Solving a partial equilibrium model in a CGE framework: the case of a BMS model

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Abstract

This paper presents a new approach to decompose an integrated computable general equilibrium (CGE) model into a partial equilibrium (PE) model and a residual CGE model. This opens up the scope for richer analysis of policy relevant matters by expending the PE sub-model in selected circumstances.

Specifically, this paper outlines how to integrate a behavioural microsimulation (BMS) model within a CGE framework. It then shows how the household module can be separated from the CGE model and how the resulting PE sub-model and CGE model can be solved iteratively, so that the equilibrium is identical to that of the integrated model. The paper focuses on two challenges that arise when linking and solving two different models: how to find a convergent solution and how to ensure it is the true general equilibrium solution. This involves ensuring that databases and theory in both models are consistent and fit exactly with each other. Some cases may require the use of a slack variable to account for temporary inconsistencies between the two models.

This approach has the potential to extend the range and quality of the analysis of policy relevant issues. For example, using a microsimulation framework for the household sub-model, makes a richer analysis of the household sector transfer and tax system possible. Further work is required to investigate whether the general framework used for this specific partitioning of the structure of the model can be applied to structural partitions that relate to the other parts (eg the inter-industry part) of a CGE model.

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Computable general equilibrium (CGE) models are very useful for analysing the impacts of policy changes on the economy as a whole. Since CGE models are normally built on the basis of input-output tables, many sub-sectoral details are often lost in aggregation. For example, households and specific industry sectors. For microeconomic analysis at the sub-sectoral or sub-regional level, the most useful tool is the partial equilibrium (PE) model. Due to the different characteristics of individual sectors or regions analysed, the types of PE models vary widely, ranging from behavioural microsimulation (BMS) models with large samples of households to detailed industry-specific models with individual firms and their unique cost structures or technologies.

Researchers have tried to bring the two types of models together to overcome their respective limitations. There are two major approaches. One is the so-called top-down approach, in which the simulation results of a CGE model are transmitted as shocks to a PE model, usually to gain a sense of distributional effects. For example, a microsimulation (MS) model, is used to calculate impacts on individual households. However, any household responses in terms of consumer demands and factor supplies are not accounted for in the CGE model. Despite this limitation, this approach remains popular among researchers, mainly due to its simplicity and ease of implementation. [1]

The other approach is more sophisticated and is known as the top down-bottom up (TD-BU) approach. In addition to the top-down effects from a CGE model, this approach also takes into account the “bottom-up” responses to the initial CGE shocks of individual households from a behavioural microsimulation (BMS) model. This feedback effect enables the CGE model to account for behavioural responses from the BMS model. Typically, the CGE model receives quantitative feedback from the BMS model, while the BMS model receives price shocks from the CGE models. This approach requires iterative simulations until the solution converges.

Savard (2003) links a CGE model and a BMS model to simulate the distributional impacts of a tariff reduction on a large number of households in the Philippines (see also, Bourguignon and Savard, 2008). A sample of similar studies includes: Fredriksen et al (2007) who combine a dynamic microsimulation model of government pension expenditures and a large CGE-model of the Norwegian economy to estimate the extent to which two reforms of the

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1 Early examples can be found, for example, in Dixon et al. (1996) and Polette and Robinson (1997), who link a MS model, STINMOD, developed by NATSEM, with a CGE model, MONASH, developed by the Centre of Policy Studies, to simulate the impacts of microeconomic reforms on the income distribution of Australian households.

public pension system might improve fiscal sustainability and stimulate employment. Aaberge et al (2007) link a micro model of household labour supply with a CGE model of the Norwegian economy to analyse the impacts of population ageing. Both studies solve the models iteratively, with changes in labour supply determined by the micro model and used by the CGE, which determine changes in wage rates, cash transfers and capital income.

Savard notes, however, an important limitation: "nothing guarantee[s] a converging solution to be found; therefore it must be validated and numerically checked for the introduction of each new hypothesis" (Savard 2003).[2] Ten years later, in a survey article on MS modelling Cockburn et al. (2014) concluded similarly that the 'main shortcoming of this technique is that convergence is not guaranteed and must be verified for each simulation'.

The lack of convergence typically originates from the models having different behavioural assumptions or different data. Typically, a BMS model includes consumer demands for each household, whereas, in a CGE model, consumer demand is modelled for a single representative household. These two demand systems can be difficult to reconcile. If the two databases are not fully consistent, a convergent solution can be even more difficult to reach.[3] More importantly, Colombo (2010, p.90) notes, "data inconsistencies between the micro and the macro datasets can also affect results seriously […] one is left unable to distinguish which is the part of the resulting change that is due to feedback effects and which is the part due to data inconsistencies." This is a serious problem for policy analysis.

The issue of data inconsistency can only be resolved by adjusting the datasets. Data reconciliation actually paves the way for developing a more 'integrated' approach, in which a PE sub-model is integrated fully into a CGE model. Although the idea of integrated models is not a new one, only limited attempts have been made to build fully integrated models to date.[4] Some practical difficulties prevent this approach from being adopted widely. One of the difficulties is the problem of dimensionality: the micro dataset can be too large, making the integrated model difficult to solve. A more serious problem is the difficulty of introducing complex functional forms or discrete rules and conditions, commonly used in PE models, into a CGE framework, particularly when based on a percentage change framework. Typically, CGE models are built with smooth, 'well-behaved' functions to make them tractable. Introducing the kind of complexities that can be found in PE models can make a CGE model intractable—or just too slow to solve to be of much use in a policy environment. This is why, despite being compelling in principle, in practice, many integrated models have been decomposed into two separate models and solved iteratively.

For instance, Rutherford *et al*. (2004) integrated over 30,000 household types in a CGE model to analyse the distributional impacts of Russia's accession to the WTO. As the

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2 Savard (2003) finds that "convergence was difficult with using the 'Almost Ideal Demand System'" (footnote 8 on page 8, Savard 2003).

3 Savard and Annabi (2004) provide a checklist to help find a convergent solution with the TD-BU approach.

4 See, for example, Cogneau and Robillard, 2001; Cockburn, 2006; Cororaton and Cockburn, 2007; Cockburn *et al*. 2008.

integrated model was too large to be solved efficiently, the household module was separated from the CGE model and the two models were solved iteratively. However, theoretical inconsistencies across the two models required the use of an algorithm to achieve convergence. At each iteration, the authors recalibrated the demand parameters in the CGE model to emulate the solution obtained from the household model.

Arntz *et al* (2005) integrate a discrete choice labour supply function into a CGE model with 26 household groups. Arntz *et al* (2006) extend this model to include 3000 household types to analyse German welfare reform proposals designed to encourage labour force participation. This model proves too big to solve as a single model so the authors separate the two models and solve the system iteratively. With only one variable (aggregate labour supply) to link, the authors found that two models converged quickly.

There seems to be a dilemma. On the one hand, the integrated model is compelling, but it cannot fully incorporate complex PE models because doing so makes the integrated model intractable. On the other hand, the iterative approach does not guarantee a convergent solution, at least when both expenditure and income sides of the household sector are excised from the main model. Obtaining a valid solution depends on trial-and-error or continuous adjustments to one model to match the other—a long procedure with an uncertain outcome. The challenge is, therefore, to find a modelling approach, which (a) is flexible enough to allow complex functional forms or conditions in a GE framework; (b) is powerful enough to solve models with large datasets quickly; (c) always produces a converging, GE solution.

In fact, if a PE model can be incorporated into a CGE framework and solved as part of an integrated model, this implies that the PE model can also be solved outside the CGE model, using iterative simulations and still produce the same solution. The success of such a solution method relies on two conditions. (a) The two databases must be fully consistent with each other. (b) The two models must be structurally dependent on each other. For example, in a CGE-BMS context, the CGE model should have no household behaviour, while the BMS model should have no price formation mechanism. The missing components of one model are exclusively determined by the results of the other. In other words, the two models are not stand-alone: they dependent on each other.

The first condition ensures that the solution is not affected by any database discrepancies while the second condition ensures that the separate PE model is still a part of the CGE model. Under these conditions, if the right linking variables are used, iterative simulations can produce a convergent solution and additional calibrating algorithms, such as those used in Rutherford *et al*. (2004) are not needed.

An integrated model provides the appropriate starting point. Take an integrated model with multiple households as an example. Once individual household behaviours are fully incorporated into a CGE framework, the household module is embodied in the CGE model and the whole system can be solved normally like any CGE model. Alternatively, the CGE model can be partitioned and the household module can be solved as a separate sub-model through a set of links with the main model, in which all household behavioural functions are taken out and only the exogenous household account remains. As households are present in both the main and sub-models, the changes in the behaviour of households in the sub-model can be fully captured in the main model, as if the sub-model still operated inside the CGE model. As a result, a convergent solution should be expected and should always be identical to that from the integrated model.[5]

More importantly, if the household sub-module can be taken out of the CGE model, it can be modified in terms of individual behaviours that are not subject to the usual CGE constraints. The sub-model can be built separately as a fully-fledged BMS model with all its complexity. For example, more flexible functional forms, discrete rules and complex conditions for tax and transfer payments at the individual household or person level can be implemented. Its variables and equations can also be written in any form, linear or nonlinear, not necessarily in the same form as the main model. However, all these modifications use a database that is fully consistent with the aggregate database used for the main model. Database and theory consistency is important to guarantee a convergent and undistorted solution.

This paper will use an integrated CGE model with a large sample of Australian households as an example to illustrate how this approach is implemented. The paper is organised as follows.

1. It uses a simple-structured CGE model and an Australian household survey dataset to build an integrated CGE model. Although this model includes a large number of households, it contains 'well-behaved' functional forms and is solved using conventional CGE solution techniques.

2. The household module is taken out of the integrated model to be used as a separate sub-model. The remainder of the integrated model is reduced to a conventional CGE model with one representative or aggregate household without behaviour. The household BMS model is then solved iteratively with the CGE model to test if a convergent solution can be found and, more importantly, if the solution is identical to that from the integrated model, a true GE solution.

3. The same database is used to build a new BMS model, in which more complex behavioural assumptions, as well as discrete real rules and conditions for income tax and transfer payments, are introduced. The new BMS model is solved iteratively with the CGE model so as to guarantee a convergent, GE solution.

The paper focusses on the solution process: how and under what conditions convergence is achieved, instead of the actual simulation results. For GEMPACK users, a new and simple approach to linking and iteratively solving separate models is introduced in the appendix. The paper concludes with some comments on the possible applications of the ideas behind the modelling approach to applications other than those requiring a more detailed treatment of the household sector.

5 Böhringer and Rutherford (2006) propose a similar approach, which decomposes an energy model from an integrated MCP (Mixed Complementarity Programming) model. This approach "puts strong emphasis on overall economic consistency and therefore makes use of a single integrated modelling framework in order to 'hark-link' bottom-up and top-down features". (Böhringer and Rutherford, 2006, p.1)

A simple CGE model integrated with a large sample of households

This section briefly outlines the database and the core equation system of a CGE model of the Australian economy integrated with a large sample of households.

The model database draws data from two sources: a 104-industry input-output table and household survey data for 9,774 sample households. The method used to reconcile the two datasets, is detailed in the appendix.

The model consists of 38 equations, which are essential for the model's general equilibrium solution.[6] A full list of the core equations is provided in appendix table A3. This section highlights only some key components.

Main model

The model specifies the behaviours of three types of economic agents:

- A representative firm in each industry, which purchases intermediate inputs and factor services to produce goods or services. For given demand for its products and input prices, the firm uses constant-return-to-scale (CRTS) technology to minimise the costs of production.
- A household sector, which plays three roles: as factor owner, it receives factor incomes from firms; as consumer, it spends income on goods and services to maximise its utility; and as investor, it allocates savings to form new capital.
- A government sector, which collects revenue from taxes on goods and services and from household and spends income on transfers to households and on purchasing goods and services.

The price system consists of three layers:

- Basic price = unit cost of production, including intermediate and factor input costs, plus production tax; the basic price of imported goods include import tariffs.
- Producer price = basic price plus transport and sales margin services.
- Purchaser price = producer price plus indirect taxes.

The basic structure of the model can be summarised in the behaviours of two groups of users: users of intermediate inputs (producers) and users of final products (household, government and investor).

Industrial firm behaviour has five components (refer to table A1).

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6 This list includes all aggregate macro accounting balancing conditions, but does not include variables and equations that are used exclusively for presentational purposes, such as equations aggregating quantities or averaging prices. The reporting variables can be added to the models without affecting its solution.

- *Industry demand for domestic and imported intermediate inputs*: a CES cost-minimising demand function of two variables – the given purchasers' prices and the industry demand for the composite good (eq. 3).
- *Industry demand for composite intermediate inputs*: a Leontief cost-minimising demand function of a given industry's output (eq. 16).
- *Industry demand for factor inputs*: a CES cost-minimising demand function of two variables – factor basic prices and the industry output (eqs. 18-21).

The input-output table used for this model is a *product-industry* table, where some industries produce multiple products. To model this production structure, a CET supply of multiple products is introduced in each industry (eq. 26), which is a function of the industry's total output, $\boldsymbol{X_{(i)}}$, and the basic prices of the products produced, $\boldsymbol{P_{(c,"dom")}}$.. The general equilibrium output of each industry $\boldsymbol{X_{(i)}}$ is determined in eq. 27 by linking the associated CET price index with the basic price of the industry $P^{ind}_{(i)}$. The general equilibrium basic price of each product, $\boldsymbol{P_{(c,"dom")}}$, is determined in eq. 29 by its market clearing condition.

Final users' behaviours (household, government and investor) have three components.

- *Final user's demand for domestic and imported goods*: a CES cost-minimising demand function of two variables – the given purchasers' prices and final user's demand for the composite good (eq. 3).
- *Final user's demand for composite goods*: the functional form varies depending on the final user concerned. For investor in an industry, it is a Leontief cost-minimising demand function of capital formation. For government, its demand is assumed to be exogenously given. For households, it is an aggregation of individual household demands to be defined in the household module below (eq. 16)
- *Final user's income and expenditure*: as indicated earlier in the model data section, three final users all have their expenditures fully paid for by their respective incomes, and, therefore, all have balanced budgets (eq. 13).

As government and inventory demands are exogenously given, two budget constraint equations are needed (eq. 17). Accordingly, government saving rate $\boldsymbol{r^{gsav}}$ and household saving rate for inventory $\boldsymbol{r^{hsav}_{("stk")}}$ are set as endogenous to balance their respective budgets.

The foreign demand for exported goods is defined in a constant elasticity of demand function (eq. 9).

Household module

There are two components in the household module. On the income side, each person's incomes from various sources within a household are aggregated. Four types of household personal income are defined.

- *Factor income*, composed of labour income from different occupation categories and from capital (eq. 30).

- *Taxable income*, composed of factor income plus various benefits received from the government and transfer income from other households, dirved from the HES-SIH datasets (eq. 31).

- *Disposable income*, defined as taxable income, net of income tax (eq. 32).

- *Expenditure income*, referring to the part of household disposable income that is used for current consumption (eq. 33). The remainder is household saving (eq. 34).

On the expenditure side, household demands for composite goods are derived from a Cobb-Douglas utility function, which is a function of household income and goods purchaser prices (eq. 35).

Household endowments of factors can be aggregated and linked to the variables of factor supplies in the main model (eqs. 36-40). The household supplies of factors are assumed to be fixed in the integrated model. This assumption will be relaxed in a separated BMS model to be introduced later in this paper.

In this model, all endogenous variables are uniquely defined by a single equation, except three sets of *undefined* variables, which are not defined by any equation.

- Industry output $X_{(i)}$ and product basic price $P_{(c,"dom")}$.
- Government saving rate r^{gsav} and household saving rate for inventory $r^{hsav}_{("stk")}$.
- Factor basic prices: labour (by occupation) $P^{lab}_{\_i(o)}$ and capital $P^{fac}_{\_i("cap")}$

The determination of the first two sets of the *undefined* variables has already been discussed above. The basic prices of labour and capital are determined by their respective market clearing conditions in eqs. 24 and 25.

As a result, the model has equal numbers of endogenous variables and equations, implying a valid closure and a unique solution.

Taking the household model out and solving it as a BMS model iteratively in a CGE framework

Once the integrated model is completed, the household module can be solved as an integral component of the CGE model. Even with a simple household module, which includes a behavioural responses only on the expenditure side, the integrated model still is a useful tool for household income distribution analysis. It can readily produce distributional impacts from a policy simulation. Once the simulation is completed, the results for individual households can be processed to suit different needs. For example, household results can be grouped into different categories according to their income levels, or along any of the characteristics recorded in the survey datasets. Such information is particularly valuable for assessing policy implications for specific groups.

This section focuses on an alternative way of solving the household module. It shows that the household module can be taken out of the CGE model, and solved as a external component of the same CGE framework by exchanging information between the two models iteratively. The household module actually becomes independent, and can be used independently like any BMS model for PE analysis or policy simulation.

There is, however, an important difference between the conventional CGE solution method and the alternative solution method. In the former, the interactions between the components of a CGE model are simultaneous. In the latter, it becomes possible to view the converging process, which gives users control over the solution process. In particular, the iteration process can be stopped at each step to analyse intermediate results. Users can diagnose problems (eg lack of convergence), take measures to rectify them , and restart the process. This is particularly useful when linking sophisticated BMS models with, for example, discrete behavioural functions or rules that may prove difficult to solve.

Separating the household module from the CGE model

The household module is included in a separate section of the integrated model in table A3 (equations 30 to 40). It is therefore easy to take it out of the CGE model system to be redefined as a separate BMS model (table 1). The module consists of two sections: one is used to define various types of household income and the household demands for goods and services. The other calculates a set of aggregate variables to be used in linking with their counterparts in the CGE model. In this implementation, factor supplies, income tax rates and real transfer payments are all assumed to be fixed to compare solutions between the integrated approach and the iterative approach. These assumptions will be relaxed in the following section where the household labour supply and more realistic income tax rules are introduced in the BMS model.

Table 1 **Core equations for individual households in the BMS model**

5. Household income and expenditure (30-35)

(30) Factor income from factor f for person n in household h

$$Y^{hfac}_{(f,h,n)} = \begin{cases} \sum_o \mathbf{P}^{lab}_{\_i(o)} \mathbf{X}^{hlab}_{(o,h,n)} & (f{=}lab;\ h\in\text{HOU};\ n\in\text{PER}) \\ \mathbf{P}^{fac}_{\_i(f)} \mathbf{X}^{hcap}_{(h,n)} & (f{=}cap;\ h\in\text{HOU};\ n\in\text{PER}) \end{cases}$$

(31) Taxable income for person n in household h

$$Y^{htinc}_{(h,n)} = \sum_f Y^{hfac}_{(f,h,n)} + \sum_{b\in B23} \mathbf{P}^{t}_{\_c("hou")} \mathbf{X}^{hben}_{(b,h,n)} \qquad (h\in\text{HOU};\ n\in\text{PER})$$

(32) Disposable income for person n in household h

$$Y^{hdinc}_{(h,n)} = Y^{htinc}_{(h,n)} (1 - \mathbf{t}^{hou}_{(h,n)}) \qquad (h\in\text{HOU};\ n\in\text{PER})$$

where $\mathbf{t}^{hou}_{(h,n)}$ is the rate of a tax on the taxable income of person n in household h.

(33) Expenditure income for household h

$$Y^{heinc}_{(h)} = \sum_n Y^{hdinc}_{(h,n)} (1 - s^{hou}_{(h)}) \qquad (h\in\text{HOU})$$

where $s^{hou}_{(h)}$ is the rate of household saving, an exogenous variable.

(34) Changes in saving rate for household h

$$\frac{d(s^{hou}_{(h)})}{s^{hou}_{(h)}} = \frac{d(\mathbf{s}^{hou}_{\_s})}{\mathbf{s}^{hou}_{\_s}} \qquad (h\in\text{HOU})$$

(35) Household h demand for composite good c

$$Q^{hexp}_{(c,h)} = f\,(Y^{heinc}_{(h)}, \mathbf{P}^{t2}_{\_s(c,"hou")}) \qquad (c\in\text{COM};\ h\in\text{HOU})$$

6. Linking household variables with CGE aggregates (36-40)

(36) Aggregation of household supplies of capital stock

$$X^{cap} = \sum_h \sum_n \mathbf{X}^{hcap}_{(h,n)}$$

(37) Aggregation of household supplies of labour occupation o

$$X^{lab}_{(o)} = \sum_h \sum_n \mathbf{X}^{hlab}_{(o,h,n)} \qquad (o\in\text{OCC})$$

(38) Aggregation of household demands for composite good c

$$Q_{\_s(c,"hou")} = \sum_h Q^{hcom}_{(c,h)} \qquad (c\in\text{COM})$$

(39) Average household income tax rate

$$t^{hou} = \frac{\sum_h \sum_n Y^{htinc}_{(h,n)} \mathbf{t}^{hou}_{(h,n)}}{\sum_h \sum_n Y^{htinc}_{(h,n)}}$$

(40) Aggregation of household real benefits

$$X^{ben} = \sum_{b\in B23} \sum_h \sum_n \mathbf{X}^{hben}_{(b,h,n)}$$

Source: table A3.

Initially, the household saving rates, $s^{hou}_{(h)}$, are determined by the household survey data. It is assumed that in the iterative simulations, they are assumed to follow the changes in the saving rate of the aggregate household in the CGE model, where it is treated as a fixed proportion of household income.

With the household module taken out, the CGE model has been reduced to a conventional model with a single household, without behaviour, with income and expenditure defined in the four equations shown in table 2. The behaviour of the aggregate household is determined as an aggregation of the results from the BMS model.

Table 2 **Linking variables for the aggregate household in the CGE model**

2. Household incomes and demands for composite goods (11-16)

(11) Household factor income

$$Y^{fac} = \sum_{o} \boldsymbol{P}^{\boldsymbol{lab}}_{\boldsymbol{\_i(o)}} \boldsymbol{X}^{\boldsymbol{lab}}_{\boldsymbol{(o)}} + P^{fac}_{\_i("cap")} \boldsymbol{X}^{\boldsymbol{cap}} (1 - \boldsymbol{r}^{\boldsymbol{ksav}})$$

(12) Household disposable income

$$Y^{hou} = (Y^{fac} + P^{t}_{\_c("hou")} \boldsymbol{X}^{\boldsymbol{ben}}) (1 - \boldsymbol{t}^{\boldsymbol{hou}})$$

(13-1) Household expenditure income

$$E_{("hou")} = Y^{hou} (1 - s^{hou}_{\_s})$$

(16-2) Household demand for composite good c

$$Q_{\_s(c,"hou")} = \boldsymbol{Q}^{\boldsymbol{hcom}}_{\boldsymbol{\_h(c)}}$$

Source: table A3.

The household model, defined in table 1, takes changes in all prices and in the average saving rate from the CGE model – these variables are exogenous from the point of view of the household model. The aggregate household in the CGE model takes aggregated changes in all quantities and changes in the average income tax rate from the household model as exogenous shocks. The two models are linked between the variables in the budgets of two aggregate households (figure 1).

In the two databases, both aggregate household budgets are initially balanced, since the data are fully compatible: disposable income, net of savings, equals the expenditure. Disposable income is equal to factor income (WL) plus transfer payments (b), net of income tax (t). When the household module is separated from the CGE model, the budget constraint for the aggregate household in the CGE model is broken because its income and expenditure sides are separately determined by shocks from the household model. In iterative simulations, delayed responses of the two models to the shocks from their counterparts results in temporary discrepancies in the household budgets. To monitor the discrepancies in aggregate household accounts between the two models, a slack variable H needs to be introduced in the CGE model,

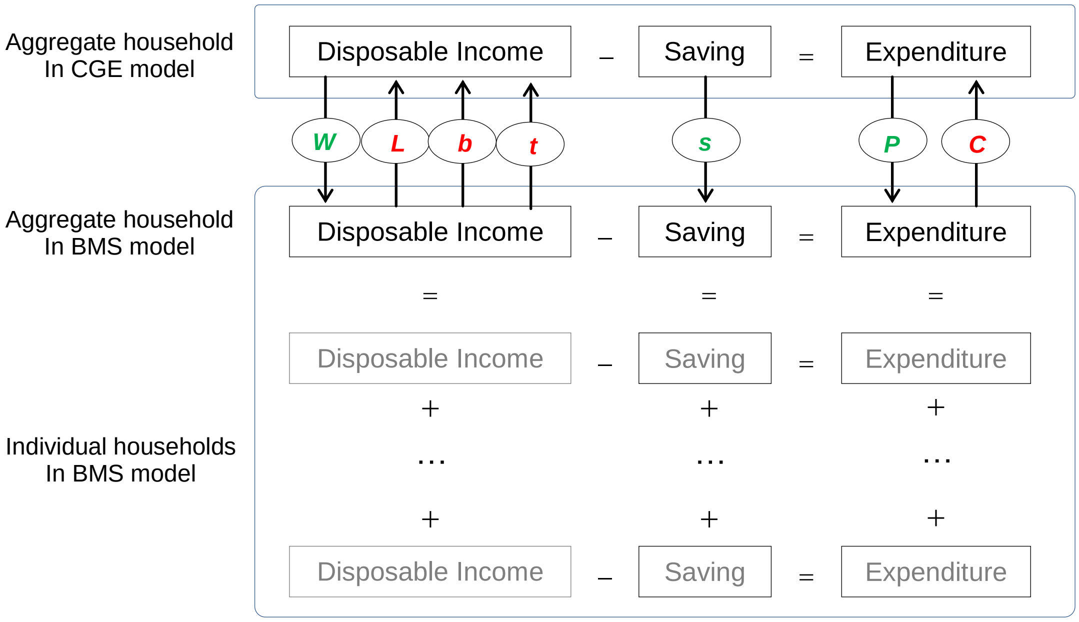
$$H = E_{("hou")} - P^{t}_{(c,"hou")} \boldsymbol{Q}^{\boldsymbol{hcom}}_{\boldsymbol{\_h(c)}}$$

This slack variable accounts for any differences between income and expenditure. It plays two important roles in iterative simulations.

- It can be used to monitor convergence and verify accuracy. A diminishing value of the slack variable indicates a converging process. Once convergence is reached, the household budgets in the two models should be equal, so that the general equilibrium value of the slack variable should be zero.

- An increasing value of the slack variable is a sign of divergence in the two models solutions. In this case, the slack variable can be swapped with the saving rate in the CGE model. The slack variable is set as exogenous to fix the budget balance, while the saving rate is set as endogenous to capture the budget discrepancy to be transmitted to the BMS model to keep its household budget in line with that in the CGE model.

Figure 1 **Variables linking the BMS and the CGE models**



Some models are "well behaved", in that they readily converge to a solution through iterative simulations without the need to force budget realignment with a slack variable. Many factors affect a model's behaviour in iterative simulations; these include assumptions about household behaviour, supply and demand elasticity values, database structures and model closures. As shown later, if two linked models are not well-behaved, the slack variable becomes crucial in bring the two models to a convergent solution.

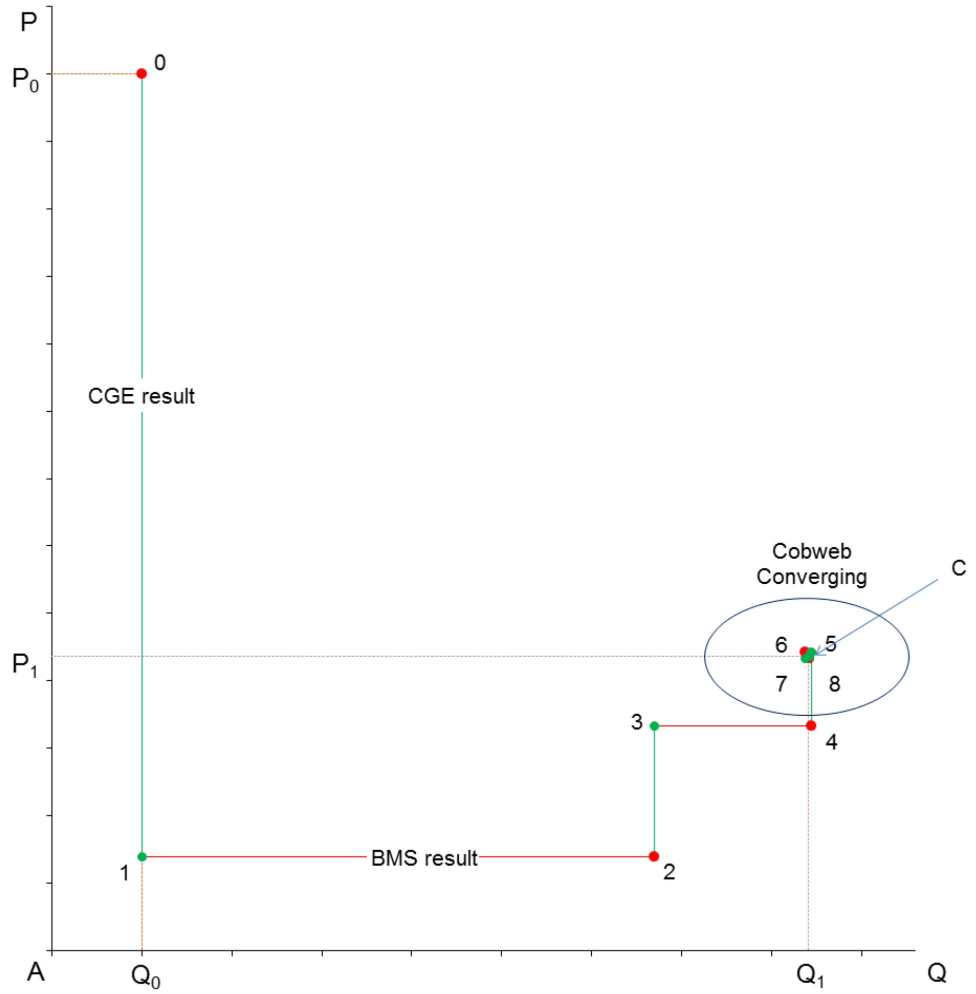
Solving the household model in a CGE framework iteratively

Although linked with the CGE model in both database and theory, the BMS model is an external component of the CGE model. The two linked models can be solved iteratively to produce a unique solution. No additional slack variable is used in the CGE model. The household saving rate is used as a slack variable and defined as endogenous; the changes in the saving rate are transmitted to the BMS model, ensuring that expenditure and income in the BMS are the same as projected in the CGE model.

A test simulation is conducted by initiating a shock from the CGE model to the BMS model. First, a 10 per cent reduction in import tariffs is imposed in the CGE model and the responses in goods and factor prices of the CGE model are used as the shocks to the BMS model. The responses in household consumptions of the BMS model are then used as feedback to shock the CGE model, together with the original tariff shocks. This iterative process is repeated until the solutions converge.

The experiment shows that it takes only five iterations [7] for the two models to converge. The convergence process in the goods market is plotted in figure 2a. The initial household expenditure is at point 0, which is the same for both models. As household demands for all composite goods are fixed exogenously in the CGE model, the reduction in the import tariffs changes only prices to point 1. As household demand is assumed not to respond to the tariff changes, the fall in goods prices is more severe than would otherwise be the case. Compared with point 0, point 1 also represents a large fall in household expenditure.

Figure 2a **Converging process in goods market: import tariff shocks from CGE model to BMS model**



The decreases in the prices of composite goods are then used as shocks to the BMS model, resulting in an increase in the household demand for composite goods to point 2. This demand increase is attributable to the fall in goods prices. But household income is also reduced because transfer payments are indexed to the consumer price index (CPI). When

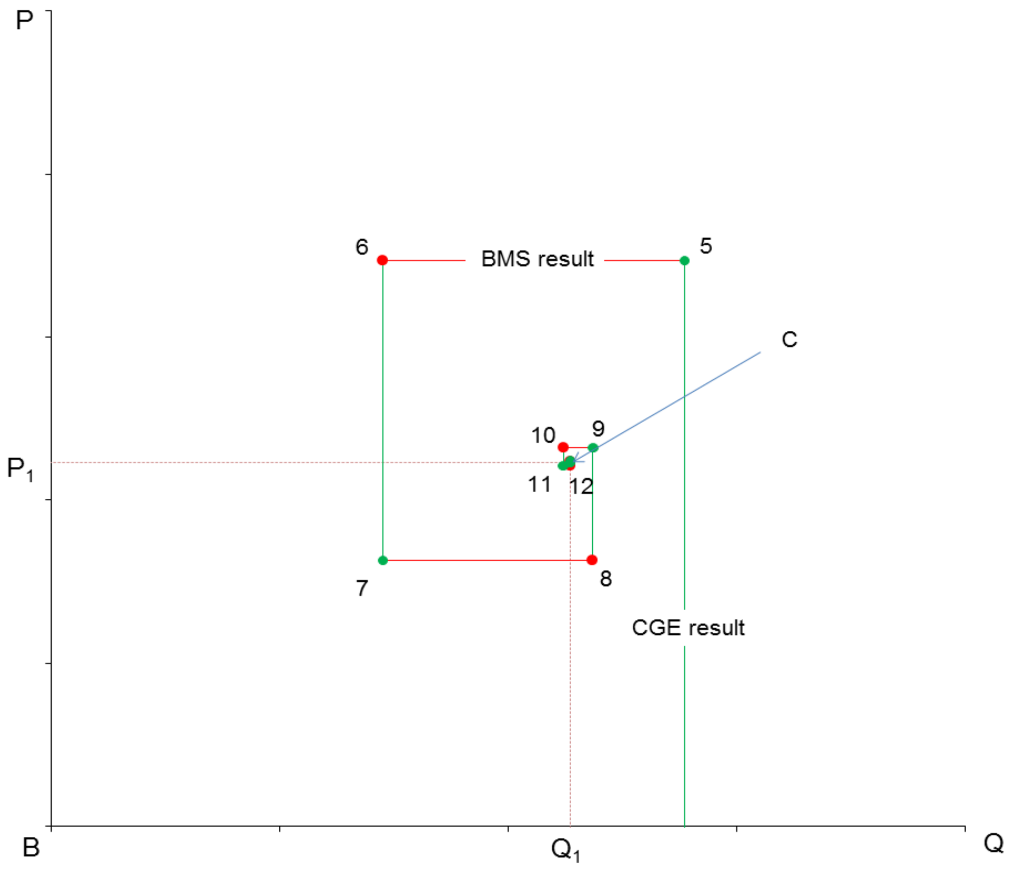
[7] One round of iteration includes one simulation from each model.

goods prices fall, household incomes also fall, offsetting the positive effects of the price fall on household demands.

In the second iteration, the increases in household demands from the BMS model are used to shock the CGE model. With this demand increase, the same tariff shocks result in a less severe fall in prices than in the first round, as shown at point 3. Due to the indexation of transfer payments, household income is also higher than in the previous iteration. With these price changes, household in the BMS model respond by increasing their demand to point 4.

After the first two iterations, the two models enter a cobweb-type of interactions (figure 2b, an enlarged display of the circled area in figure 2a). After a few more iterations, the two models converge to an equilibrium solution at point C. This is also the final equilibrium solution for the two models.

Figure 2b **Cobweb converging toward equilibrium in goods market: import tariff shocks**



The two figures show that convergence is quick and smooth, implying that both models are well-behaved.

This converging process has two phases. In the first phase, the process follows one direction, while in the second phase, it follows a cobweb pattern. The first phase involves some realignments of household incomes between the two models. Once the income discrepancies, created by the lagged adjustments between the two models, are reduced to a relatively low level, the cobweb phase begins. The phase of income realignment is crucial for the cobweb convergence. In the above example, the income realignment is achieved without the introduction of a slack variable because the saving rate in the household database small (only 2.2 per cent according to the survey dataset). This implies that in both models, household expenditures account for almost the entire disposable income so that it is not necessary for the changes in the low saving rate to be transmitted back to the BMS model. This is because the price feedback from the CGE model captures all the changes required for the BMS model to produce a consistent response, including the required adjustments in household income. This is because, with a low saving rate, changes in prices determine changes in expenditure and changes in disposable income that are of similar order of magnitude. As a result, the income realignment between two models can be achieved through the CGE price changes alone, and a slack variable is not required.

To verify whether this convergent solution is a true GE solution,[8] the same simulation is conducted with the integrated model. The results from the iterative model are found to be identical to those from the integrated model. This confirms that the iterative approach can be used as an alternative solution method to the integrated approach.

An advantage of the iterative approach is that it breaks the solution process into a number of small steps. Each step is an intermediate step toward the final general equilibrium solution. This approach makes the process of solving a CGE model visible: users can actually see how a solution is reached or what causes a solution to collapse. The latter is especially important for solving sophisticated PE models that might embody unconventional assumptions. These assumptions or functional forms may result in unusual behaviour for the aggregated economic agents in the CGE model. The iterative simulation method can help researchers to assess whether such complex behavioural assumptions are warranted and consistent with other parts of the framework, or whether modifications are required to reach a solution.

There are three possible outcomes in the iterative simulation of the two linked models: convergence, divergence and oscillation. Convergence is the desired outcome, consistent with the two models running as if they were a fully integrated model. Oscillation and divergence occur when the BMS model is not well-behaved. This is especially the case, when conditional or discrete rules or functions are introduced, in which case, corner solutions or regime-switches can occur, resulting in oscillation. Solution can then swing between two possibilities. In most cases, this is driven by model theory, which might need to be reconsidered.

Divergence is also a possible outcome. This may be caused by inconsistency in theory, such as contradictory or inconsistent assumptions. Although divergence is an indication that the models are not well-behaved in iterative simulations, it does not necessarily mean that the models are theoretically unsound. Divergence could happen with perfectly normal models. As shown later in the paper, if, supply and demand curves in a market happen to take a particular shape, iterative simulations could lead to a diverging cobweb. Divergence is not necessarily an indication of infeasible or no solution. Many cases of divergence can be

8 As Colombo has mentioned, such a convergent solution may not be free from distortions (2010).

overcome by correcting structural inconsistencies or account imbalances between the two models. More on these issues will be discussed later in this paper.

Divergence is a problem, but it may be solved once the inconsistencies in the model theories are located and rectified. From a policy perspective, oscillation may or may not be a problem. If the model's assumptions and theories are correct, the oscillation indicates that several outcomes are possible. Sometimes, this may shed useful light on our understanding of the true impacts of a policy change.

Such an insight cannot be gained if a conventional solution method for CGE models is used, because the simulation will collapse and produce no solution at all.

Building and solving a more flexible PE model with the same database

The separation of one sector from a CGE model allows for more flexible structures to be introduced in this sector. It is even possible to build an entirely new model on the same database. This is because, once the sub-model is separated from a CGE model, it becomes free from CGE model constraints. Complex functional forms, not acceptable in a CGE model, can be introduced in the sub-model. The only condition for this sub-model to be solved in the original CGE model's framework is the necessity for the databases to be fully consistent. No matter how the structure and the behavioural assumption of the sub-model are modified, they have to be calibrated back to the same database. This database ensures the sub-model is can still be solved in the CGE framework.

To introduce more sophisticated behaviours for economic agents and more realistic policy settings, the sub-model might be written in a format that differs from that of the CGE model. For example, the sub-model can be written in non-linear form, or levels, instead of in percentage change form.

In this section, the household module is replaced by a more flexible BMS model, with flexible labour supply behaviour and a complex income tax schedule. This is the type of model that might be required to analyse a change in tax policies.

Two examples of a flexible BMS model

In the remainder of this section, two examples are used to illustrate how flexible functions are introduced in a BMS model and the model can still be solved iteratively in a CGE framework.

Experiment 1: introducing household labour supply

In the BMS model database, some households have persons who are employed and receive labour income. In the household survey, employment status is classified as "employed", "unemployed" and "not in the labour force". The employed are further classified as "full-time employed" or "part-time employed". For each employed person, there is information on occupation (eight categories), weekly working hours (up to 50 hours) and annual labour income. This rich information in the household database allows for labour supply behaviour to be defined at an individual level as a function of the reported hourly wage rate as well as policy-related factors that affect disposable labour income, such as income tax rates and various transfer payments.

In a simple form, the supply of occupation *o* by person *n* in household *h* can be defined as a function of the market wage rate, net of a personal income tax plus any transfer payments the person may have received, as follows,

$$X^{lab}_{(o,h,n)} = \alpha^{lab}_{(o,h,n)} \, (P^{tlab}_{(o,h,n)})^{\varepsilon_{(o)}}$$

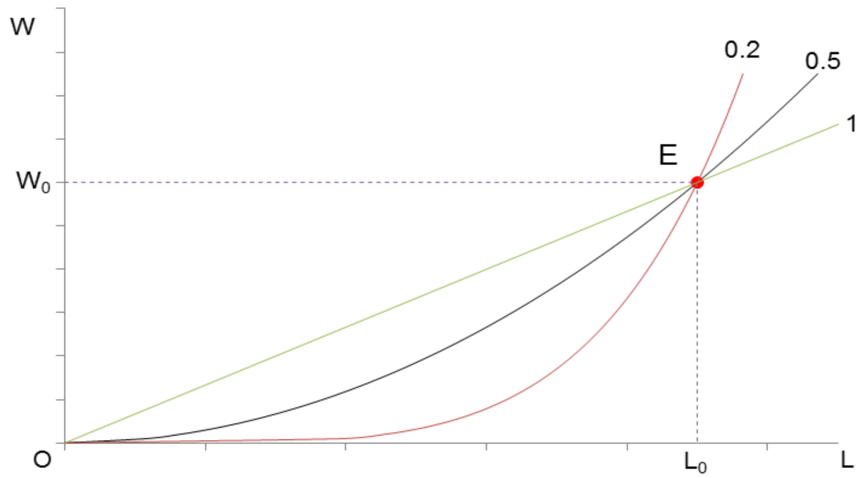
where $P^{tlab}_{(o,h,n)}$ is the post-tax wage rate,

$$P^{tlab}_{(o,h,n)} = P^{lab}_{(o,h,n)} \, (1 - t^{lab}_{(h,n)}) \, (1 + \sum_{g} r^{ben}_{(b,h,n)})$$

and, ε is the elasticity of labour supply with respect to wage changes and $\alpha^{lab}_{(o,h,n)}$ is the average wage per hour.

This labour supply behaviour can be illustrated in figure 3. Point E shows equilibrium weekly hours in a particular occupation and the hourly post-tax wage rate, for an individual. The curvature of the supply curve is determined by the parameter ε. When ε = 1, the labour supply is linear, proportionate to wage changes. Lower the parameter values produce more curved, labour supplies is curved, implying less responsiveness of labour supply to wage changes around E. There is also a maximum number of working hours allowed in each week. Once reached that limit, the supply curve becomes vertical, implying no more response in labour supply as the wage increases.

Figure 3 **Labour supply curves**

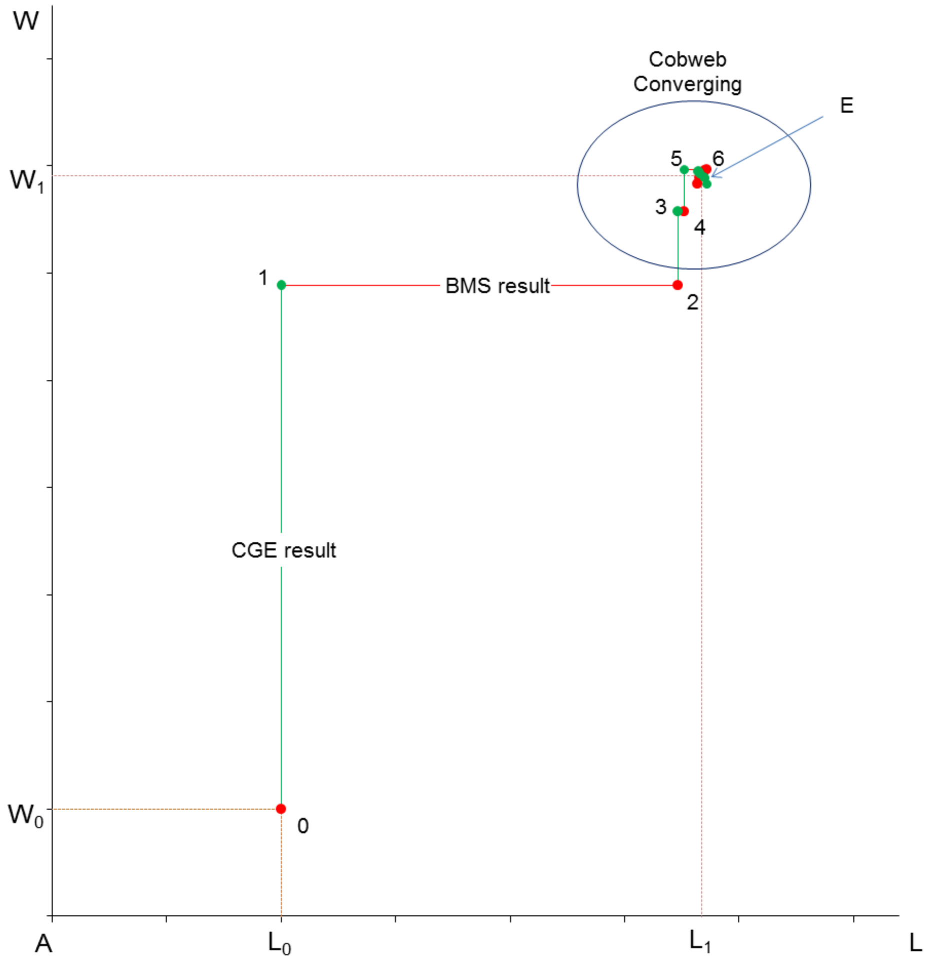


This labour supply responds positively to price signals. However, unlike the functions in a CGE model, this function is not continuous: it is truncated at a specified maximum limit, 50 hours per week or classified as “fully employed” in the survey dataset. For those fully employed persons who have already reached the maximum working hours, they will be unable to respond to further wage increases. Even for those who are close to full employment, a slight rise in wage rate may induce them to hit the limit. Such possibilities may cause the model not to behave well under certain conditions. The iterative solution method is flexible enough to handle such a constraint on behaviour in the BMS model.

In the first test simulation, a 10 per cent cut in the tariff rates for imports is introduced again. This time, the labour supplies of individual persons follow the behaviour specified above in the labour supply function. The labour supply elasticity is set as 0.5 for all occupations. Unlike the previous simulation in which labour supplies were fixed, the introduction of labour supply functions implies more flexible supply-side responses and, therefore, more rounds of iteration may be needed for the two models solutions to converge. Overall, a converged solution is reached between two models after 11 rounds of iterative simulations.

Figure 4a shows the converging process in the aggregate labour market. As before, the first three simulations realign the household budgets (figure 4a) and put the labour-wage interaction on a converging cobweb trajectory (figure 4b).

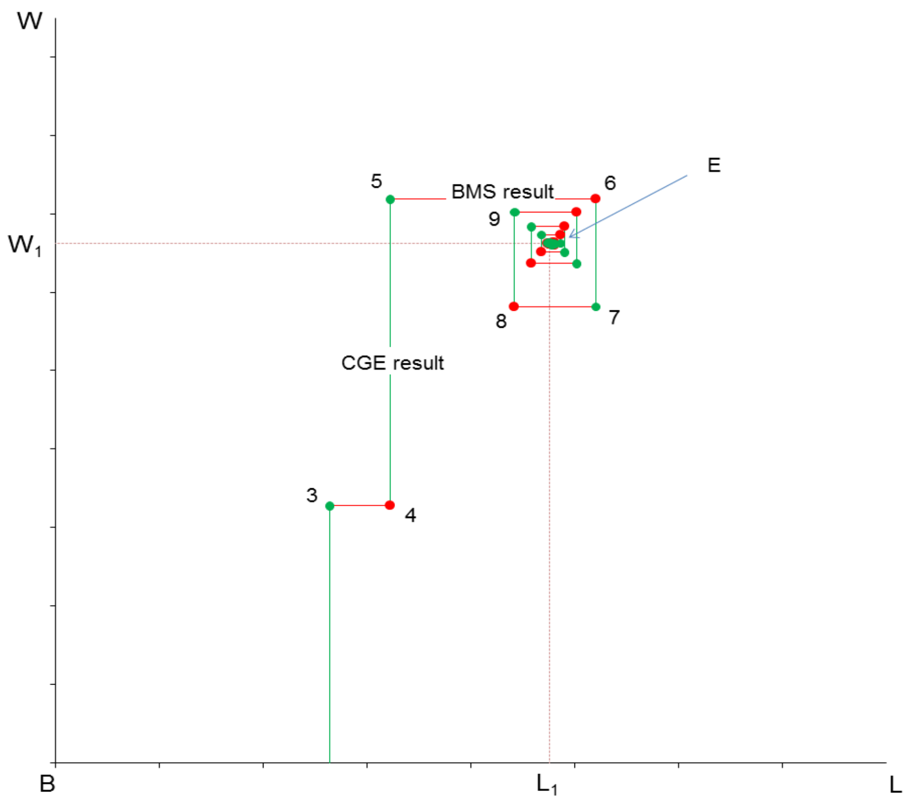
Figure 4a **Converging process in labour market: import tariff shocks from CGE model to BMS model with flexible labour supply**



Although an equilibrium appear to be reached at E, a tiny oscillation is still detectable at E. The model results show that a tiny gap exists in labour supply (0.000001%) as well as in wage rate (0.000002%). This is caused by a tiny oscillation in one of the eight occupation labour markets, “Community and service workers”. The negligible oscillation amounts to 0.000012% for this occupation and 0.00003% for the corresponding wage rate.

These results show another advantage of using the iterative solution method: it not only allows users to see whether a converging solution exists or not, but also to what extent the solution has converged. In this case, for example, the two model solutions are not perfectly matched, but it may be good enough for policy analysis, because the remaining oscillation is too tiny to be of any relevance. However, such an imperfect solution can never be produced by a conventional solution method because even a tiny oscillation is big enough for the simulation to collapse.

Figure 4b **Cobweb converging toward equilibrium in labour market: import tariff shocks with flexible labour supply**



Experiment 2: introducing marginal rates of income tax

Partial equilibrium models are particularly useful to analyse policies that affect individuals or sectors. In a BMS model, the policy changes can be simulated at the individual or household level. The impacts of these policy changes, calculated at the individual and household level, can then be aggregated into specified groups, or to regional or national levels to understand the wider impacts. The iterative approach of linking such a BMS model with a CGE model allows detailed micro analysis to be extended to a general equilibrium framework.

The household survey contains rich information on personal income taxes and various types of transfer payments at the detailed personal or household level. However, in a CGE model, many of these tax and transfer payment are difficult to specify, because the rules and conditions, on which they are based, are simply too complex and do not always comply with the requirements for solving a CGE model.

A separate BMS model, built in levels, provides the flexibility required to incorporate the complex rules and conditions of a tax and transfer system. In the following, some detailed marginal rates of income tax are used to illustrate how a such a BMS model is solved iteratively with a CGE model.

Australia has a progressive income tax system. There are five marginal tax rates, applied to different income ranges, as shown in table 3. In the household survey datasets, individual tax payers report the amount of income tax paid. These income tax payments can be recalculated using these marginal rates, so that simulation can be conducted to analyse the impact of changing income tax rates on individual persons or households as well as on the economy as a whole.

Table 3 **Progressive marginal rates of income tax and shocks**

Income from	to	tax rate	Change	New rates
0	18,200	0%		
18,201	37,000	19%	5%	0.1995
37,001	80,000	32.5%	5%	0.34125
80,001	180,000	37%	10%	0.407
180,001	above	45%	10%	0.495

Source: Australian Taxation Office (https://www.ato.gov.au/rates/individual-income-tax-rates/) and author calculations.

More specifically, the estimated income tax, paid by a person in a household, is calculated according to his taxable income $Y^{tinc}_{(h,n)}$ and the applicable marginal tax rates as follows.

$$T^{inc}_{(h,n)} = \text{If } (Y^{tinc}_{(h,n)}<18200, 0) + \text{if}(18201< Y^{tinc}_{(h,n)}<37000, (Y^{tinc}_{(h,n)}-18201)\times 0.19)$$

$$+ \text{ If } (37001<Y^{tinc}_{(h,n)}<80000, (37000-18201)\times 0.19 + (Y^{tinc}_{(h,n)}-37001)\times 0.325)$$

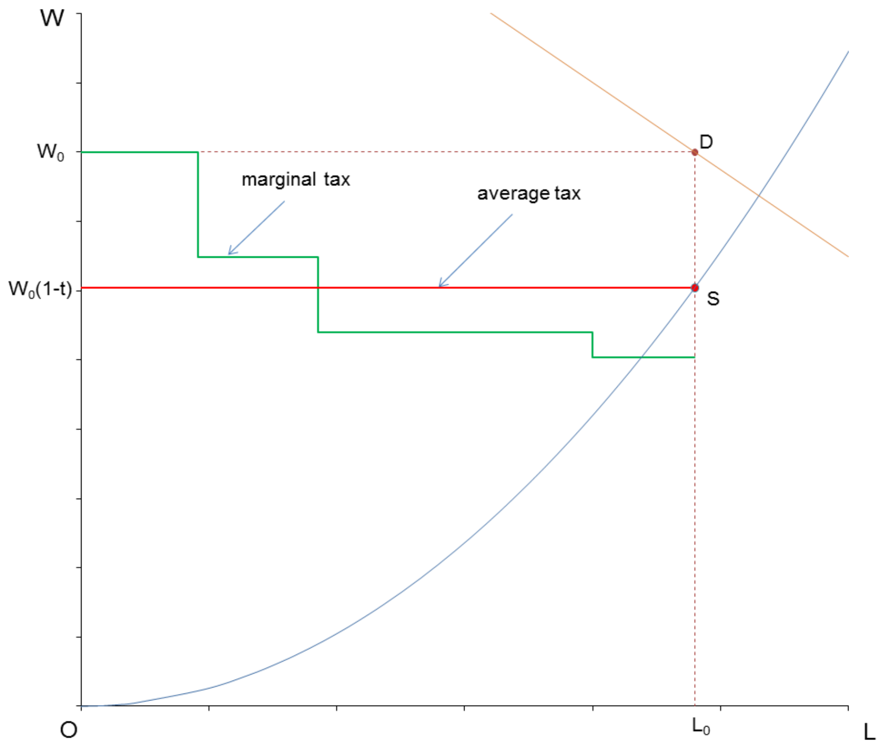
$$+ \text{ If } (80001<Y^{tinc}_{(h,n)}<180000, (37000-18201)\times 0.19+(80000-37001)\times 0.325+(Y^{tinc}_{(h,n)}-80001)\times 0.37)$$

$$+ \text{ If } (180001<Y^{tinc}_{(h,n)}, (37000-18201)\times 0.19+(80000-37001)\times 0.325+(180000-80001)\times 0.37+(Y^{tinc}_{(h,n)} -180001)\times 0.45) + R^{inc}_{(h,n)}$$

This equation shows that the observed income tax, paid by a person, is defined as the sum of the estimated tax plus a residual $R^{inc}_{(h,n)}$, which captures the difference between the observed and estimated income tax payments. The rate of this residual tax is assumed to be fixed, independent of any policy changes. As a result of the above setting, the household income tax, observed in the survey dataset, becomes a function of a policy variable, the marginal tax rates.

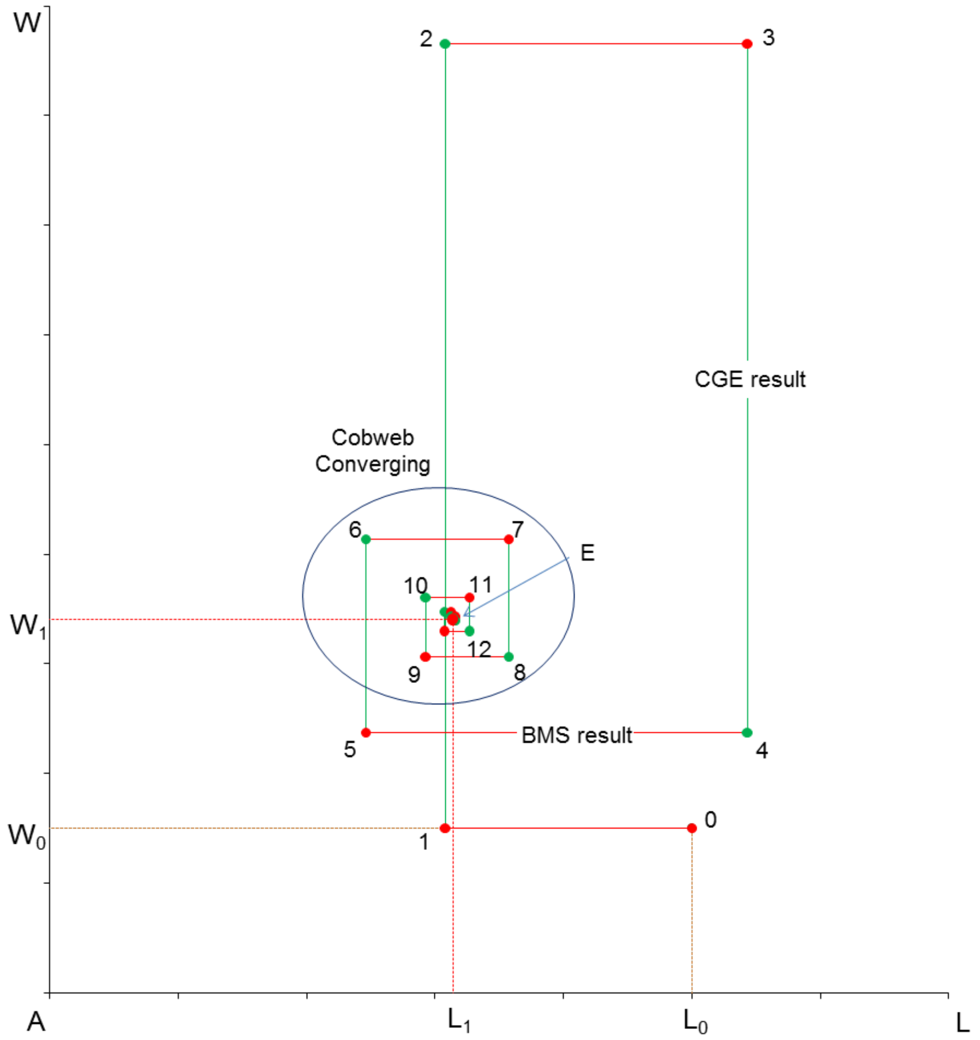
The impact of income tax on the labour supply of a particular person is illustrated in figure 5. The observed market wage rate is W_0 and the associated labour supply is L_0. This person's income tax is paid according to a given set of progressive marginal rates. This person's average after-tax wage rate is W_0 (1– t). The labour supply curve goes through point S and the labour demand curve goes through point D. The gap between the two points is the average rate of income tax (t).

Figure 5 **Equilibrium in labour market with income tax**



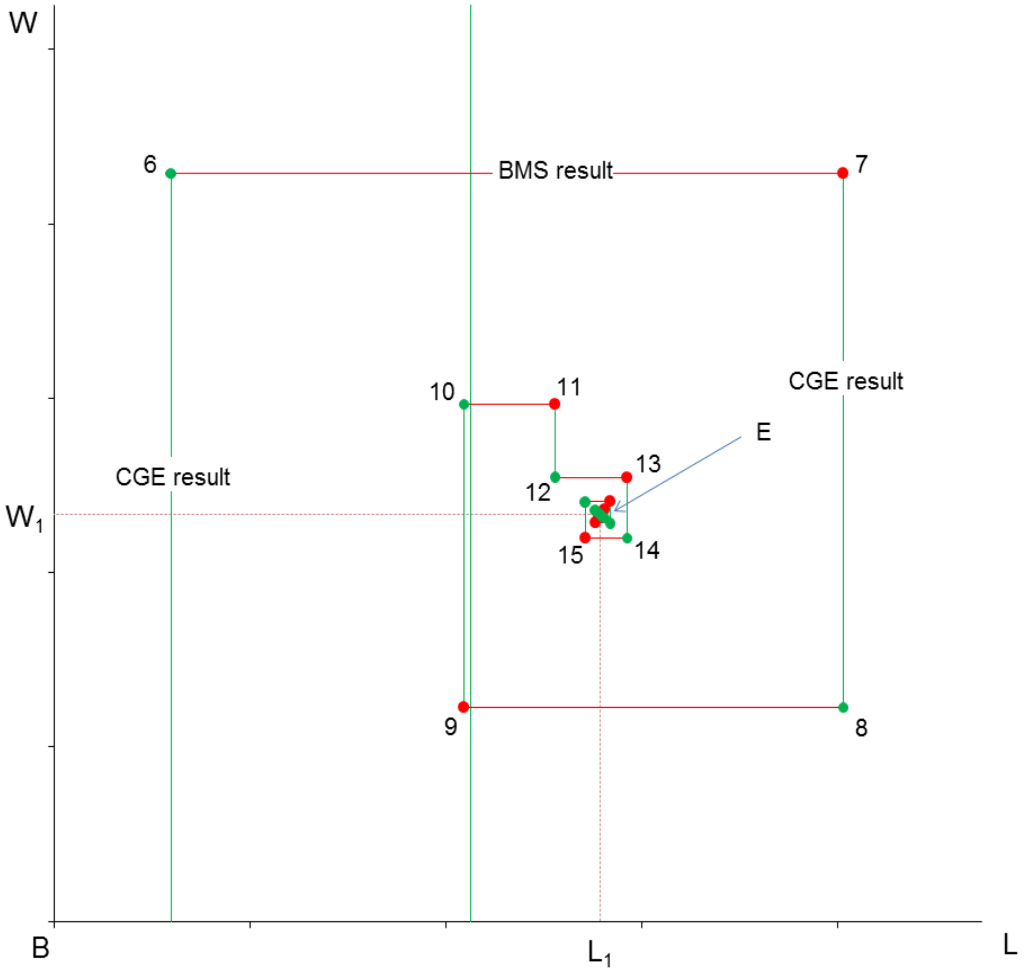
In this section, the income tax schedule is assumed to change according to the shocks that produce the new tax rates in the last column of table 3. The process of convergence in the labour market is shown in figure 6a with the enlarged final convergence in table 6b. The increase in income tax reduces the post-tax wage rate and discourages labour supply. As a result, the total labour supply moves downward along the aggregated labour supply curve. On the other hand, the increase in income tax increases the wage rate for firms and reduces their demand for labour, as shown in the figure. As before, after realigning household budgets between the two models in the first three iterations, the simulations enter the cobweb stage, converging toward equilibrium E.

Figure 6a **Converging process in labour market: marginal tax shocks from BMS model to CGE model with flexible labour supply**



The conditional functions and progressive marginal tax rates are not an obstacle to convergence. The discrete functions and complex rules are implemented at individual or household levels. When they are aggregated to the national level to be transmitted to the CGE model, the detailed discontinuities largely cancel out, so in this case, the functions in the CGE model are 'well-behaved', and the solution is easy to find.

Figure 6b **Cobweb converging toward equilibrium in labour market: marginal tax shocks with flexible labour supply**



5. Not well-behaved models: the role of a slack variable

The models discussed so far can all be described as well-behaved. This is because the models interact smoothly with each other and move quickly toward a convergent solution without the help of a slack variable. This implies that the relations between the supply and demand of both models satisfy the requirements for cobweb adjustments to converge. This appears to be an ideal situation in the application of the iterative approach. However, not all models are well-behaved. This section will present a case of a not so well-behaved model. It uses a different integrated CGE model with a different household survey dataset to illustrate how to solve problems of divergence.

The model and database are taken from an integrated CGE model of the Australian economy, described in Zhang (2015). As before, the household module is taken out of the integrated model to be used as a separate BMS model. And then, in the integrated CGE model, the household demands for composite goods are turned off and linked with the simulation outcome of the BMS model. In the BMS model, all prices of goods and factors are determined by the simulation outcome of the CGE model.

The simulation is a one-per-cent increase in the supply of the occupation ‘labourers’ by all households that report someone with this occupation in the survey dataset. The simulation is first conducted with the integrated model, using a conventional CGE model solution method.

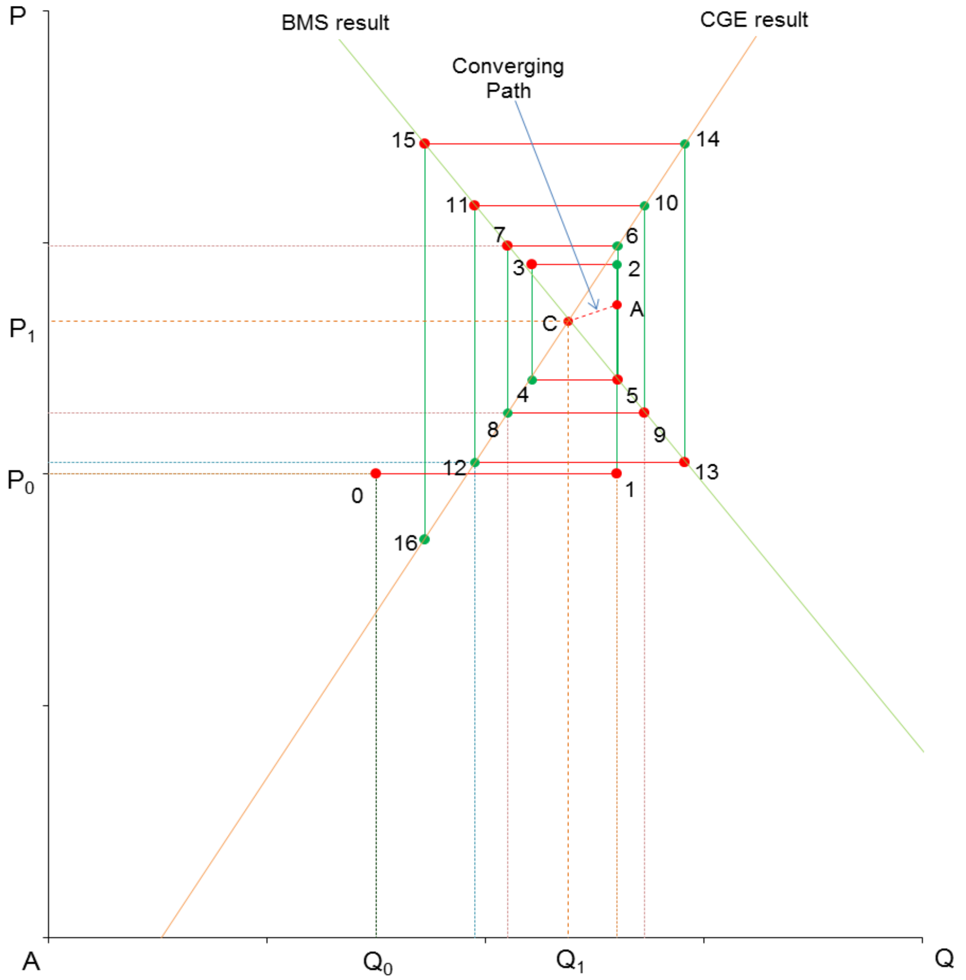
The same experiment is then conducted with the two models linked. As in the previous sections, no slack variable is used, deliberately, to show how the two models behave in the iterative process. The interactions between the two models in the aggregate goods markets, the quantitative and price responses, are plotted in figure 7a.

It is clear from the figure that the two models are not well-behaved: the initial shock causes the quantitative responses of the BMS model and the price responses of the CGE model to move away from each other. These diverging responses follow a clear cobweb pattern. These diverging responses can be explained by the particular shape of the supply and demand curves. In this case, the supply curve, derived from the responses of the CGE model, is too steep, relative to the demand curve, derived from the responses of the BMS model. It seems clear from the figure that the two models may never converge. This peculiar behaviour is attributable to a relatively large saving rate (about 30 per cent) in the household database. This implies that the price feedback from the CGE model alone cannot align the household budgets across the two models.

However, the integrated model has already proved the existence of a solution. In fact, the figure reveals the final equilibrium in the goods market, which is given by the intersection of the supply and demand curves at point C. It indicates that these two models do have a convergent solution. The question is then how the two models can be solved iteratively to reach the observed equilibrium at point C. The answer lies in the choice of a slack variable.

In this instance, the chosen slack variable is the household saving rate. In the integrated model, household saving rates are exogenous. When the equations for household demands are turned off in the integrated model and set as exogenous to be determined from outside the CGE model, a new set of equations can be introduced to balance the household budgets. To close the model, household saving rates must be set as endogenous. These saving rates are adjusted by a slack variable to account for possible discrepancies between the household budgets in the CGE model and the BMS model. Such discrepancies arise from differences between the household expenditures in the two models.

Figure 7a **Diverging responses in goods market without a slack variable: labour supply shocks from BMS model to CGE model**

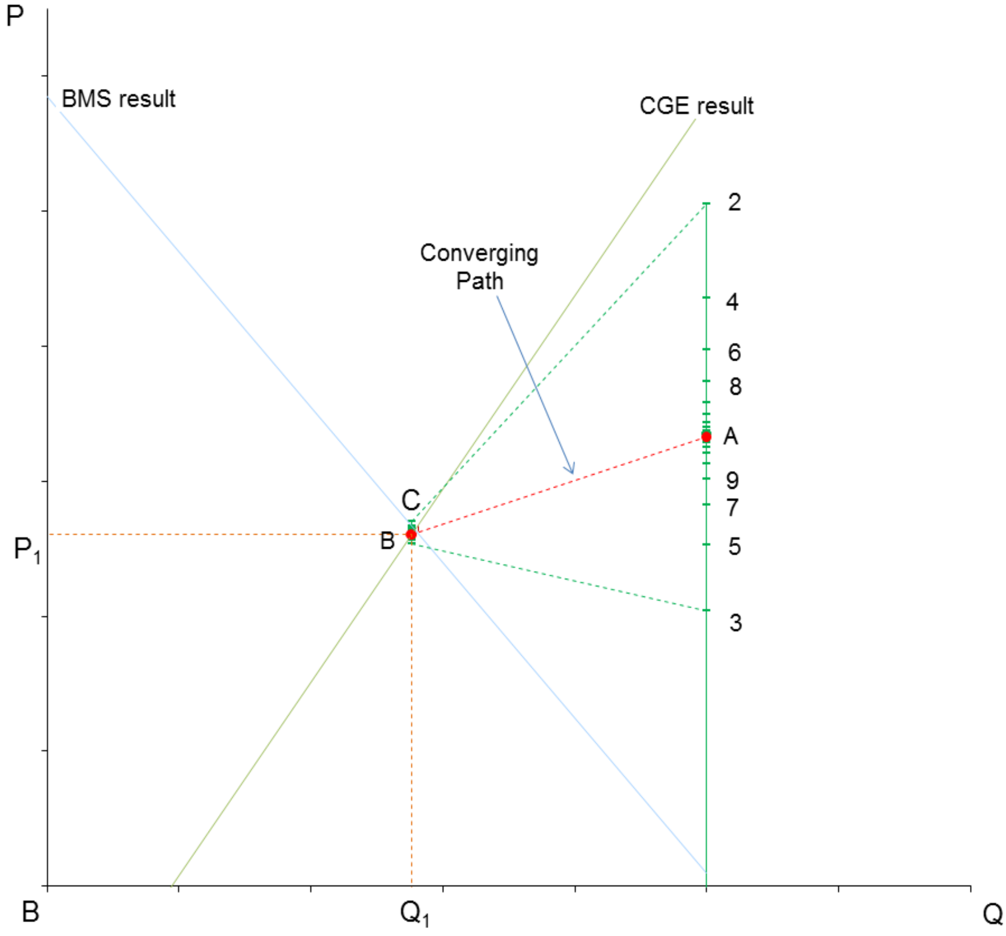


In the iteration process, endogenous changes in household saving rates are transmitted from the CGE model to the BMS model so that the household budgets in the BMS model are realigned constantly with the aggregate household budget in the CGE model to ensure the household decisions in the BMS models are made on the same income basis as in the CGE model. This income realignment prevents the response of the BMS model from following the cobweb diverging path. With a slack variable in place in the BMS model, the two models are solved iteratively again. The new results for the goods market are plotted in figure 7b.

In the first BMS simulation, with goods prices fixed, an increase in the supply of the 'labourers' occupation increases the demand for composite goods from point 0 to point 1. When this outcome is transmitted to the CGE model, the average goods price rises to point 2. Instead of moving to point 3 on the demand curve, as shown in figure 6a, the quantitative response of the BMS model is offset by the changes in the saving rate from the CGE model: it now moves from point 2 to point 3 down on the CGE price response line. In the following simulations, there will be no quantitative responses in the goods market from the BMS model: it will only move up and down along the CGE price response line until it reaches point A. At this stage, the change in the saving rate is not equal to zero, implying that the current solution is not a final one and further simulations are required.

To bring the saving rate to zero, the next step is for the BMS model to shock all household saving rates to zero and then restart the iterative simulations until a new solution is reached. The new equilibrium point will be on a path from point A, closer to the final equilibrium C. The new equilibrium saving rate is smaller than the first one. If it is still not equal to zero, the same process needs to be repeated again. This process should continue until the value of the saving rate is reduced to zero. When the goods market reaches its equilibrium at point C, a convergent solution for the two models is also reached. It can be proved that this final solution from the iterative simulations is identical to the one obtained from the integrated model. This confirms that this solution is a true general equilibrium solution.

Figure 7b **Converging path in goods market with slack variable**



This example shows how a slack variable brings the two not-well-behaved models to a converging solution. Without the slack variable adjustments in household budgets, the two models would never reach a solution through iterative simulations, even though the integrated model has shown that such a solution exits. The use of a slack variable also helps explain how the observed cobweb diverging process is formed. This is simply because, without the slack variable, household income discrepancies between the two models accumulate with each iteration, driving the solutions further apart and away from the final equilibrium.

6. Some concluding remarks

This paper has used an integrated CGE model to show how a section of the model can be separated from the integrated model and still to be solved under the same CGE framework. It has also shown, using examples, how an iterative simulation method can solve two linked models to produce a convergent solution. Compared with conventional CGE solution methods, this new approach is more flexible and provides an alternative way of introducing, through separate sub-models, complex functional forms and behavioural assumptions into a general equilibrium framework.

As these sub-models are outside the CGE model structure, the solution time for solving a sub-model and its linked CGE model can be substantially reduced. Take the first experiment of tariff reduction, presented in this paper, as an example. With an Intel Core i5 computer, it takes the GEMPACK software 3 minutes and 44 seconds to produce a one-step solution for the integrated CGE model. Under the new approach that combines the two models in one TABLO code,[9] it takes only 3 seconds to produce a one-step solution. If ten rounds of iterative simulations are required for a convergent solution, it still takes only 30 seconds to complete. The reason for such a sharp difference in solution time is because the large BMS model is treated as a data processing task, rather than as a closed system like a GE model, which means that the matrix to invert remains small and no need to store large amounts of temporary outputs in memory.

Although in this paper the solution method is applied to the household module of a CGE model, the principles that underlie the method could apply broadly. It is possible to build a PE sub-model for some other sector of a CGE model and still solve it as part of the CGE model. The household sector is typically easier to separate out than some of the industry sectors because its links can be clearly defined and are relatively straight-forward, but the principles discussed here are likely to be useful in decomposing other industries from a CGE model and solving the system iteratively. This can lead to more flexible CGE models, better adapted to addressing specific policy questions at sub-sectoral levels.[10]

Unlike the CGE model that has developed some commonly accepted structure, PE models are very diverse in data and theory. How to link a particular PE model to a CGE model and solve it successfully with the CGE model will require some effort. That said, the approach presented in this paper provides some direction to facilitate this.

[9] See appendix for an effective GEMPACK implementation.

[10] For example, it would be possible and relatively straightforward to link MARKAL models with CGE structures.

References

Aaberge R, Colombino U, Holmøy E, B Strøm and Wennemo T. (2007) 'Population ageing and fiscal sustainability: integrating detailed labour supply models with CGE models'. In Harding, A. and Gupta, A. (eds.) *Modelling Our Future: Social Security and Taxation* (Volume I), Elsevier, Amsterdam, pp.259-290.

Arntz, M., Boeters, S., Gürtzgen, N. and Schubert, S. (2006) "Analysing welfare reform in a microsimulation-AGE model: the value of disaggregation", Conference paper presented at the 8th Nordic Seminar on Microsimulation Models, Oslo, Norway, June 8-9.

Australian Bureau of Statistics, (2012) "Household Expenditure Survey and Survey of Income and Housing 2009-10", (6503.0).

Australian Bureau of Statistics, (2015) "Australian National Accounts: Input-Output Tables, 2012-13", (ABS 5209.0.55.001).

Böhringer, C., and Rutherford, T. (2006). "Combining Top-Down and Bottom-up in Energy Policy Analysis: A Decomposition Approach", ZEW Discussion Paper No. 06-007.

Bourguignon, F., and L. Savard, (2008) "Distributional effects of trade reform: an integrated macro-micro model applied to the Philippines", Chapter 6 in Bourguignon, F., M. Bussolo and L.A. Pereira da Silva, (eds.) *The Impact of Macroeconomic Policies on Poverty and Income Distribution: Macro-Micro Evaluation Techniques and Tools*, Palgrave Macmillan and the World Bank, Washington DC.

Cockburn, J. 2006, 'Trade liberalisation and poverty in Nepal: a computable general equilibrium micro simulation analysis', in Bussolo M and Round J (Eds.), *Globalization And Poverty: Channels And Policies*, London: Routledge, 171-194.

Cockburn, J., Corong, E. and Cororaton, C. 2008, 'Poverty effects of the Philippines' tariff reduction program: insights from a CGE model', *Asian Economic Journal*, 22(3), 287–317.

Cockburn, J., L. Savard and L. Tiberti, (2014) "Macro-micro models", in O'Donoghue, C. (ed.) *Handbook of Microsimulation Modelling*, (Contributions to Economic Analysis, Vol. 293), Emerald Group Publishing Ltd., Chapter 9, pp.275-304.

Cogneau, D. and Robillard, A. S. 2001, 'Growth distribution and poverty in Madagascar: learning from a microsimulation model in a general equilibrium framework', TMD Discussion Paper 61, International Fool Policy Research Institute, Washington, D.C..

Colombo, G., 2010. "Linking CGE and microsimulation models: a comparison of different approaches", *Journal of Microsimulation*, 3(1) 72-91.

Cororaton, C. and Cockburn, J. 2007, 'Trade reform and poverty-lessons from the Philippines: a CGE microsimulation analysis', *Journal of Policy Modeling*, 20(1), 141-163.

Dixon, P., M. Malakellis and T. Meagher, (1996) "A microsimulation/applied general equilibrium approach to analysing income distribution in Australia: plans and preliminary illustration". Preliminary Working Paper No. IP-67, March, Centre of Policy Studies and the Impact Project, MONASH University.

Fredriksen, D., Massey Heide, K., Holmoy, E. and Foldoy Solli, I. (2007). "Macroeconomic effects of proposed pension reforms in Norway", In Harding A and Gupta A (Eds.) *Modelling Our Future: Social Security and Taxation* (Volume I), Elsevier, Amsterdam, pp.107-142.

Polette, J., and Robinson, M. (1997) "Modelling the impact of microeconomic policy on Australian families". Discussion Paper 20, National Centre for Social and Economic Modelling, University of Canberra.

Rutherford, T. D. Tarr and O. Shepotylo (2004) "Poverty effects of Russia's WTO accession: modelling 'real' households and endogenous productivity effects", World Bank Policy Research Working Paper 3473, World Bank. Available at www.worldbank.org/trade/russia-wto.

Savard, L., (2003) "Poverty and income distribution in a CGE-household micro-simulation model: top-down/bottom up approach", Working Paper 03-43, CIRPEE, Montréal.

Savard, L., and N. Annabi, (2004) "Note on convergence problems with large CGE microsimulation models", mimeo.

Zhang, Xiao-guang, (2015) "Incorporating household survey data into a CGE model", presented at the 17th GTAP Conference, Melbourne, June 17.

Appendix

The database of an integrated model

The model is built on the basis of Australian input-output table 2012-13 (ABS, 2015) and the 2009-10 Household Expenditure Survey (HES) and Housing and Income Surveys (HIS) (ABS, 2011). The input-output table includes 104 industries and two factors of production, labour and capital. An aggregated version of this table is shown in figure A1.

Industries (aggregated in the first column) use domestically produced and imported intermediate goods in production. Domestically produced goods (the first row) are used as intermediate inputs by industries and by other users: investors by industry, a representative household and government. Some products are exported, and some can be used as transport margins, which are added to the basic price of products.

Figure A1 **Aggregated input-output table in basic prices (AUD million)**

	1 ind	2 hou	3 gov	4 inv	5 stk	6 exp	7 mgn	Total
1 dom	1159672	566012	265156	327917	-2034	270578	272793	2860094
2 imp	167734	79915	2878	63035	3387			316949
3 mgn	94733	127542	2672	20227	-87	27706		272793
4 GST	3624	37415	0	8235	0	1090		50364
5 tax	17831	25502	0	13609	652	0		57594
6 sub	-6636	-5463	0	-1154	-14	-616		-13883
7 fac	1371108							1371108
8 ptx	52028							52028
Total	2860094	830923	270706	431869	1904	298758	272793	4967047

Notes: Basic values of imports (316949) include import tariffs (3233).

Source: Compiled from *Australian National Accounts: Input-Output Tables, 2012-13* (ABS 5209.0.55.001).

As the main owner of primary factors, the household receives factor incomes. Household primary income is divided between consumption expenditure and savings. Government collects various taxes and uses this income to pay for expenditures on goods and services.

Investment is partly funded by domestic savings. The gap between investment and saving is net foreign investment (NFI) inflow, which should be equal to the country's trade deficit.

The data on household income and expenditure are extracted from the 2009-10 HES and SIH (ABS, 2011). The HES-SIH data are contained in two unit record data files: one file contains expenditure data on 9,774 sample households; the other file contains data on income sources for the 17,919 individual members of the sample households. The household data are mapped to the income sources and commodity categories of the CGE model.

On the income side, HES-SIH income items are aggregated in to 34 types of personal income items, including wage incomes from 8 labour occupations, capital income, 23 government benefits, and income taxes. In households with multiple income earners, incomes from each source are consolidated within each household.

Private income sources include wages and salaries from eight occupations, and non-wage or capital income from business and investments. Government transfer payments include family allowances, unemployment benefits and age pensions. Table A1 displays the allocation of the HES-SIH sources of income to the CGE database items.

On the expenditure side, there are 614 items recorded under the Commodity Code (ComCode) system in the HES data file. Among them, 485 can be aggregated into corresponding commodities in the input-output table. The rest of them need to be allocated to more than one commodity in the input-output table. In the absence of additional information about the composition of these commodities, the value shares of the commodities in the input-output table are used to allocate the expenditures of these items.

Table A1 **Sources of household income in CGE model and HES-SIH**

CGE model	*HES and SIH*	
Wages for eight occupations (same as those in HES-SIH)	Managers and administrators; Professionals; Technicians and trade workers; Community and personal service workers;	Clerical and administrative workers; Sales workers; Machinery operators and drivers; Labourers;
Capital	Own unincorporated business; Investment;	Superannuation/Annuity/Private pension and other regular sources
Transfer payments from government	Austudy/Abstudy; Age pension; Carer allowance; Carer payment; Disability pension; Disability support pension; Family tax benefits; Newstart allowance; Other pensions and allowances; Overseas pensions and benefits; Parenting payment; Baby bonus payment;	Partner allowance; Service pension; Sickness allowance; Special benefit; War widows pension; Widow allowance; Wife pension; Youth allowance; Utilities allowance; Senior supplement; Pension supplement;
Direct taxes	Incom taxes	

Once the preliminary processing of the HES data is complete, a five-step procedure reconciles the differences between the two databases (Zhang, 2015). As the HES expenditure and income data are expressed in purchaser prices, the input-output data used should also be in purchaser prices.

1. Use the HES weights to inflate the sample household data to the level for the whole population.
2. Scale the aggregated HES expenditure to be consistent with the total household expenditure in the input-output table.
3. Use the same scaling factor to scale the aggregated HES primary incomes.
4. Adjust each HES household’s expenditure shares, while keeping total expenditure on each product constant, so that the aggregation of all HES expenditures is consistent with

the commodity consumption pattern of the representative household shown in the input-output table.

5. In each industry in the input-output table, adjust factor income shares (labour by occupation and capital), while keeping the total value of factor income constant, so that the aggregated labour and capital income (net of capital savings) in the input-output table are consistent with the aggregated labour and capital incomes in the household data.

The aim of the second and third steps of adjustments is to preserve the original relativities between households in the survey data. The aim of the fourth and fifth steps is to keep the budget balances of individual households unchanged. A bi-proportional adjustment technique, such as RAS, or some entropy-based method, is required to carry out the last two steps.

A comparison between factor income in the 2012-13 input-output table and household income in the 2009-10 HES-SIH data is shown in table A2. A scaling factor of 1.3634 is used to update the 2009-10 HES-SIH data to the level consistent with the 2012-13 input-output table.

Table A2 **Comparison between input-output and HES-SIH aggregates (AUD million)**

Scaling factor = 1.3634

	Input-output 2012-13	*HES-SIH scaled*	*HES-SIH 2009-10*
Expenditure	830924	830924	609468
Income	1371109	850486	623816
Labour income	744282	744282	545918
- Managers and administrators	132202	132202	96968
- Professionals	229510	229510	168341
- Technicians and trade workers	101273	101273	74282
- Community & personal service workers	45706	45706	33525
- Clerical and administrative workers	91417	91417	67052
- Sales workers	43234	43234	31712
- Machinery operators and drivers	50821	50821	37277
- Labourers	50118	50118	36761
Capital income	626826	154400	113250
Benefits (23)		106875	78391
Income Tax (-)		155071	113742
Savings	(540185)	19562	14348

Note: Capital income in the input-output table is divided between household (154400) and firm saving (472426).

Table A3 **The core equations of a single country-single region (SCSR) model using ABS 2012-13 input-output table and 2009-10 HES data**

1. Demands for domestic and imported goods (1-10)

(1) Domestic price of imported good *c*, exclusive of duty

$$P_{(c,\text{"}imp\text{"})} = \boldsymbol{P^{cif}_{(c)}}\ \boldsymbol{ex} \qquad (c\in\text{COM})$$

where $\boldsymbol{ex}$ is the nominal exchange rate (domestic currency value per unit of foreign currency), and $\boldsymbol{P^{cif}_{(c)}}$ is the world price of import *c*.

(2) Basic price of good *c* from src *s* for usr *u*

$$P^{b}_{(c,u,s)} = \begin{cases} \boldsymbol{P_{(c,\text{"}dom\text{"})}} & (c\in\text{COM};\ u\in\text{USR};\ s=dom) \\ P_{(c,\text{"}imp\text{"})}\ (1 + \boldsymbol{t^{imp}_{(c,u)}}) & (c\in\text{COM};\ u\in\text{USR};\ s=imp) \end{cases}$$

where $\boldsymbol{t^{imp}_{(c,u)}}$ is the *ad valorem* rate of duty on imported good *c* for usr *u*, and $\boldsymbol{P_{(c,\text{"}dom\text{"})}}$ is the basic price of domestic good *c*, an *undefined* variable.

(3) CES demand of usr_stk *u* for good *c* from src *s* (exclusive of *stk*)

$$Q_{(c,u,s)} = CES\ (Q_{\_s(c,u)},\ P^{b}_{(c,u,s)},\ P^{b}_{\_s(c,u)}) \qquad (c\in\text{COM};\ u\in\text{USR\_stk};\ s\in\text{SRC})$$

(4) CES price for composite good *c* for usr *u*

$$P^{b}_{\_s(c,u)} = \begin{cases} CES\ (P^{b}_{(c,u,\text{"}dom\text{"})},\ P^{b}_{(c,u,\text{"}imp\text{"})}) & (c\in\text{COM};\ u\in\text{USR\_stk}) \\ \dfrac{1}{\sum_s \boldsymbol{Q_{(c,u,s)}}} \sum_s \boldsymbol{Q_{(c,u,s)}}\ P^{b}_{(c,u,s)} & (c\in\text{COM};\ u=\text{stk}) \end{cases}$$

(5) Demand of good *c* for usr *u* for margin good *m*

$$Q^{mgn}_{(c,u,m)} = Leontief\ (Q_{\_s(c,u)}) \qquad (c\in\text{COM};\ u\in\text{USR};\ m\in\text{MCM})$$

(6) Producers' price of composite good *c* for usr *u*

$$P^{t\_}_{(c,u)} = \frac{1}{Q_{\_s(c,u)} + \sum_m Q^{mgn}_{(c,u,m)}}\ (Q_{\_s(c,u)}\ P^{b}_{\_s(c,u)} + \sum_m Q^{mgn}_{(c,u,m)}\ \boldsymbol{P_{(m,\text{"}dom\text{"})}}) \qquad (c\in\text{COM};\ u\in\text{USR})$$

(7) Purchasers' price of composite good *c* for usr *u*

$$P^{t}_{(c,u)} = P^{t\_}_{(c,u)}\ (1 + \sum_g \boldsymbol{t^{dom}_{(c,u,g)}}) \qquad (c\in\text{COM};\ u\in\text{USR})$$

where $\boldsymbol{t^{dom}_{(c,u,g)}}$ is the *ad valorem* rate of domestic tax *g* on composite good *c* for usr *u*.

(8) Producers' price of exported good *c*

$$P^{exp}_{(c)} = \frac{1}{Q^{exp}_{(c)} + \sum_m Q^{emgn}_{(c,m)}}\ (Q^{exp}_{(c)}\ \boldsymbol{P_{(c,\text{"}dom\text{"})}} + \sum_m Q^{emgn}_{(c,m)}\ \boldsymbol{P_{(m,\text{"}dom\text{"})}}) \qquad (c\in\text{COM})$$

(9) Foreign demand for exported good *c*

$$Q^{exp}_{(c)} = \boldsymbol{E^{exp}_{(c)}}\ (P^{fob}_{(c)})^{-\varepsilon_{(c)}} \qquad (c\in\text{COM})$$

where $\boldsymbol{E^{exp}_{(c)}}$ is foreign expenditure on export *c*, denoted in foreign currency, and $\varepsilon_{(c)}$ is the constant elasticity of demand for exported good *c*.

(10) Foreign price of exported good *c*

$$P^{fob}_{(c)} = \frac{1}{\boldsymbol{ex}}\ P^{exp}_{(c)}\ (1 + \sum_g \boldsymbol{t^{exp}_{(c,g)}}) \qquad (c\in\text{COM})$$

where $\boldsymbol{t^{exp}_{(c,g)}}$ is the *ad valorem* rate of tax *g* on exported good *c*.

2. Final user incomes and demands for composite goods (11-17)

(11) Household factor income

$$Y^{fac} = \sum_o \boldsymbol{P^{lab}_{(o)}}\ X^{lab}_{(o)} + \boldsymbol{P^{cap}}\ X^{cap}\ (1 - \boldsymbol{s^{cap}})$$

where s^{cap} is the share of capital income firms retain for investment.

(12) Household disposable income

$$Y^{hou} = (Y^{fac} + P^{t}_{\_c("hou")}\ X^{ben})\ (1 - t^{hou})$$

(13) Expenditure income of fusr u

$$E_{(u)} = \begin{cases} Y^{hou}\ (1 - s^{hou}_{\_s}) & (u{=}hou) \\ (Tax\ Revenue)\ (1 - s^{gov}) & (u{=}gov) \\ Y^{hou}\ s^{hou}_{("inv")} + (Tax\ Revenue)\ s^{gov} + P^{cap}\ X^{cap}\ s^{cap} + Y^{fdi} & (u{=}inv) \\ Y^{hou}\ s^{hou}_{("stk")} & (u{=}stk) \end{cases}$$

where s^{gov} is government saving rate and $s^{hou}_{\_s}$ is household saving rate,

$$s^{hou}_{\_s} = s^{hou}_{("inv")} + s^{hou}_{("stk")}$$

where $s^{hou}_{("inv")}$ and $s^{hou}_{("stk")}$ are household saving for investment and change in inventory, respectively.

(14) Net foreign direct investment (in domestic currency)

$$Y^{fdi} = P^{t}_{\_c("inv")}\ Q^{fdi}$$

where Q^{fdi} is the real inflow of net foreign direct investment.

(15) Purchasers' price index for usr u

$$P^{t}_{\_c(u)} = \frac{1}{\sum_{c} Q_{\_s(c,u)}} \sum_{c} Q_{\_s(c,u)}\ P^{t}_{(c,u)} \qquad (u \in \mathrm{USR})$$

(16) Demand of usr u for composite good c

$$Q_{\_s(c,u)} = \begin{cases} Leontief\ (X_{(u)}) & (c \in \mathrm{COM};\ u \in \mathrm{IND}) \\ Q^{hcom}_{\_h(c)} & (c \in \mathrm{COM};\ u{=}hou) \\ Leontief\ \left(\dfrac{E_{(u)}}{P^{t}_{\_c(u)}}\right) & (c \in \mathrm{COM};\ u{=}inv) \\ \sum_{s} Q_{(c,u,s)} & (c \in \mathrm{COM};\ u{=}stk) \end{cases}$$

where $X_{(u)}$ is the output of industry u, an *undefined* variable, and $Q_{\_s(c,"gov")}$ and $Q_{(c,"stk",s)}$ are exogenous variables.

(17) Budget constraint for government and inventory (=>s^{gov}, $s^{hou}_{("stk")}$)

$$E_{(u)} = \sum_{c} (P^{t}_{(c,u)}\ Q_{\_s(c,u)}) \qquad (u{=}gov,\ stk)$$

3. Industry outputs, prices and demands for factors (18-25)

(18) CES demand of industry i for labour occupation o

$$Q^{lab}_{(o,i)} = CES\ (Q^{lab}_{\_o(i)},\ P^{lab}_{(o)},\ P^{fac}_{("lab",i)}) \qquad (o \in \mathrm{OCC};\ i \in \mathrm{IND})$$

where $Q^{lab}_{\_o(i)}$ is the demand of industry i for composite labour,

$$Q^{lab}_{\_o(i)} = Leontief\ (Q^{fac}_{("lab",i)}) \qquad (i \in \mathrm{IND})$$

(19) Basic price of factor f in industry i

$$P^{fac}_{(f,i)} = \begin{cases} CES\ (P^{lab}_{(o_1)},\ \ldots,\ P^{lab}_{(o_8)}) & (f{=}lab;\ i \in \mathrm{IND}) \\ P^{cap} & (f{=}cap;\ i \in \mathrm{IND}) \end{cases}$$

where $P^{fac}_{("lab",i)}$ is a CES price index for composite labour, $P^{lab}_{(o)}$ and P^{cap} are the equilibrium prices of labour occupation o and capital, respectively, both of which are *undefined* variables.

(20) CES demand of industry i for factor f

$$Q^{fac}_{(f,i)} = CES\,(Q^{fac}_{\_f(i)}, P^{fac}_{(f,i)}, P^{fac}_{\_f(i)}) \qquad (f\in\text{FAC};\ i\in\text{IND})$$

where $Q^{fac}_{\_f(i)}$ is the demand of industry i for composite factor,

$$Q^{fac}_{\_f(i)} = Leontief\,(\boldsymbol{X_{(i)}}) \qquad (i\in\text{IND})$$

(21) CES price index for composite factor in industry i

$$P^{fac}_{\_f(i)} = CES\,(P^{fac}_{(\text{"cap"},i)}, P^{fac}_{(\text{"lab"},i)}) \qquad (i\in\text{IND})$$

(22) Unit cost of output in industry i

$$C^{ind}_{(i)} = \frac{1}{\sum_c Q_{\_s(c,i)} + \sum_f Q^{fac}_{(f,i)}} \sum_c Q_{\_s(c,i)}\, P^{t}_{(c,i)} + \sum_f Q^{fac}_{(f,i)}\, P^{fac}_{(f,i)} \qquad (i\in\text{IND})$$

(23) Basic price of output in industry i

$$P^{ind}_{(i)} = C^{ind}_{(i)}\,(1 + \boldsymbol{t^{ind}_{(i)}}) \qquad (i\in\text{IND})$$

where $\boldsymbol{t^{ind}_{(i)}}$ is the *ad valorem* rate of a tax on output of industry i.

(24) Total supply of labour occupation o (equal to its demand in equilibrium) (=> $\boldsymbol{P^{lab}_{(o)}}$)

$$X^{lab}_{(o)} = \sum_i Q^{lab}_{(o,i)} \qquad (o\in\text{OCC})$$

(25) Total supply of capital (equal to its demand in equilibrium) (=> $\boldsymbol{P^{cap}}$)

$$X^{cap} = \sum_i Q^{fac}_{(\text{"cap"},i)} \qquad (f\in\text{FAC})$$

4. Industry supplies of multi-products (26-29)

(26) CET supply of good c from industry i

$$X^{com}_{(c,i)} = CET\,(\boldsymbol{X_{(i)}}, \boldsymbol{P_{(c,\text{"dom"})}}, P^{ind}_{(i)}) \qquad (c\in\text{COM};\ i\in\text{IND})$$

(27) CET price index for composite goods (equal to $P^{ind}_{(i)}$ in equilibrium) (=> $\boldsymbol{X_{(i)}}$)

$$P^{ind}_{(i)} = CET\,(\boldsymbol{P_{(c_1,\text{"dom"})}}, \ldots, \boldsymbol{P_{(c_m,\text{"dom"})}}) \qquad (i\in\text{IND})$$

(28) Total supply of good c from all industries

$$X^{com}_{\_i(c)} = \sum_i X^{com}_{(c,i)} \qquad (c\in\text{COM})$$

(29) Total sales of good c (equal to $X^{com}_{\_i(c)}$ in equilibrium) (=> $\boldsymbol{P_{(c,\text{"dom"})}}$)

$$X^{com}_{\_i(c)} = \begin{cases} \sum_u Q_{(c,u,\text{"dom"})} + Q^{exp}_{(c)} & (c\in\text{NCM}) \\ \sum_u Q_{(c,u,\text{"dom"})} + Q^{exp}_{(c)} + \sum_n \sum_u Q^{mgn}_{(n,u,c)} + \sum_n Q^{emgn}_{(n,c)} & (c\in\text{MCM}) \end{cases}$$

5. Household income and expenditure (30-35)

(30) Factor income from factor f for person n in household h

$$Y^{hfac}_{(f,h,n)} = \begin{cases} \sum_o \boldsymbol{P^{lab}_{(o)}}\, \boldsymbol{X^{hlab}_{(o,h,n)}} & (f=lab;\ h\in\text{HOU};\ n\in\text{PER}) \\ \boldsymbol{P^{cap}}\, \boldsymbol{X^{hcap}_{(h,n)}} & (f=cap;\ h\in\text{HOU};\ n\in\text{PER}) \end{cases}$$

where $\boldsymbol{X^{hlab}_{(o,h,n)}}$ is labour income from occupation o and $\boldsymbol{X^{hcap}_{(h,n)}}$ is capital income.

(31) Taxable income for person n in household h

$$Y^{htinc}_{(h,n)} = \sum_f Y^{hfinc}_{(f,h,n)} + \sum_{b\in B23} P^{t}_{\_c(\text{"hou"})}\, \boldsymbol{X^{hben}_{(b,h,n)}} \qquad (h\in\text{HOU};\ n\in\text{PER})$$

where $\boldsymbol{X^{hben}_{(b,h,n)}}$ is the entitlement of real benefit b.

(32) Disposable income for person n in household h

$$Y^{hdinc}_{(h,n)} = Y^{htinc}_{(h,n)}\,(1 - \boldsymbol{t^{hou}_{(h,n)}}) \qquad (h\in\text{HOU};\ n\in\text{PER})$$

where $\boldsymbol{t^{hou}_{(h,p)}}$ is the rate of a tax on the taxable income of person n in household h.

(33) Expenditure income for household h

$$Y^{heinc}_{(h)} = \sum_{n} Y^{hdinc}_{(h,n)} (1 - r^{hsav}_{(h)}) \qquad (h \in \mathrm{HOU})$$

(34) Changes in saving rate for household h

$$\frac{d(r^{hsav}_{(h)})}{r^{hsav}_{(h)}} = \frac{d(s^{hou}_{\_s})}{s^{hou}_{\_s}} \qquad (h \in \mathrm{HOU})$$

(35) Household h demand for composite good c

$$Q^{hcom}_{(c,h)} = f\,(Y^{heinc}_{(h)}, P^{t2}_{\_s(c,\text{"}hou\text{"})}) \qquad (c \in \mathrm{COM};\ h \in \mathrm{HOU})$$

6. Linking household variables with CGE aggregates (36-40)

(36) Aggregation of household supplies of capital stock

$$X^{cap} = \sum_{h} \sum_{n} \boldsymbol{X^{hcap}_{(h,n)}}$$

(37) Aggregation of household supplies of labour occupation o

$$X^{lab}_{(o)} = \sum_{h} \sum_{n} \boldsymbol{X^{hfac}_{(o,h,n)}} \qquad (o \in \mathrm{OCC})$$

(38) Aggregation of household demands for composite good c

$$Q^{hcom}_{\_h(c)} = \sum_{h} Q^{hcom}_{(c,h)} \qquad (c \in \mathrm{COM})$$

(39) Average household income tax rate

$$t^{hou} = \frac{\sum_{h} \sum_{n} Y^{htinc}_{(h,n)} \boldsymbol{t^{hou}_{(h,n)}}}{\sum_{h} \sum_{n} Y^{htinc}_{(h,n)}}$$

(40) Aggregation of household real benefits

$$X^{ben} = \sum_{b \in B23} \sum_{h} \sum_{n} \boldsymbol{X^{hben}_{(b,h,n)}}$$

Notes:

1. Variables in red colour are set as exogenous.

2. Variables in blue colour are *undefined*.

3. Equations, shaded in yellow colour, are *general equilibrium conditions*, used to solve *undefined* variables.

Table A4 **Sets used in the model and database**

Sets	*Definitions*
COM(1,…,*m*):	Commodities (indexed by *c*)
IND(1,…,*m*):	Industries (indexed by *i*)
MCM(1,…,*n*):	Margin commodities (indexed by *m*)
NCM(= COM − MCM):	Non-margin commodities (indexed by *c*)
FUSR(*gov*, *hou*, *inv*, *stk*):	Final users of commodities (indexed by *u*)
USR(=IND + FUSR):	Users of commodities (indexed by *u*)
USR_stk(=USR − *stk*):	Users of commodities, exclusive of *stk* (indexed by *u*)
SRC(*dom*, *imp*):	Domestic/import sources of commodities (indexed by *s*)
FAC(*lab*,*cap*):	Factors of production (indexed by *f*)
OCC(1,…,8):	Labour occupation (indexed by *o*)
SAV(*inv*,*stk*):	Saving destination (indexed by *s*)
HOU(1,…,9774):	Sample households (indexed by *h*)
PER(1,…,6):	Persons in a household (indexed by *p*)
B23(1,…,23):	Transfer payments (indexed by *b*)
C2(c55,c110):	Two transfer payments between households (indexed by *c*)

A new approach to iterative simulations

For GEMPACK users, CGE models are written in percentage change format. If a sub-model is written in levels, a different program has to be chosen. Although many such programs are currently available, linking and solving two models written in two different programs can be a challenging task.

In fact, there is a better way of doing iterative modelling for GEMPACK users. The CGE model in percentage changes and the sub-model in levels, can actually be written in a single TABLO code. The CGE model is defined in the normal way using “Variable” and “Equation” statements, while the BMS model is defined using “Coefficient” and “Formula” statements in a separate section. The model can be run as any CGE model, producing two files.

1. A solution file with the results in percentage changes for all endogenous variables in the CGE model. Variables that are used as input into the BMS model, such as goods and factor prices, are written to a Header Array file using SOTOHT, to be used as shocks to the BMS model in the following interation.

2. A Header Array file that contains coefficients produced by the BMS model, that are to be used as shocks to the CGE model. It is easy for TABLO to convert coefficients to percentage change forms so that they can be read by the Command file as shocks to the CGE model in the following round of simulation. These are mainly those variables of quantitative changes, such as household demands for goods and supplies of factors, as well as changes in policy variables, which typically affect values, such as income tax rates and transfer payments.

A batch file can be used to automate the procedure so that, in each simulation, both models can run in parallel. The CGE model reads shocks from a Header Array file, produced from the BMS model in the previous simulation, and produces a solution file to be written in a new Header Array file, which is used in turn to shock the BMS model in the following iteration. During the same simulation, the BMS model reads in shocks to prices from a Header Array file, produced from the CGE model in the previous simulation, and writes its responses in terms of changes to quantities in a new Header Array file for the CGE model in the following simulation.

This approach allows the BMS model to be written in levels within the same code as the CGE model. More complex functional forms, such as conditional or discrete functions, and unconventional behavioural assumptions, such as switching between different options, can be implemented alongside a CGE system. It can also save computation time significantly.