begin{array}{c} \text{ratio between exports} \\ \text{of commodity and} \\ \text{GDP for country} \end{array} \right]}{\left[\begin{array}{c} \text{ratio between exports} \\ \text{of commodity and} \\ \text{GDP for world} \end{array} \right]} = \frac{\frac{E_{r,c,t}}{\sum_{r'} E_{r',c,t}}}{\frac{GDP_{r,t}}{\sum_{r'} GDP_{r',t}}} = \frac{\left[\begin{array}{c} \text{country share in} \\ \text{world exports} \\ \text{of commodity} \end{array} \right]}{\left[\begin{array}{c} \text{country share} \\ \text{in world GDP} \end{array} \right]}; \quad 0 < ra_{r,c,t} < +inf$$

For non-Comtrade sectors, the definition is the same except for that exports are replaced by value-added. Across all sectors, the required data is quite readily available. Some characteristics of the above definition of *density* and its components are worth noting:

- a. revealed advantage (*ra*) is a function of the ratio between the exports of commodity *c* and GDP (with both exports and GDP measured at constant prices), not the export share. *ra* for *c* may increase as the result of an increase in this ratio for *c* and an increase for some other commodity (to which it is linked by a positive *proximity*).
- b. *rasym02* is continuous between 0 and 2. Also in a simulation context with a limited number of sectors, it is sensitive to sector developments over time.
- c. *density* of *c* is a function of *rasym02* of all *c* (unless *proximity* is zero), including the commodity *c* itself.

In MAMS, *density* (below defined in activity space) is viewed as a measure of the capability of a country to be a high-productivity producer in different activities. In the model, capability is translated into total factor productivity (TFP); the ratio between current and base-year *densitycnt* for an activity *a* enters the definition of its efficiency (or TFP) term in a constant-elasticity relationship:

$$\alpha_{a,t} = \alpha_{a,0} \cdot \left(\frac{\text{density}_{a,t}}{\text{density}_{a,0}} \right)^{\varepsilon_{a,\text{density}}} \cdot \prod_z \left(\frac{\text{tfpdet}_{z,t}}{\text{tfpdet}_{z,0}} \right)^{\varepsilon_{a,z}}$$

where *z* = index of determinants of efficiency other than *density*; *t* = time periods (0 = base-year); $\alpha_{a,t}$ = efficiency of activity *a* in time *t*; $\text{tfpdet}_{a,z,t}$ = value for determinant *z* in time *t*; $\varepsilon_{a,z}$ = elasticity of efficiency of in activity *a* with respect to *z*. The other determinants include aggregate openness to foreign trade and public capital stocks.]]]