Achieving a long-run equilibrium in the dynamic GTAP model[1]

Paul Gretton
East Asian Bureau of Economic Research and Centre of European Studies,
Australian National University

Abstract

A dynamic version of the GTAP model of the global economy became available in 2012. The dynamic version known as GDyn, introduced partial adjustment mechanisms for capital accumulation and a dynamic accounting of capital-finance and related income flows between regional households and firms, and a global trust. In long-run equilibrium, the model rates of return are to be equal and constant over time. In practice, illustrative results presented with the release of GDyn show the equilibrium conditions are not satisfied. This paper confirms this model property. It finds that this gives rise to model instability which limits the use of GDyn for the analysis of economic growth within a neoclassical framework. To achieve model stability and overcome this limitation, modelling of rates of return and capital-finance flows is further developed within the GDyn framework to satisfy the stated longer-run neoclassical equilibrium conditions. Results of the revised model demonstrating a stable, long-run equilibrium are reported together with an illustrative policy simulation of a productivity improving technological change in a medium-sized open economy. Some key areas for further research are identified.

JEL: C68, E17, E21, F11, F21, F43.

Key Words: Economic Growth, Dynamic Base Lines, Trade Policy Analysis

1 Introduction

A dynamic version of the GTAP model became available in 2012 as an accessible public domain general purpose modelling technology (Ianchovichina and Walmsley 2012). The dynamic version known as GDyn, introduced partial adjustment mechanisms for capital accumulation and a dynamic accounting of capital-finance and related income flows between regional households and firms, and a global trust. The objectives of the development were to 'provide a better treatment of the long-run within a GTAP framework and provide a way of tracing the evolution of the global economy through time' (p, 12). The applicability of the model was demonstrated in I&W 2012 through a series of simulations on topics as diverse as trade reform, growth and investment, climate change, natural resource use, technological change and demography benchmarked using a 1995 data base.

An important advantage of the GTAP and the GDyn models for economic research and policy analysis is that they are fully documented, accessible and in the public domain, and that they

[1] This paper is a revised version of a paper presented at the 21st Annual Conference on Global Economic Analysis, 13 to 15 June 2018, Cartagena. Previous versions of the paper were presented to the National CGE Workshop, 17 August 2017, Victoria University, Melbourne and a seminar at the Australian National University on 22 May 2018. The author is grateful for helpful comments made by participants to those events.

utilize a general purpose modelling technology based on the widely available and used GEMPACK modelling software.

With regard to GDyn-type models, in a long-run neo-classical equilibrium, model rates of return on capital are to be equal and constant over time. In practice, illustrative results presented with the release of GDyn show these equilibrium conditions are not satisfied with the rates of return on capital falling without bounds (Ianchovichina and McDougall 2012). This paper confirms this model property. It also finds that this gives rise to model instability which limits the use GDyn for the analysis of economic growth within a neoclassical framework. It is of particular relevance for analyses involving longer time periods or with a focus on the domestic/foreign sourcing of funds for capital accumulation by regions. This adds to the operational burden of using the model as users seek to manage the modelling and interpretative implications of falling rates of return on capital and model instability. These factors act as a disincentive to the wider use of GDyn.

Golub and McDougall (2012) and Walmsley and McDougall (2012) introduced ‘saving’ and ‘complementarity’ versions of GDyn to stabilise net foreign investment positions and returns in GDyn through endogenising saving and placing a lower bound on gross investment, respectively. The methods sought to bound variation in target variables. The methods worked within the partial adjustment mechanisms for rate of return determination, capital accumulation and capital financing of the basic model without overcoming the underlying limitation that rates of return on capital could fall without bounds.

To overcome this limitation, modelling of rates of return and capital-finance flows is further developed within the GDyn framework to satisfy the stated long-run neoclassical equilibrium conditions. Under these conditions, the actual rates of return on capital are matched with a predetermined target rate through a partial adjustment mechanism. Optimizing behaviour of economic agents is also introduced to mediate between the demand for, and supply of domestically sourced and foreign sourced funds for capital accumulation. These innovations combine to achieve model stability. Results from the revised model, termed GDyn-F, demonstrate a stable, long-run equilibrium.

The neo-classical stability conditions ascribed to the GDyn model are set out in section 2 together with illustrative simulations using the original model theory (termed GDyn-2012 in this paper) with only time shocked to assess model projections against these stability conditions. The simulations are undertaken using a data base with 6 diverse regions — Australia, China, Japan, the United States, EU-28 and the Rest-of-the-World — and 13 industry sectors — two agriculture, one mining, four manufacturing, five services plus ownership of dwellings. Five primary factor inputs are modelled — Land, Natural resources, Skilled labour, Unskilled labour and Capital (appendix A).

Section 3 details the theory included in GDyn-F for rate of return determination and for the mediation between the demand for and supply of domestically sourced and foreign sourced funds for capital accumulation. It should be emphasised that this theory is introduced using the terminology and architecture of original implementation. The following section reports projections again with only time shocked to examine the stability of the revised theory. A benchmark reporting time horizon of 100 years is adopted based on background simulations out to 200 years.

The following two sections report modelling base cases out to 2050 based on CEPII projections of value added, population, and employment of skilled and unskilled workers, projected using GDyn-2012 and GDyn-F. This is followed by a policy simulation of a one-off 1 percent productivity improving technical change in a medium-sized open economy (Australia).

The final section concludes the paper suggesting some areas for further investigation. The illustrative simulations show that through the revised model, it is possible to trace out the time scale of the effects of policy and other economic changes over the long run. It is possible to utilise the modified model to examine the impacts of population, workforce participation and productivity assumptions on global growth and the convergence of economies over time.

2 Long-run equilibrium

In GDyn investment theory, in a long-run equilibrium, the model expected, target and actual rates of return on capital are conceptually all equal and constant over time (Ianchovichina and McDougall 2012, p.68). In equilibrium, the change in the normal rate of growth of capital ($KHAT$) is also constant over time and equal to zero with changes only occurring with the passage of time, that is, with no change in the growth variables of population, labour force participation or productivity. That is:

$$RORGEXP(r) = RORGTARG(r) = RORGROSS)(r), \forall r \qquad (2.104)$$

$$\dot{RORGEXP}(r) = \dot{RORGTARG}(r) = \dot{RORGROSS}(r) = 0, \forall r \qquad (2.105)$$

$$KHAT(r) = 0, DKHAT(r) = 0, \forall r \qquad (2.106)$$

Source: Ianchovichina and McDougall 2012, pp. 68-69.

These conditions imply a constant gross investment to capital ratio for each region. With a common depreciation rate assumed across regions, the long run equilibrium gross investment to capital ratio will also be common across regions. A long-run equilibrium is reached through a partial adjustment process whereby gross investment adjusts to equate the expected rate of return with the target rate, with capital simultaneously adjusting to equate the expected and actual rates of return. With the expected rate of return of each region determined by the ratio of projected capital in next period relative to the current period and the rate of return, long-run equilibrium in GDyn can be likened to equilibrium in a multi-region q-investment theory model.

Ianchovichina and McDougall (2012) tested the long-run properties of the model over a 100-year period using a three region aggregation. The initial data for the period 1992 revealed substantial differences in regional rates of return ($RORGROSS$) and normal rates of growth in capital ($KHAT$). These data differences were depicted as representing a world in disequilibrium. The 100-year simulation represented changes in the economies occurring only on account of the passage of time and the elimination of regional risk premiums (implemented through the adoption of a common target rate of return across regions ($RORGTARG$). Each rate of return — $RORGROSS$, $RORGEXP$ and $RORGTARG$ — is modelled as being variable over time with the target rate ($RORTARG$) remaining common across regions in any one year.

The GDyn model projections reported by Ianchovichina and McDougall (2012) show that the model gross rate of return converges gradually to the target rate of return (figure 2.5) and that errors in expectations are progressively eliminated (figure 2.6) across the three regions modelled. The projected normal rate of growth of capital, after a period of oscillating adjustment, follows a downward trend broadly aligned across regions, but without converging to the long-run equilibrium value of zero (Figure 2.7). The investment to capital ratios across the regions follow a declining path, also after a period of oscillating adjustment (figure 2.8). While the figures showed a gradual movement towards the target steady-state values, these were not reached after 100 years.

More broadly, it was reported that GDyn projections of the capital account and net foreign liabilities are highly volatile and under the model implementation can grow without bound. With variable savings rates between regions, it was also noted that under the model implementation there can be projections of excessive investment with rates of return falling without bound (Ianchovichina and McDougall, p. 78). In the trial simulations reported, for example, projected rates of return remained on a clear downward path, even after 100 years while projections of the normal growth of capital had not converged to the long-run equilibrium value of zero (figures 2.5 and 2.7, referenced above).

Exploratory simulations conducted by the author using first a GDyn 2004 data base and then a 2011 data base with six regions provided results that broadly aligned with the results reported in Ianchovichina and McDougall (2012). In particular, the projections showed a declining trend in the actual rate of return on capital (figure 1, top left panel). These 2011-based projections also showed somewhat erratic oscillating adjustments for the error in expectations (the natural log of the ratio of the expected rate of return to the gross rate), the normal rate of growth of capital and the investment to capital ratio. In contrast to the Ianchovichina and McDougall (2012) analysis using a 3 region data base which extended at least to 100 years, the simulations using a six region data base terminated with error in period 41 (that is, the calendar year 2052) out of a 200 year target simulation period.[2] When the model simulations terminated, the actual rate of return converged toward the data base rate of depreciation of 4 percent (figure 1, top left panel). At that rate of return, regional economies are not generating an economic return to domestic and foreign owners of capital.

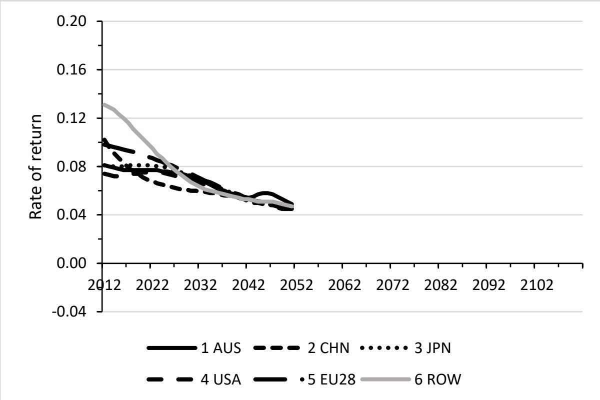
The initial intention of the test simulations was the deployment of GDyn for policy analysis using different levels of regional aggregation, to help trace out the time path of policy changes traditionally considered within the comparative static framework of GTAP. The possibility of examining broader long-term economic growth issues was also a possibility. However, downward trajectory of the expected, target and actual rates of return in contravention of the

[2] Parallel simulations using 2004, 2007 and 2011 data, differing aggregation and parameter setting also terminated but with different simulation periods. Testing showed that parameter settings that dampened the amplitude of oscillations tended to also extend the simulation period, while settings that increased the amplitude of oscillations tended to shorten the simulation period.

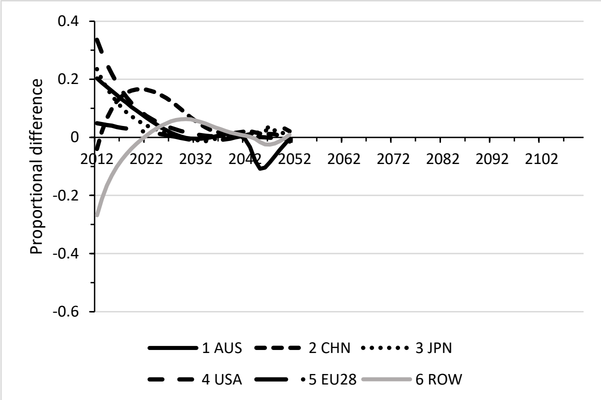
neo-classical stability conditions ascribed to the model and lack of model stability, diverted attention from these broader issues towards the model theory and modifications that would meet the equilibrium conditions and provide model stability.

Figure 1: GDyn-2012 model projections with only time shocked do not meet stated neo-classical long-run stability conditions, 2011 based

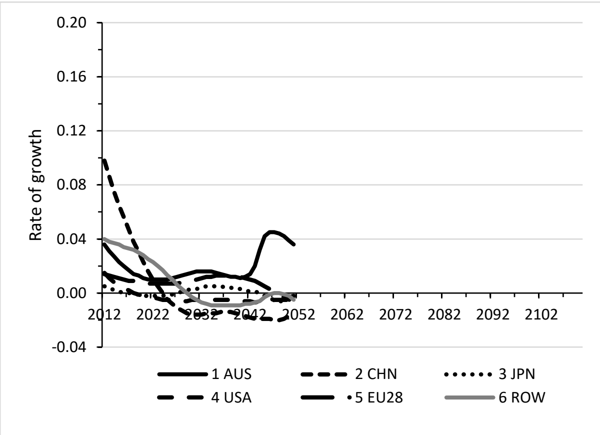
Projected actual rates of return (RORGROSS) trends down



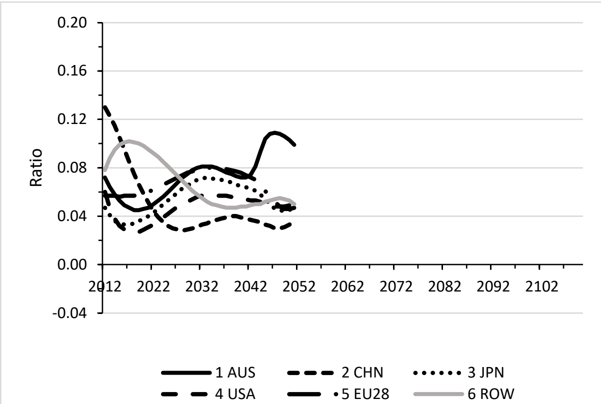
Error in expectations (the ratio ERRRORG) are not eliminated



Price-neutral rate of growth in the capital stock (KHAT) oscillates without converging to zero



Investment to capital ratio (IKRATIO) oscillates without converging



Source: Author estimates based on figures 2.5 to 2.8 Ianchovichina and McDougall (2012).

3 Modifying GDyn to achieve long-run equilibrium

Within the broad theoretical architecture of the GDyn model, two elements are considered to achieve long-run equilibrium:

- extend the GDyn investment rule to include an exogenous treatment of the target rate of return ($RORGTARG$); and
- modify the treatment of international capital flows to include optimizing behaviour by regional firms and households, respectively.

Extend the investment rule

The long-run neo-classical stability conditions set for GDyn are not met, at least in part, because each model rate of return variable is endogenously determined in model simulations. In this set up, each new investment cycle drives the actual rate of return down and this decline is transmitted to the model expected rate and target rates of return which also decline towards the rate of depreciation on capital (figure 1, top left panel). Moreover, in GDyn theory there is no readily apparent way of exogenising a rate of return variable, for example, to allow for normal returns to the owners of capital and changes in country institutions and risk. With each GDyn rate of return measure represented by a coefficient and a variable (that is, the percentage change in that coefficient) (table 1), the introduction of an exogenous treatment would target one of those variables.

Table 1: **Rate of return coefficients and variables in GDyn**

Rate of return concept	*Model treatment*
Actual gross rate of return Coefficient: $RORGROSS(r)$ Variable: $rorga(r)$ percentage change in coefficient	$rorga$ endogenous and variable across regions — varies with the real returns to capital by region. $RORGROSS$ calculated from data by dividing the gross return on capital during a period by the level of capital at the beginning of the period.
Expected rate of return Coefficient: $RORGEXP(r)$ Variable: $rorge(r)$ percentage change in coefficient	$rorge$ endogenous and variable across regions — modelled based on the relation: $RORGEXP = RORGROSS * [K(1)/K(0)]\ \hat{}\ -rorgflex$, where $rorgflex$ is a parameter of adaptive expectations. If planned capital ($K(1)$) is above base capital $K(0)$, the expected rate of return is lower relative to base rate of return by the (negative) power of the parameter $rorgflex$ (set at 10 in GDyn). $RORGEXP$ updated by $rorge$.
Target rate of return Coefficient: $RORGTARG(r)$ Variable: $rorgt(r)$ percentage change in coefficient	$rorgt$ endogenous and uniform across regions — modelled as adjusting towards $rorge$, while capital stocks vary by region to align $rorge$ with (a variable) $rorgt$. $RORGTARG$ updated by $rorgt$.

Source: Ianchovichina and McDougall (2012).

From the above, $RORGROSS(r) = RORGTARG(r)$ in long-run equilibrium. Noting that gross returns for a region includes a common risk neutral entrepreneurial income component ($RNETCOMM$), a regional institutions and risk premium component ($RRISK(r)$) and a depreciation on region capital ($RDEPN(r)$) component, the value of the gross rate of return can be depicted as:

$$RORGROSS(r) = RNETCOMM + RRISK(r) + RDEPN(r)$$

The simulations reported above considered the case of the elimination of regional risk premiums implemented through the adoption of a common target rate of return across regions, so that $RORGTARG(r) = RNETCOMM + RDEPN(r)$. The gradual compression of the target rate of return in the simulations to the common depreciation rate then implies the elimination of a return to entrepreneurship. Arguably, this return is determined outside of the model and should be treated as exogenous. With an exogenous setting of a target rate (either with or without a regional institutions and risk component), the regional gross and expected returns would converge to that target rate over time. This long run rates would be contain an entrepreneurial income element and be above the depreciation rate.[3]

In order to make the target rate exogenous, it is necessary to first augment the model to enforce the condition that the expected rate of return adjusts to be equal to the required rate of return, as represented by the model target rate. This is achieved by augmenting the original partial adjustment relationship:

$$erg\_rorg(r) = LAMBORG(r) * \left(rorgt(r) - rorge(r)\right)$$

where $erg\_rorg(r)$ denotes the percentage point change in the expected rate of return relative to the percent change in the target rate and $LAMBORG(r)$ is the model coefficient of partial adjustment between the percentage changes in the expected and target rates of return (Inachovichina and McDougall 2012, equation 2.86, p. 59). In this model, an increase in the expected rate of return relative to the target rate would raise the magnitude of $erg\_rorg(r)$ and through this, the incentive for investors to invest.

An appropriate augmentation would allow investors to react to the proportional difference in the *actual* rates of return. This may be incorporated in the theory through the term:

$$LAMBORG(r) * ERRRORGT(r)$$

where $ERRRORGT(r)$ is the proportional difference between the target and expected rates of return, that is $ERRRORGT(r) = loge\left(RORTARG(r)/ROREXP(R)\right)$. Including this additional term in the partial adjustment relation into GDyn gives:

$$\begin{aligned} erg\_rorg(r) = {} & LAMBORG(r) \\ & * [rorgt(r) - rorge(r) + LAMBORG(r) * ERRRORGT(r)] \end{aligned}$$

In this formulation, investors respond both to relative differences in changes in the target and expected rates of return (by the rate $LAMBORG(r)$) and the actual rates (by the square of the rate $LAMBORG(r)$). The second test enables the target rate of return to be defined exogenously.

The setting of regional risk premiums in simulations is achieved in the augmented framework by determining values for the target rate outside of the model. This is enabled by setting $rorgt(r)$ exogenously. To do this, an additional equation needs to be added to define the percentage change in the target rate of return:

$$rorgt(r) = srorc(r) + srorc\_r$$

where $srorc(r)$ is a region-specific shift term on the target rate of return while $srorc\_r$ is a common global shift term on the target rate. The region-specific shift term is useful for considering issues of financial market policy or environmental changes that affect region-specific risk and is naturally exogenous.

[3] It should also be noted that this is consistent with the transversality condition of a growth model with consumer optimization (the Ramsay model) which required that the steady state net rate of return on capital exceeds the growth rate in population and productivity (Barro and Sal-i-Martin 1995, p. 73). This requirement is consistent with a long-run equilibrium gross return in GDyn simulations lying above the depreciation rate.

Closing the model with the extended treatment of investment

The common global shift term $srorc\_r$ is not naturally exogenous. In order to fix $rorgt(r)$, the global shift term needs to be swapped with another global variable. The variable chosen is a shift term on the global capital stocks $sqkworld$, a region-generic shift term on capital in the model equation linking capital formation with the provision on capital services (GDyn **Equation** KBEGINNING in box 1 below). With this closure change, $RORGTARG(r)$ is fixed for all regions. Importantly, the predetermined value for the target rate of return for each region is not eliminated in dynamic simulations.

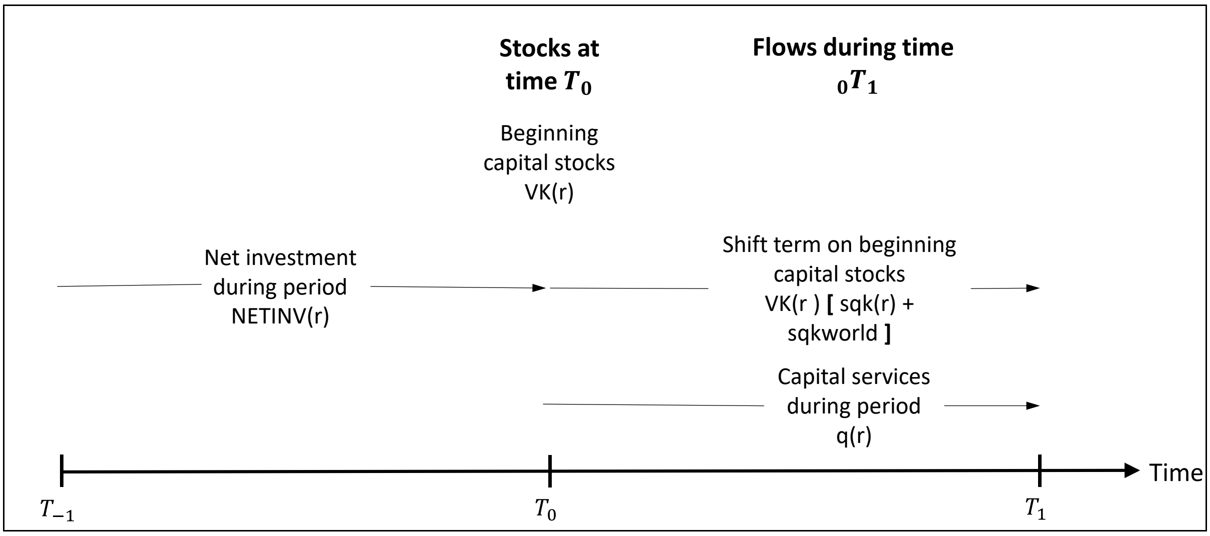
Box 1: **TABLO code for equation associating capital services with changes in capital stock**

```
Coefficient (ge 0)(all,r,REG)
VK(r) # value of capital stock, in region r #;

Variable (all,r,REG)
qk(r) # Capital stock located in region r #;

Coefficient (all, r, REG)
NETINV(r) # regional NET investment in region r #;

Variable (all, r, REG)
sqk(r) # Arbitrary region-specific shock to capital stock #;

Variable
sqkworld # Arbitrary region-generic shock to capital stock #;


Equation KBEGINNING # associates change in cap. services with change
in cap. stock (HT 56) # (all,r,REG)
VK(r) * qk(r) = 100 * NETINV(r) * time
                + VK(r) * [ sqk(r) + sqkworld ];
```

Source: GDyn TABLO code.

In GDyn, capital formation is represented by the coefficient for net investment ($NETINV(r)$) in the right hand side of **Equation** KBEGINNING, with the level of net investment defined (separately) as equal to gross investment less depreciation on the stock of capital. The value of this coefficient is taken from the base-period data base and represents net investment during the previous period (figure 2). The value of capital is measured at the beginning of the current period, that is, the end of the base period. The value is also taken from the base-period data base. All else equal, if $sqk(r)$ and $sqkworld$ are treated exogenous as in the original model closure, the growth in capital services (in percentage changes) in the current period, should by definition, be the equal to the proportional increase in capital stocks.

Figure 2: **Accounting for net investment, capital stocks and capital services in GDyn**



Source: Based on GDyn theory and data base.

The endogenous treatment of $sqkworld$ in the extended model, apart from achieving a technically valid model closure, recognises that there can be a timing gap between when gross investment occurs and when it enters into production as the productive input ‘capital services’ (that is, $q(r)$) or it is scrapped due to normal or unforeseen events. That is, not all new capital need be available for use in production for the full period in which it is produced and not all capital need remain in production until its depreciated value is effectively zero according to the diminishing balance depreciation method adopted in GDyn (and common in this type of model). This is reflected by the relation:

$$Capital\ services\ =\ Net\ investment\ +\ Shift\ term\ for\ adjustment\ costs.$$

In the present application, $sqkworld$ is adopted as the model representation of the shift term.

In the GDyn 2011 data base, gross investment amounts to about 7 percent of the value of capital globally while the standard depreciation rate is 4 percent. With global savings equal to global gross investment in GDyn (that is, $S = I$), global saving also amounts to about 7 percent of capital in the data base. This leaves gross investment of about 3 percent of capital to enter as a productive input in a future period or be absorbed as an adjustment cost.[4] With saving assumed

[4] There may be uncertainty about the actual scale of adjustment costs and how they may vary with changing economic circumstances and time. Gretton and Fisher (1997) on the basis of Australian input-output data, suggested that the average value of fixed assets at time of final sale or scrapping could be about 7.5 percent of acquisition costs. Leamer (1988) suggested the value could be around 13 percent. GDyn does not include a theory to account for the scrapping of assets short of the effective working life of assets implied by the diminishing balance depreciation method. Other global models adopt different depreciation rates, for example the MIRAGE model adopts a depreciation rate of 6 percent (Decreux and Valin (2007), Fontagne *et al.* (2013)). Variability of the adjustment cost term as the global economy approaches a long-run equilibrium, also reflects the possibility that the nature of an equilibrium depends on the costs of approaching and maintaining it, including adjustments of work routines and tasks (Lucas 1967, Hamermesh and Pfann 1996). Moreover, aggregate variations are influenced by the cross sectional distribution of firm responses (Caballero (1996). A short-term change in saving need not therefore be translated immediately into a change in capital services. McKibbin and Vines 2000 argued that adjustment costs are likely to vary with the scale of new investment and are important in economic models used for policy analysis of the effects of economic change. GDyn does not include a theory for adjustment costs on account of war, civil disturbance, technological change, policy and

to be a fixed proportion of income in GDyn, this would suggest that $sqkworld$ could converge to around 3 percent in the long-run with only time varying. These data may be interpreted as indicating:

- that there are adjustment costs in capital accumulation and use not embodied in the database measure of depreciation or the model partial adjustment theory;
- that the data base and model theory are not calibrated to the special case where saving and investment is proportional to depreciation (so that the gross investment to capital ratio is equal to the model depreciation rate) or the model converges to such an equilibrium in the long-run; or
- a combination of these possibilities.

The current extension by retaining the GDyn specifications of the relationship between capital services and net investment incorporates the first of these possibilities. It would be potentially worthwhile to extend the modelling of the utility maximizing behaviour of regional households beyond the current Cobb-Douglas paradigm to allow the apportionment of household income between consumption and saving to vary and through this for gross investment to capital ratios to vary. This matter left for further consideration.[5][6]

The augmented theory and model closure meets all market clearing and neo-classical stability conditions for GDyn. [7]

---

other economic changes outside of the convex adjustment costs implied by the diminishing balance depreciation method and the depreciation rate adopted in the model.

[5] Golub and McDougall 2012, review household saving behaviour in dynamic GTAP and suggest an extension to GDyn that enables saving rates to vary within bounds indicated by historical data. Another possibility that may have merit would be to relax the Cobb-Douglas utility maximizing behaviour of households in GTAP and GDyn to allow regional households to transform between consumption and saving on the basis of relative prices (for example, by using the familiar constant elasticity of transformation approach).

[6] The relationship between savings, real capital investment and the current account balance is characterised in GTAP by: $S - I = X - M + R = NL$ where *S* represents gross saving of the national household, *I* investment in capital goods net of depreciation, *X* and *M* exports and imports respectively, and *R* net remittances and transfers aboard (Hertel and Tsigas 1997). *NL* represents net lending aboard. In GDyn, estimates are made for investment income on finance for real capital (prefixed by *YQ* in model theory). Following the GTAP convention, all other remittances are set to zero and national 'saving' is derived as a residual. Inspection of the GDyn (and GTAP) data base indicates that this convention result in negative values being recorded against 'saving' for some regions. Possible sources of negatives include remittances of residents working abroad, royalties on intangible assets, foreign aid, insurance claims and income on reserve assets. It is implicit in the treatment that any direct investment reinvested earnings recorded in income flows are matched by capital flows. The model theory is written on the assumption that the saving is positive.

[7] Another approach may have been to seek to fully replace the GDyn investment theory with an alternative theory such as that applied by Dixon and Rimmer (2002). This approach adopts an inverse logistic capital supply function specifying a relationship between an equilibrium expected rate of return and the capital growth rate, bounded by parameters for region (or regional industry) reflection maximum and minimum growth rates, trend normal growth (given the equilibrium rate of return) and an elasticity of investment. While the convex adjustment process parallels the GDyn approach, it adds stickiness to investment through the invocation of growth bounds based on pre-specified capital growth rates. It is also similar to the standard GDyn theory in that it assumes capital enters production in the period of investment and is retired according to the data base measures of the rate depreciation.

Modify the treatment of international capital flows

While the extended investment treatment allows for the retention of returns to entrepreneurial income in the model and, coincidentally, for a fuller treatment of adjustment costs, it does not fully resolve the stability problems characteristic of GDyn simulations. Specifically, with standard model parameters, firm income remitted to the global trust typically turns negative during simulations, causing the simulation to terminate with a data error. While it is possible to extend the time horizon of a simulation through re-calibrating the model's financial market rigidity parameters,[8] such an approach is not a robust solution that lends itself to a general purpose modelling technology.

To address this second source of instability there is a need to revise the GDyn capital-finance treatment.

In GDyn, the ownership of reproducible fixed capital in a region consists of two components: that owned by domestic residents and that owned by foreign residents. Foreign ownership is consolidated into a global trust. On the other side of the ledger, regional households hold wealth in a two-asset portfolio comprised of portfolio ownership of fixed capital in domestic firms and holdings in the global trust. The total across regions of firms' drawings on the global trust is equal to the total across regions of households' portfolio holdings in the trust.

Regional cost minimizing firms are considered to draw on funds needed for investment from domestic households and the global trust according to the relative expected cost of capital, while regional income maximizing households invest their wealth in regional firms or the global trust according to relative returns. Firm and household responsiveness to changes in financial market conditions is mediated in GDyn by two atheoretic resistance parameters intended to divide a strictly positive total value of regional wealth into positive components while staying as close as possible to the initial shares (Golub and McDougall 2102, pp. 100-101). The GDyn parameter $RIGWQH(r)$ determines the rigidity of allocation of regional wealth by the regional household, while the parameter $RIGWQ\_F(r)$ determines the rigidity of source of funding of regional firms. An underlying feature of GDyn is that regional income is shared between regional owners and the global trust in proportion to wealth holdings.

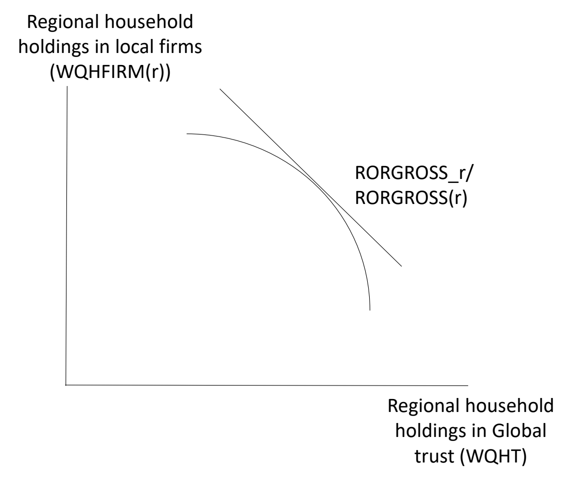
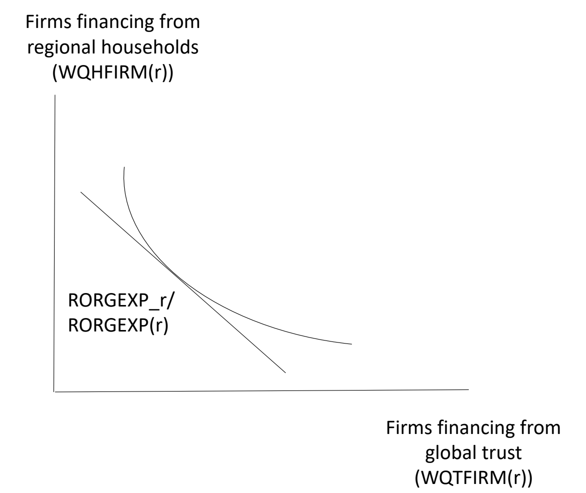
In order to directly allocate capital-finance between regions on the basis of relative returns, shift to a theoretic approach and, in doing so, achieve model stability, a standard optimising methodology is introduced. This method employs a constant elasticity of transformation (CET) function to mediate between domestic and foreign portfolio holdings by regional households on the basis of relative returns and a constant elasticity of substitution (CES) function to mediate between domestically and foreign source finance by regional firms on the basis of the relative expected cost of finance (figure 3).[9] In this depiction, it is assumed that households

[8] The re-calibration of the rigidity parameters is discussed in Golub and McDougall 2012, pp. 121-123.

[9] This approach is similar to the approach adopted in Productivity Commission (2010) in which a CET|CES formulation was introduced to the GTAP model to examine the impact of polices affecting the relative cost of bilateral investments. That implementation differs from the GDyn application in this paper in that it was comparative static and not dynamic, and implemented a disaggregation of the global trust into a 25 region by 25 region flow of funds.

respond adaptively to observed (actual) relative returns (measured by model variable $RORGROSS$) and firms respond to expected to expected costs of finance ($RORGEXP$).[10]

Figure 3: **Household and firm optimizing behaviour with a CET|CES approach to modelling capital-finance**

Households portfolio investment decisions	*Firms sourcing of funds decisions*
	

Under this approach, a CET parameter of the theoretical model replaces the GDyn atheoretic parameter $RIGWQH(r)$ while the CES parameter replaces $RIGWQ\_F(r)$ (table 2).

Table 2: **To address a second source of instability there is also a need to revise Gdyn capital-finance treatment**

GDyn-2012 – utilizes funding rigidity parameters	*GDyn-F – utilizes standard CET|CES theory*
RIGWQH(r) # rigidity of allocation of wealth by regional household #	CETCF(r) # region-specific elasticity of transformation domestic/foreign holdings #;
RIGWQ_F(r) # rigidity of source of funding of enterprises #;	CESCF(r) # region-specific elasticity of substitution domestic/foreign capital finance #;

Source: GDyn theory and author formulation.

In this modified framework, regional households' CET decisions for allocating regional wealth to the global trust (variable $wqht(r)$) is characterised by:

$$wqht(r) = wqt(r) + CETCF(r) * (rorga\_r - rorga(r)) + swqht(r)$$

[10] Another treatment would be to assume that the saving of each region accumulates in a global trust which undertakes investment equal to the projected level of saving. The incremental capital services projected to accrue with changes in investment (net of depreciation) would be allocated across regional industries according to relative competitiveness of those industries. Such an approach, for example, is adopted in the MIRAGE model (Decreux and Valin (2007), Fontagne *et al.* (2013)).The separation of inward and outward flows as in GDyn affords the possibility of recognising home-country bias in capital-finance decisions of regional households and industries. It also affords possibilities for directly modelling the impact of changes in regional institutions and risk (of importance, for example in modelling the impacts linkages between the operation of country/regional financial markets and growth (Schumpeter (1935), McKinnon (1973) and Shaw (1973) and later writers).

— GDyn equation $EQYHOLDHHDFND$ (box 2). In this model, regional households invest in the global trust $wqht(r)$, according $CETCF$ weighted average gross returns accruing to the global trust, in percentage changes, ($rorga\_r$) relative to the gross returns at home ($rorga(r)$). The returns from the global trust is calculated as the weighted sum of returns across all regional economies — $rorga(r)$. An increase in returns from the global trust relative to returns available locally, would lead the regional household to increase its holdings in the trust. Adoption of actual returns conforms with the assumption that regional households react adaptively to observed returns in budget allocation decisions, consistent with the model's regional household consumption-saving decisions. To close the model and allow for exogenous household portfolio decisions, the shift term $swqht(r)$ is included. Under the income and wealth accounting conventions of GDyn, the income and wealth shares of a region in the global trust are assumed to coincide.

Box 2: **TABLO code for Gdyn implementation of CET theory for household investment decisions, in percentage changes**

```
Coefficient (parameter)(all,r,REG) CETCF(r) # region-specific
elasticity of transformation domestic/foreign holdings #;
Variable (all,r,REG) wqht(r) # equity held by the regional hhld in
the global trust #;Variable (all,r,REG) rorga(r) # actual rate of
return #;
Variable wqt # equity held by the global trust #;
Variable (all,r,REG) rorga(r) # actual rate of return #;
rorga_r # Percentage change in actual rate of return on capital from
the global trust # ;
Coefficient (all,r,REG) WQ_THHLDSHR(r) # share of region r in the
global trust #;
Equation E_rorga_r
rorga_r = sum{r,reg , WQ_THHLDSHR(r) * rorga(r)} ;
Equation EQYHOLDHHDFND # eq'n determines equity holdings of the
regional hhld in the global trust # (all,r,REG)
wqht(r) = wqt + CETCF(r) * (rorga_r - rorga(r)) + swqht(r) ;
```

Source: Author's modified GDyn TABLO code.

Regional firms' CES decisions for determining the sourcing of funds from the portfolios of regional households (variable $wqhf(r)$) is characterised by:

$$wqhf(r) = wq\_f(r) - CESCF(r) * (rorge(r) - rorge\_r) + swqhf(r)$$

— GDyn equation $EQYHOLDHHDLCL$ in box 3.[11] In this model, regional firms source total funding requirements $wq\_f(r)$ locally according $CESCF$ weighted expected returns, in percentage changes, ($rorge(r)$) relative to the average expected returns to the global trust ($rorge\_r$). The expected returns to global capital is calculated as the weighted sum of expected returns in regional economies — $rorge(r)$. An increase in the expected cost of local finance relative to finance from the global trust would thus lead local firms to reduce financing from local households. The adoption of expected returns conforms with the assumption that regional

[11] The equation name is carried over from the original atheoretic specification.

firms react adaptively to those returns in their investment decision making (see above). To close the model and allow for exogenous funding decisions, the shift term $swqhf(r)$ is included. As noted, under the income and wealth accounting conventions of GDyn, the income and wealth shares of a region in the global trust are assumed to coincide.

Box 3: TABLO code for Gdyn implementation of CES theory for firm financing decisions, in percentage changes

```
Coefficient (parameter)(all,r,REG) CESCF(r) # region-specific elasticity of substitution domestic/foreign capital finance #;
Variable (all,r,REG) wqhf(r) # equity held by the regional household in domestic firms #;
Variable (all,r,REG) wq_f(r) # equity in firms in region r #;
Variable (all,r,REG) rorge(r) # expected gross rate of return #;
Variable rorge_r # Percentage change in expected rate of return on capital for the global trust # ;
Coefficient (all,r,REG) WQ_TFIRMSHR(r) # share of the global fund in firms in region r #;
Equation E_rorge_r
rorge_r = sum{r,reg , WQ_TFIRMSHR(r) * rorge(r)};

Equation EQYHOLDHHDLCL # eq'n determines the local equity holdings of the regional hhld # (all,r,REG)
wqhf(r) = wq_f(r) - CESCF(r) * (rorge(r) - rorge_r) + swqhf(r) ;
```

Source: Author's modified GDyn TABLO code.

Closing the capital-finance account and market clearing

With the shift terms $swqhf$ and $swqht$ exogenous, regional firms decisions to source finance locally and regional households' decisions invest in the global trust are determined, respectively. With regional firms' local finance decisions determined, firm financing from the global trust is determined, through the relation:

$$wqtf(r) = wq\_f(r) + swqtf(r)$$

where the $wqtf(r)$ is percentage change in the funds source by regional firms from the global trust, $wq\_f(r)$ is the total funding required by regional firms, as above, and $swqtf(r)$ is a naturally endogenous shift term (box 4 shows the TABLO implementation of this concept).

With household investment in global trust and firms demand for funds determined, supply of funds by regional households to local firms is also determined by market clearing relation:

$$WQHHLD(r) * wqh(r) = WQHFIRM(r) * wqhf(r) + WQHTRUST(r) * wqht(r)$$

where $WQHFIRM(r), WQHTRUST(r)$ are data base values of household wealth in regional firms and the global trust, respectively (see box 4).

Box 4: **TABLO code for Gdyn implementation of market clearing of the finance account**

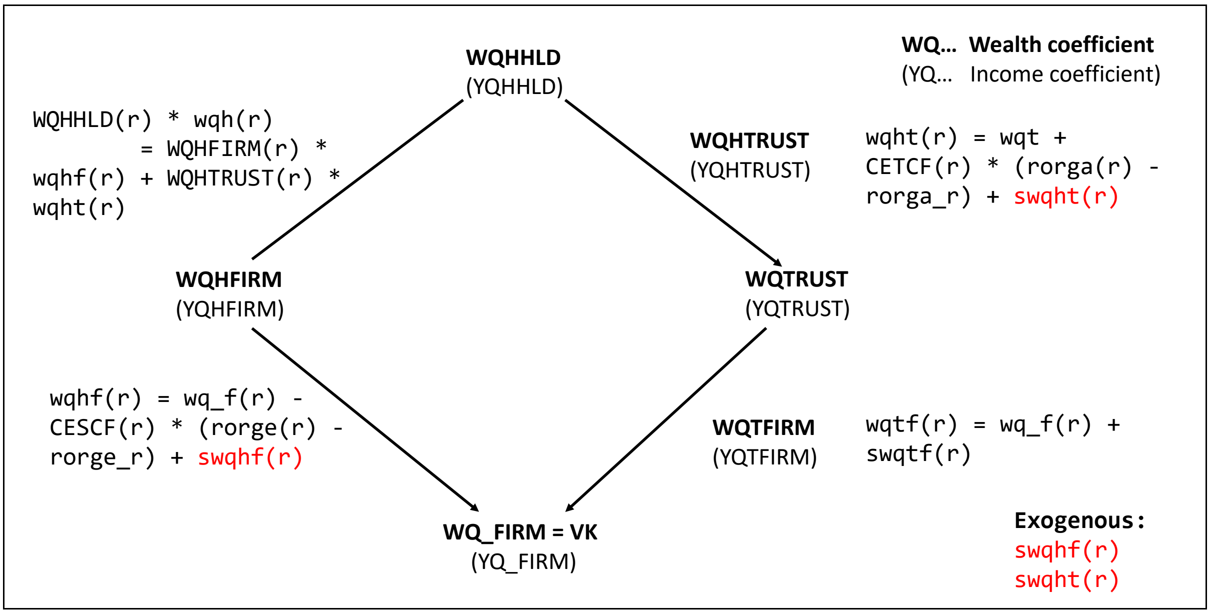
```
Equation EQYHOLDFNDHHD # eq'n determines the equity of the trust in domestic firms # (all,r,REG)
wqtf(r) = wq_f(r) + swqtf(r) ;
Variable (all,r,REG) wqh(r) # wealth of region r #;
Equation EQYHOLDWLTH # eq'n determines shift variable for wealth of the regional hhld # (all,r,REG)
WQHHLD(r) * wqh(r) = WQHFIRM(r) * wqhf(r) + WQHTRUST(r) * wqht(r);
```

Source: GDyn TABLO code.

Summary of GDyn modelling of wealth linkages with CET|CES theory

In broad terms, GDyn finance theory links the supply of funds to firms for investment in fixed capital by households and the global trust and the demand for those funds by regional firms (figure 4). In the modified theory introduced above, the demand for funds is conditioned by a CES relationship depicted on the regional-firm to regional household axis in the lower left hand section of figure 4. The supply of funds is conditioned by a CET relationship depicted on the regional household to global trust axis in the top right hand section of figure 4. Market clearing conditions are depicted on the top left hand and bottom right hand axes of figure 4. As noted, the regional shift variables $swqht(r)$ and $swqhf(r)$ are exogenous while $swtf(r)$ is naturally endogenous.

Figure 4: **Stylized representation of GDyn-F wealth linkages for a single region[a]**



[a] Corresponding income linkages shown in brackets.
Source: Based Ianchovichina and McDougall (2012), figures 2.1 and 2.2.

Selection of CES and CET parameter values

To put the modified theory into effect, it is necessary to assign values to the model CES and CET parameters. With the application of the firm and household funding and investment behaviour expressed in nominal terms, the Cobb-Douglas case would be represented by CES and CET parameter values of zero. This setting is a reasonable reference setting as it aligns with the GDyn (and GTAP) regional household aggregate utility function which specifies that total regional income is spread over private household expenditures, government expenditure and saving according to constant shares. However, this case is likely to be too restrictive over time for modelling global capital flows as it does not take account of the changing regional composition of the global economy. This shortcoming would be of particular concern for the analyses of periods of rapid compositional change or for analyses with a particularly long time horizon where substantial changes in the regional distribution of economic activity and income can occur gradually. These considerations suggest non-zero values are appropriate.

With uncertainty concerning possible values, a sensitivity approach has been adopted with four benchmark combinations, in addition to the Cobb-Douglas case (table 5). Consistent with the home country bias effect underpinning the setting of the atheoretic parameters in GDyn, the CET parameter is set at half the value of the CES parameter. This reflects the notion that regional households have a greater bias towards investing at home than regional firms have in sourcing funding for new investment from local sources.[12] The higher settings on the CES parameters provide local firms with the opportunity to shift to offshore funding sources if local investment opportunities exceed local funding possibilities (for example, during a period of capital deepening or an economic growth spurt).

Table 5: **Possible CES|CET parameter settings considered**

	CESCF	*CETCF*
Cobb-Douglas	0.00	0.00
Test 1	0.02	0.01
Test 2	0.05	0.03
Test 3	0.10	0.05
Test 4	0.20	0.10

Source: Author conjectures.

With respect to the CES parameters, values of between 0.02 and 0.2 were tested. The tests were conducted on the six region by thirteen sector data base used for the illustrative simulations in this paper. The tests indicated that the lower value parameters constrained annual variation in funding and investment flows while the parameter values at the high end encountered model stability problems due to high year-on-year variation in funding and investment flows.

[12] This convention is consistent with the balance adopted by Golub and McDougall (2012, pp. 126,7) in setting the GDyn atheoretic wealth allocation parameters.

Parameter values in test case 3 were found to afford model stability (at the level of aggregation reported) and are used in all illustrative simulations reported.

In applying the methodology in economic analysis, it would be prudent to simulate the Cobb-Douglas case together with other cases to test the sensitivity of results to alternative assumptions about the flexibility of funding and portfolio investment decisions.

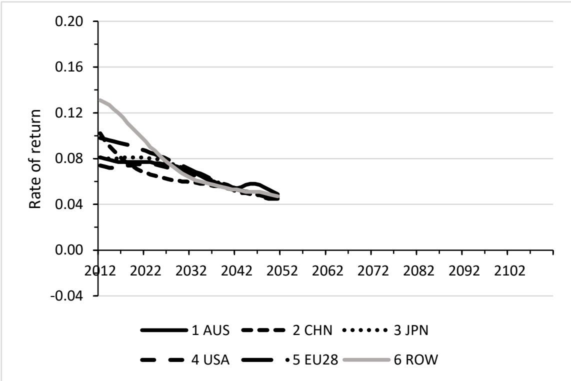
3 Simulation with capital-finance specification included

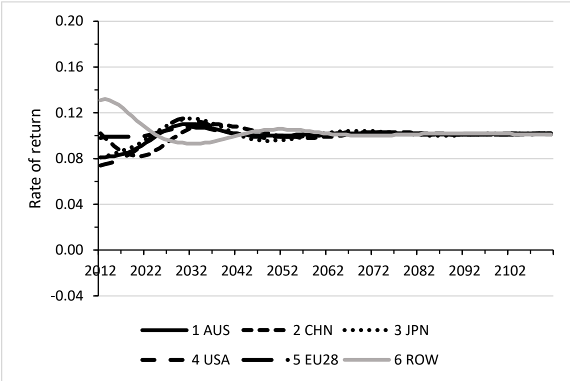
Model stability and achieving a long-run equilibrium

The simulations examining the stability of GDyn were repeated using GDyn-F and results between the two models compared. As above, the simulation represented changes in the economies occurring only on account of the passage of time and the elimination of region risk premiums. The first thing to notice is that in GDyn-F simulations, the models measure of the gross rate of return is projected to converge to a common and constant rate across regions (figure 5, top right hand panel). According to the model theory, that rate is the target gross rate of return, the measure of returns without regional risk premiums as recorded in the standard data base. With the six region, thirteen industry aggregation, deviations of the actual rate of return from the target rate are largely eliminated within 60 years, given standard GDyn adjustment parameters. This occurs after a series of oscillating adjustments by the regional economies as each region adjusts gradually through investment responses to adjustments in the expected rate relative to the actual and the (fixed) target rates of return. The model measure of the normal rate of growth of capital also converges towards zero for all regions over the same period (figure 5, bottom right hand panel). These results show that the model with modification meets the stability conditions for a long-run equilibrium. This contrasts with the original model — GDyn-2012 — which, as noted, does not (figure 5, left hand panels).

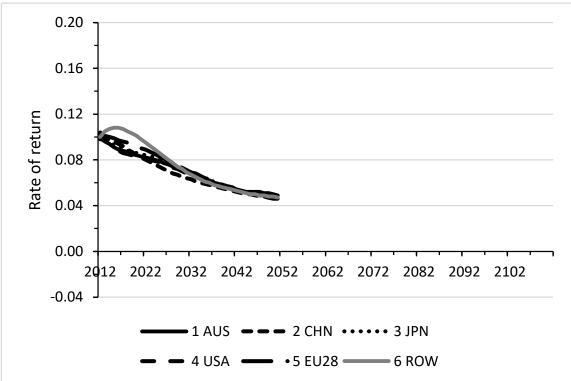
Figure 5: **Model stability conditions can be achieved through GDyn-F, 100-year simulation with only time shocked**

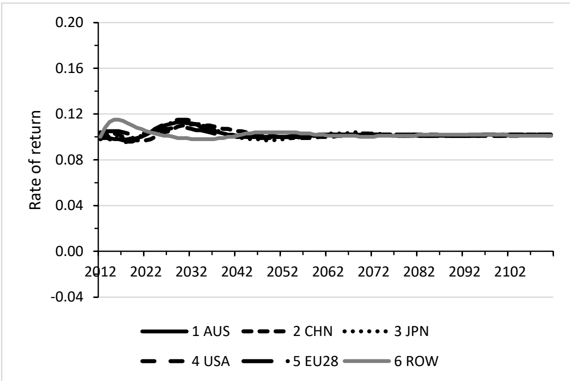
GDyn-2012 (from above)	***GDyn-F***
RORGROSS	*RORGROSS converges to RORGTARG*



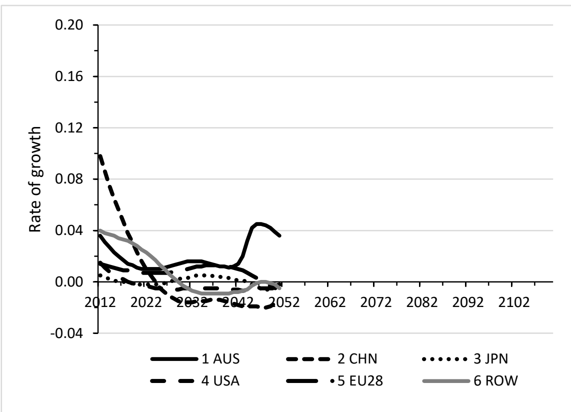


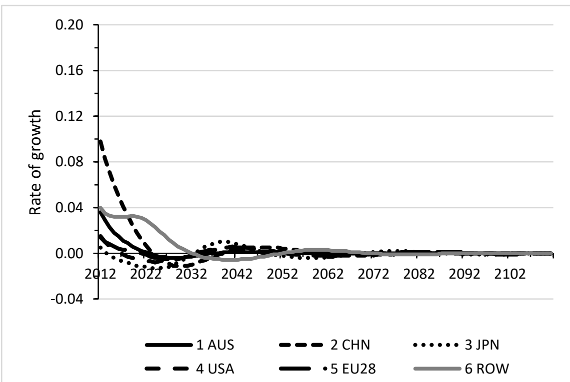
RORGEXP	*RORGEXP converges to RORGTARG*





Price neutral rate of growth in capital stock (KHAT)	*Price neutral rate of growth in capital stock (KHAT) converges to zero*





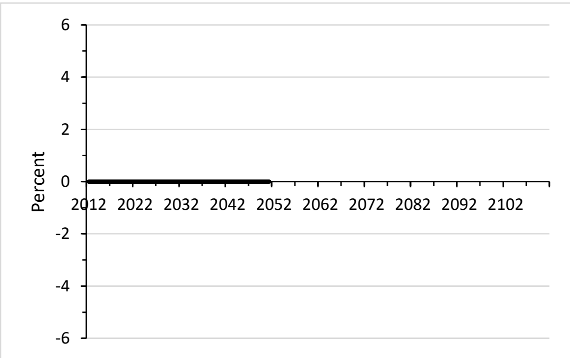
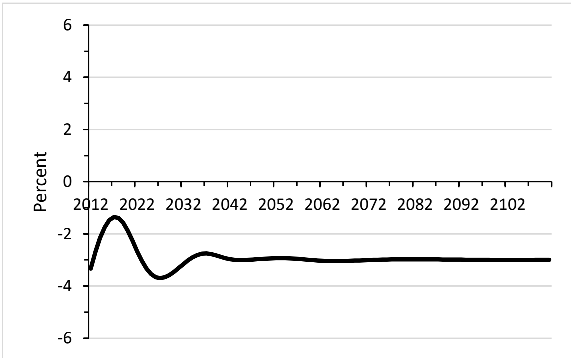
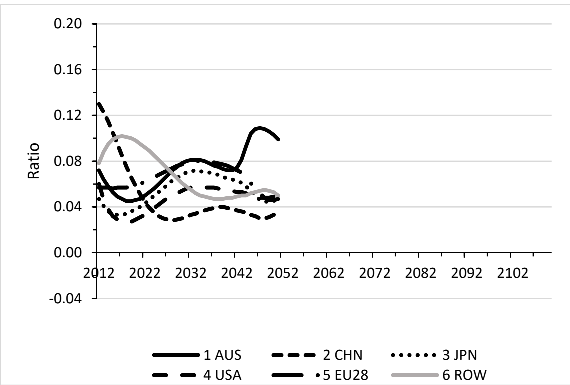
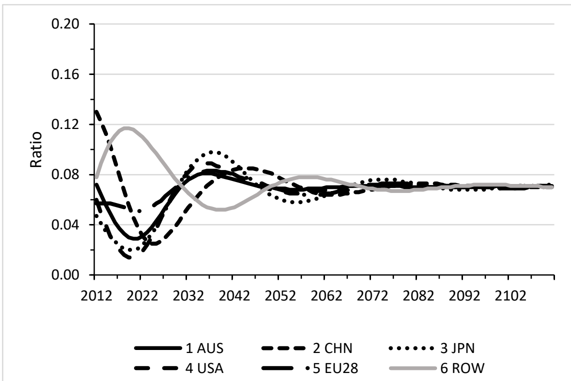
Note: Based on Ianchovichina and McDougall 2012, figures 2.5 and 2.7.

Source: Author estimates based on GDyn-2012 and Gdyn-F.

As discussed above, GDyn-F was closed by allowing the capital services adjustment shift term ($sqkworld$) to vary while fixing the model common target rate of return (through exogenising $rorgt\_r$). In the discussion of this convention, it was conjectured that the capital adjustment item would converge to around (a negative) 3 percent, that is, the difference between the depreciation rate of a uniform 4 percent and the ratio of gross investment to the value of fixed capital stocks. While the conjectured value is not enforced in the simulations, the model estimate does converge to a value of around -3 percent after a period of adjustment (figure 6, top right hand panel). The gross investment to capital ratio converges towards the conjectured value of around 0.07 (that is, 7 percent of the value of capital). An issue is whether the projected value converges to a similar reference value in a modelling base case in which output, population and employment are projected to increase over time. This case is considered below. With the capital adjustment term assumed exogenous in GDyn-2012, the value is projected as unchanged (figure 6, left hand panel). The gross investment to capital ratio does not converge.

Figure 6: **What do projections of $sqkworld$ and the gross investment to capital look like over an 100-year simulation with only time shocked?**

Percent, ratio

<table>
<tr><th>GDyn 2012</th><th>GDyn-F</th></tr>
<tr><td>Capital adjustment costs (sqkworld) (percent)</td><td>Capital adjustment costs (sqkworld) converges to around minus 3 percent</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>IKRATIO</td><td>IKRATIO converges to around 0.07 (7percent)</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Note: Based on figure 2.8 Ianchovichina and McDougall (2012).

Source: Author estimates based on GDyn-2012 and Gdyn-F.

Global trust and regional wealth shares

As noted, the atheoretic funding rigidity parameters in GDyn were set at values intended to retain the division of wealth and sourcing of funding, as close as possible to initial values. The theoretically-based projections allocate funds according to relative returns, but do not target a particular outcome. How the actual projections turn out for the simulation accounting for the passage of time and the elimination of regional risk premiums is illustrated by reference to three benchmark wealth shares.

- The share of the global trust in regional firms (model coefficient $WQ\_TFIRMSHR(r)$), with shares adding to 1 across regions. By the CES theory of the modified model, firms would substitute domestic funding of new investment for funding from the global trust if the (expected) cost of funds from the trust were lower than domestic costs, relative to initial levels.
- The regional household share in the global trust (model coefficient $WQ\_THHLDSHR(r)$), with shares adding to 1 across regional household holdings in the global trust. By the CET theory of the modified model, households would channel funds in favour of the global trust if the (actual) returns on those funds were higher than returns available from domestic firms, relative to initial levels.
- The foreign ownership share of local capital (model coefficient$WQ_FTRUSTSHR(r)$), with shares being region specific. This share reflects the overall outcome of firms' funding and investment decisions. Greater investment opportunities in a region relative to other regions (and relative to domestic savings) would tend to be accompanied by an increase in this share as local firms sought to draw on global savings to fund new investment.

The expected gross rate of return is recorded in the GDyn data base as aligning closely across regions around close to the average rate. The actual gross rate of return varies more substantially across regions. The gross rate of the return varies across regions with the gross rate estimated for the rest of the world exceeding the average while the estimates for Japan, the United States and Australia lie below the average. The estimates for China and the EU28 lie around the average.

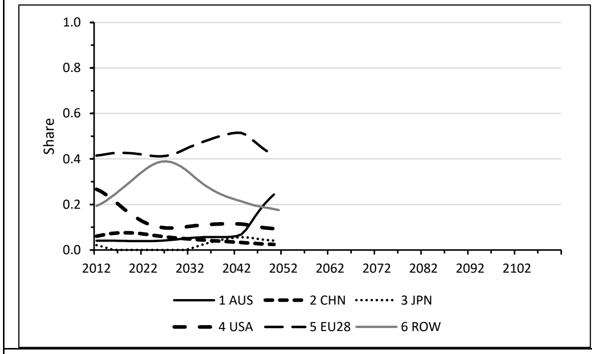
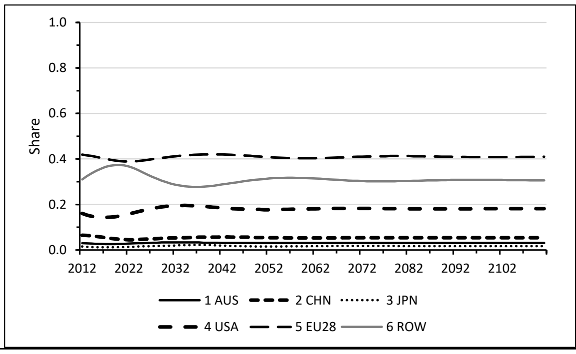
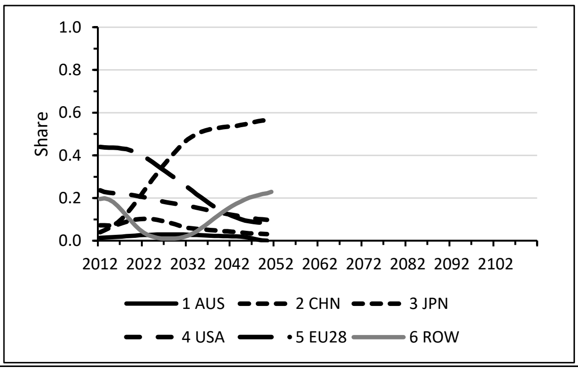
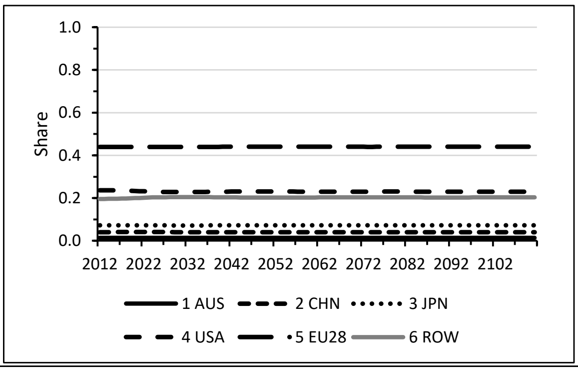
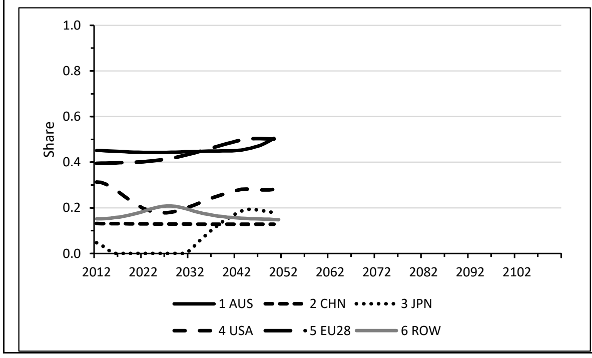
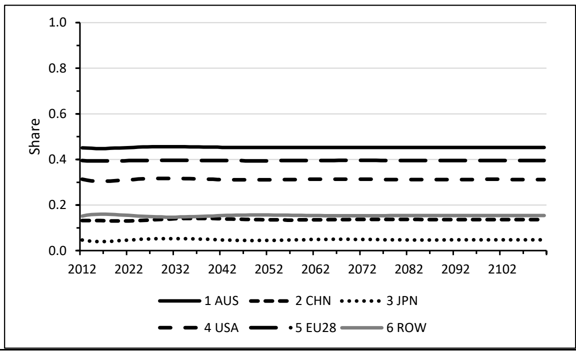
With the modelled elimination of the risk premium on the rest of the world, the planned growth in capital stocks (through variable KHAT, figure 7 bottom right hand panel) increases relative to other regions as investors react adaptively to the change. Higher expected returns locally (figure 7, centre right panel) lead local investors to look offshore for funding, raising the share of the global trust in firms in the Rest-of-the-World region (figure 7, top right hand panel).

With the modelled permanent decline in the actual gross rate of returns in the Rest-of-the-World region relative to other regions (and an increase in regional wealth), a permanent build-up of holdings by Rest-of-the-World households in the global trust occurs (figure 7, centre right hand panel). While the shifts in capital flows to and from the global trust are substantial, because of the home country effect, the net effect of these changes only translate, in this simulation, to relatively modest variations in the foreign ownership shares of local capital (figure 7, bottom right hand panel).

Although there are some similarities between the projected results using the modified GDyn model and the original model, the original model terminates before a long-run equilibrium is reached. As such the results from the original model do not provide a point of comparison for the longer run effects of the elimination of regional risk with the passage of time.

Figure 7: **Model base-line projections of global trust and regional household wealth shares in an 100-year simulation with only time shocked**

Shares

GDyn-2012	***GDyn-F***
Share of the global trust in regional firms (Regional shares add to 1)	*Model coefficient WQ_TFIRMSHR(r)*
	
Regional household share in global trust (Regional shares add to 1)	*Model coefficient WQ_THHLDSHR(r)*
	
Foreign ownership share of local capital (Region specific shares)	*Model coefficient WQ_FTRUSTSHR(r)*
	

Note: GDyn-F parameters: CET 0.05; CES 0.10.

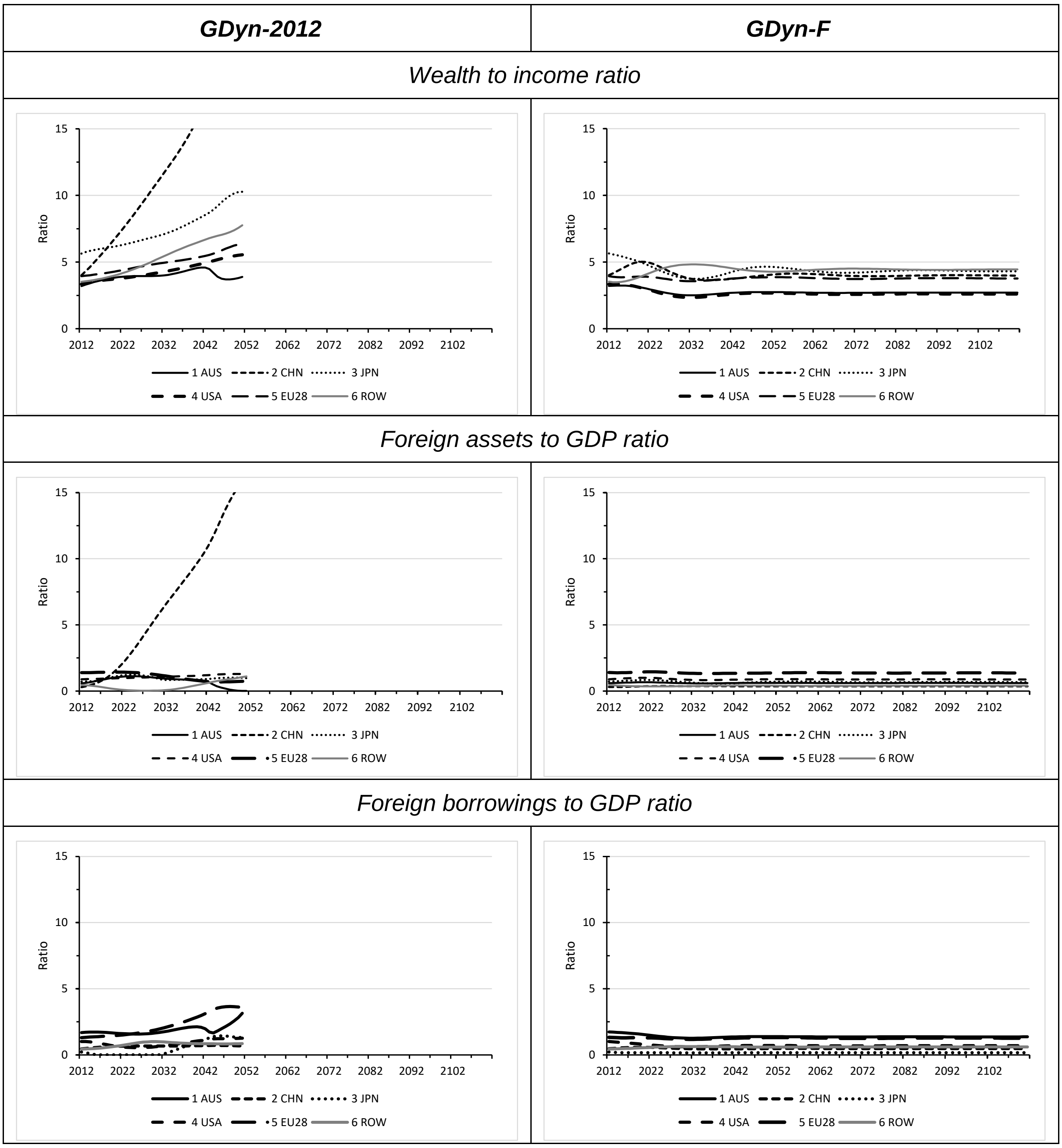
Source: Author estimates based on GDyn-2012 and GDyn-F.

Wealth and asset shares

For completeness, this section reports projections of national wealth as a ratio of regional income, and foreign assets and borrowing as a share of GDP for each of the six regions in the model data base. These comparisons follow comparisons in Golub and McDougall (2012) made in the context of the introduction of atheoretic endogenous saving behaviour to regional households as a means of stabilising the target ratios. The main finding from the additional results of the modified model is that the retention of the entrepreneurial income component of gross rate of return (through exogenising the model target rate of return) and the introduction of theoretically based optimisation behaviour to regions firms and household in their funding and investment decisions, yields more stable results than afforded by the original model (figure 8).

One result for the modified model warranting attention is the permanent changes in the wealth to income ratios across regions (figure 8, top rate hand panel). For the Rest-of-the-World region, the elimination of region-specific risk (through a reduction in the regional risk premium), was projected to lead to an increase in the level of wealth to regional income associated with the projected accumulation of capital and capital deepening. The elimination of country-specific risk through an increase in the regional risk premia, as for the United States, Japan and Australia, was projected to lead to capital shallowing and a lowering of the level of wealth relative to income. These results testify to the importance of the choice of the regional target rate of return in model simulations.

Figure 8: **National wealth as a ratio of income and foreign assets/ borrowings as a share of GDP in a 100 year simulation with only time shocked**



Notes: GDyn-F parameters: CET 0.05; CES 0.10. Based on figures 14.9 and 14.12 in Golub and McDougall (2012). The reference figures show net foreign assets as a share of GDP. This figure shows the inflow and outflow components.

Source: Author estimates based on GDyn-2012 and Gdyn-F.

4 Simulation of a stylised baseline

A baseline in a dynamic framework such as GDyn depicts how regions in the global economy could be expected to evolve over a given period of time in the absence of a policy or other economic change of interest. Baselines are generally projected using assumptions about possible changes population, labour market participation and productivity.[13] A baseline may also take into account the potential impacts of key policies that have already been agreed, or are expected to occur, as well as other influences on regional economies or global activity such as educational attainment, environmental considerations, propensity to save, assessment of regional institutions and risk changes in taste. While such a baseline is intended to inject additional realism into projections of the economic effects of policy and other economic changes, such projections can also be undertaken against a baseline with only time changing (such as represented by the projections reported above). Such projections would abstract from the interactions of target change with any additional effects of ongoing economic change.

Baseline projections and model closure

The focus of the illustrative simulations reported in this and the next section is on a baseline that incorporates projections of GDP, population, skilled labour and unskilled labour for each of the six regions modelled. The CEPII projections provided with the standard GDyn database are used. These projections cover the 39-year period from 2011, the database year, out to 2050.

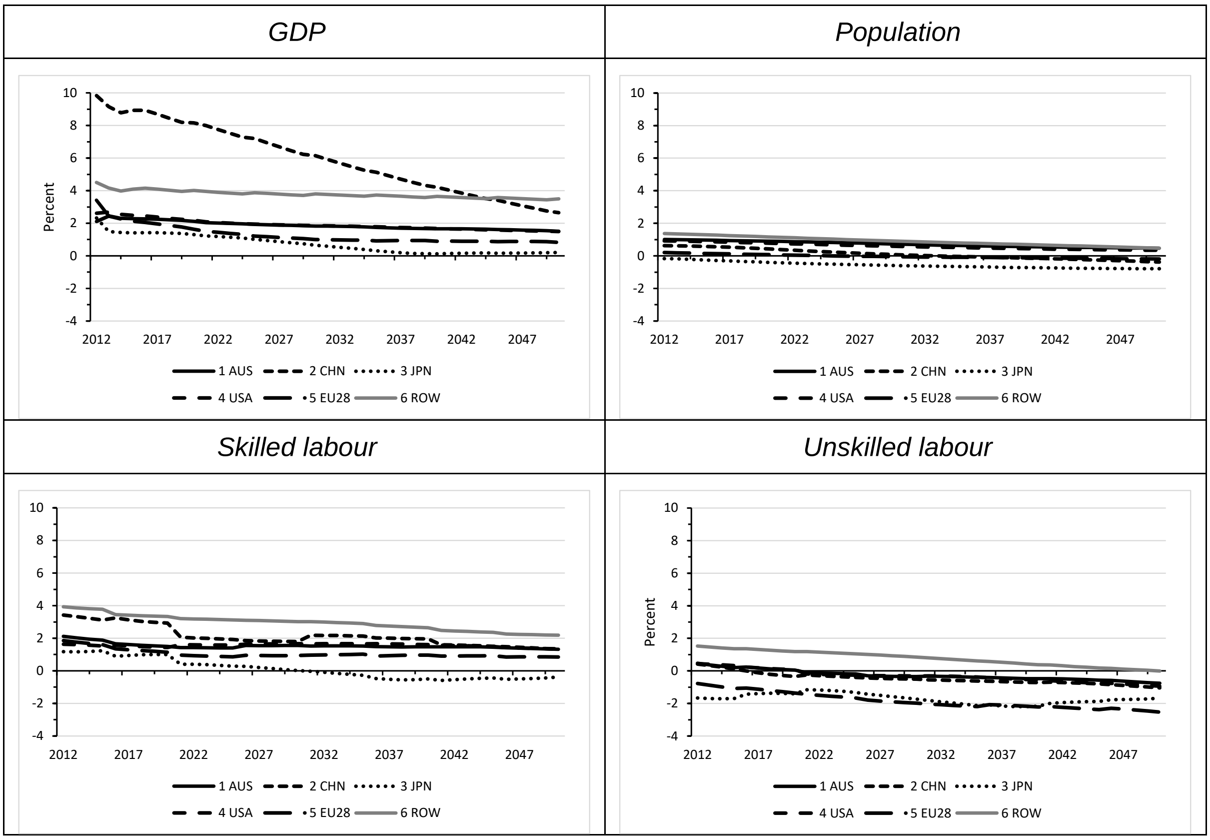
The projections reflect an expectation that GDP growth in China and the regions covered by the Rest-of-the-World grouping will continue at rates above those for Australia, Japan, the United States, and the EU28 economies, with projected growth rates for China and Japan, in particular, tapering off (figure 9, top left panel). Population growth rates for all regional groups are projected to decline, while growth rates for Japan are projected to be negative through the period and to turn negative for China and the EU28 economies (figure 9, top right panel). Growth rates for Skilled labour and Unskilled labour inputs are also projected to decline, more so for the Unskilled group (figure 9, bottom left and right panels). For this later group, growth rates are projected to negative for all regions except the Rest-of-the-World (figure 9, bottom right panel).

With GDP growing ahead of labour inputs, the growth in labour productivity needed would accrue from productivity growth and capital deepening. With Skilled labour inputs growing ahead of (or contracting less than) Unskilled labour inputs, growth would characteristically be biased towards activities that are more intensive in the use of Skilled labour or that are the more capital intensive.

[13] For example, see Productivity Commission (2010) and Gabbitas and (2017) for a discussion of the application of a baseline (or reference case) at a national level for the evaluation of national economic reform, and Walmsley *et al.* 2012) for a detailed description of a baseline in GDyn.

Figure 9: **Baseline projections of GDP, population and labour input growth by region, GDyn projections sourced to CEPII, 2012 to 2050**

Percentage change



Source: CEPII estimates provided with GDyn_V36, file: Projectionsforthe112_v3.zip downloaded from GTAP webpage 26 June 2016.

In GDyn, growth in regional populations and labour inputs are naturally exogenous.[14] With GDP usually determined by the model, that is, naturally determined endogenously, it is necessary to swap this activity variable with another in baseline simulations. Following Walmsley *et al.* (2012), GDP by region is swapped with the region-specific technological change variable for 'non-accumulable endowment commodities', that is, land, natural resources and labour.[15] Under this treatment, technological change is determined endogenously while gross investment is also determined endogenously by the investment theory of the model whereby regional firms vary gross regional investment to: eliminate errors in expectations; align actual and target rates of return; and gradually align the actual with the normal rate of capital growth. An historical validation of changes in gross investment, installed capital and capital services in the years that the projection period overlaps with years for which data are available is beyond the scope of the methodological focus of this paper.

The baseline simulations recognise that regional risk may not be eliminated over the projection period. In the absence of other information, it is assumed that difference in country institutions

[14] Through model variables: pop, $qfactsup[SkLab]$ and $qfactsup[UnskLab]$.

[15] Technological change in non-accumulable endowment commodities is represented by the model variable $afereg(r)$. The model closure swap is therefore: $\boldsymbol{swap}\ afereg\ = qgdp$; $!\, en = ex$.

and risk is measured by the difference between the global average and the regional gross rates of return in the initial 2011 database.[16]

Model baseline projections

In any one year, output may increase through the addition of labour and capital inputs or through technological and organisational change. With the year-on-year labour input growth projected to decline across regions both absolutely and relative to projected output growth (figure 9), growth in output would be more dependent on growth in capital inputs and technological and organisational change. Reflecting this dependence, technical change in 'non-accumulable endowments' and fixed capital (and hence capital services in production) are projected in GDyn-F to increase, but overall, at higher rates than labour input and output growth (figure 10, right-hand panels, first and second rows). At the regional level, the convergence of the high growth rates for China towards the average for the diversified regions of the United States and the EU-28 are accompanied by a progressively lowering of the projected growth in fixed capital and technology for China (figure 10, right hand column, first and second rows). In contrast, the above average output growth rates projected for the Rest-of-the-World region are projected to be accompanied by similarly higher growth rates in fixed capital and technological and organisational change (figure 10).

The projections provided by the GDyn-2012 model do not appear to follow this regular pattern, with some outliers appearing (figure 10, left-hand panels, first and second rows). The GDyn-2012 model simulation terminated with an error after 28 of the 39-year projection period (that is, in 2039 compared to the projection period end-year of 2050).

Consistent with simulations reported above, the gross rate of return is projected to decline in GDyn-2012 simulations (figure 10, bottom left panel). This contrasts with GDyn-F model projections of relatively stable gross rates of return by region (figure 10, bottom right panel). The GDyn-F projections thus retain a component for entrepreneurial income and regional risk.

Concerning the global capital adjustment cost variable ($sqkworld$), it was asked above whether projections for the variable would converge to around a negative 3 percent in a baseline simulation using the modified model. The illustrative projections indicate such convergence is indeed characteristic of the modified model (figure 11, top right hand panel). An issue is whether and how such adjustment costs are embodied in historical data and whether those costs could be modelled more explicitly in the GDyn framework. Resolution of this issue is beyond the scope of this paper, which is focused on achieving a stable long-run equilibrium and an improved general purpose platform for dynamic modelling in GDyn.

The projections also indicate that in a baseline characterised by variable growth in output between regions combined with capital (and technology) deepening, regions are likely to tend towards region-specific gross investment to capital ratios that are above current levels (figure 11, bottom right hand panel). A similar longer-term picture is provided by GDyn-2012 projections, although those projections are not uniform and do not reach the end of the projection period leaving some uncertainty as to the final outcomes implied.

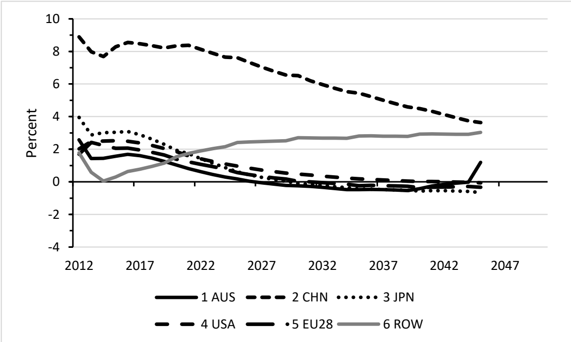
[16] This assumption is implemented by setting the model target rate of return by region $RORTARG(r)$ equal to the gross rate ($RORGROSS(r)$) in 2011.

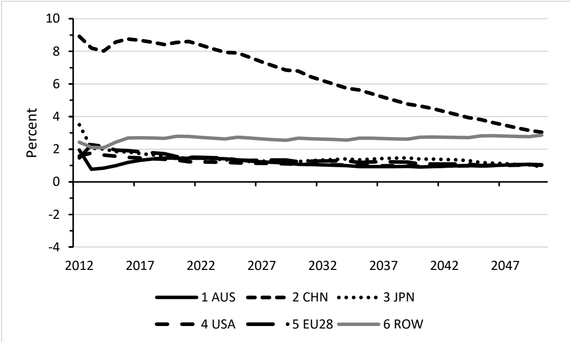
Figure 10: **Model baseline projections of primary factor technical change, fixed capital and gross rate of return, 2012 to 2050**

Percentage change, rate

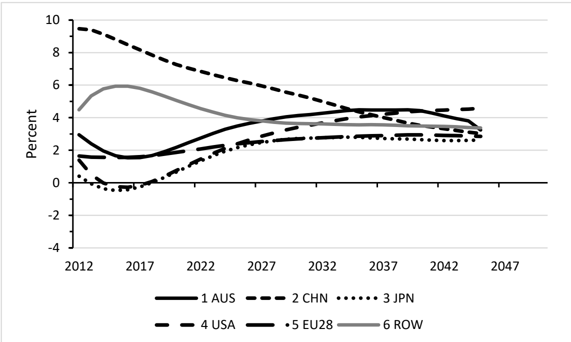
GDyn-2012	***GDyn-F***

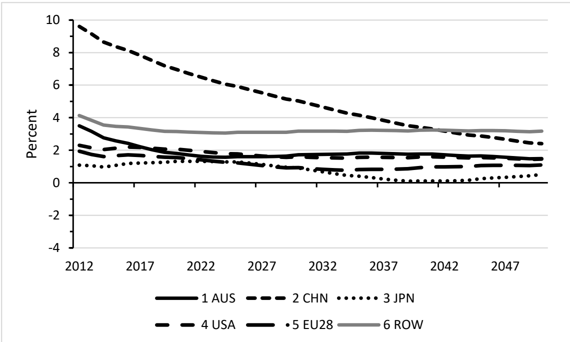
Primary factor technical change in non-accumulable endowments (% change)



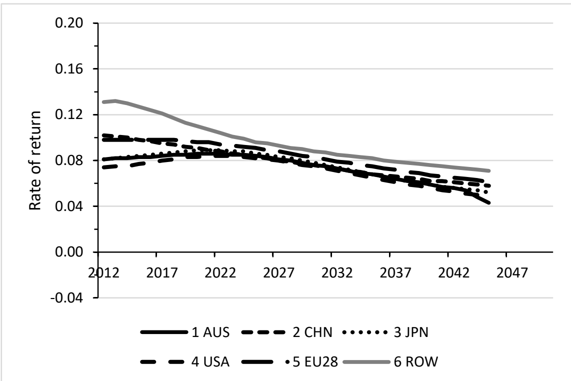


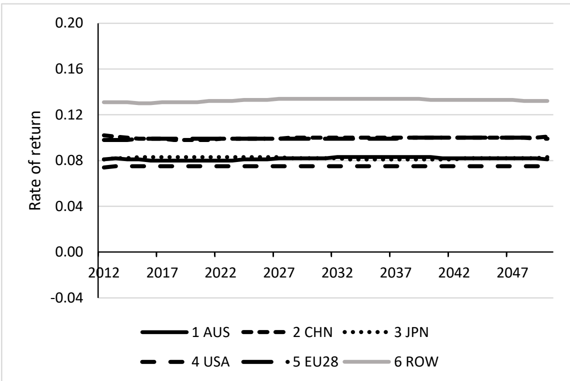
Growth in fixed capital (% change)





Gross rate of return



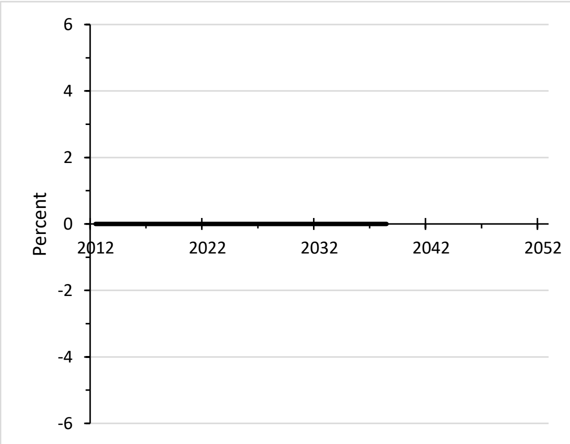


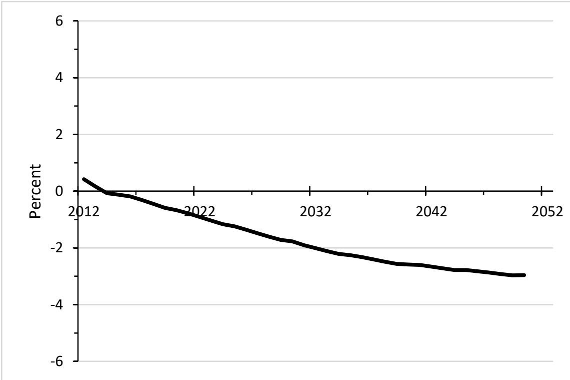
Note: GDyn-F parameters: CET 0.05; CES 0.10.

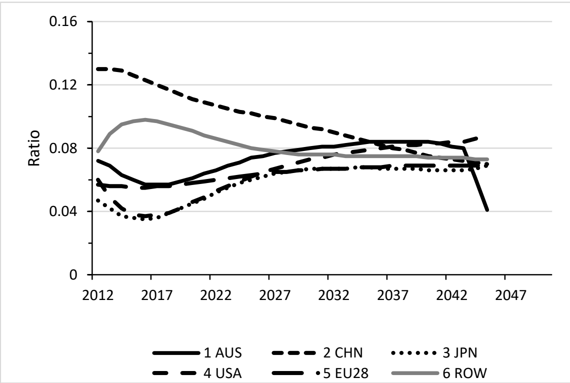
Source: Author estimates based on GDyn-2012 and GDyn-F.

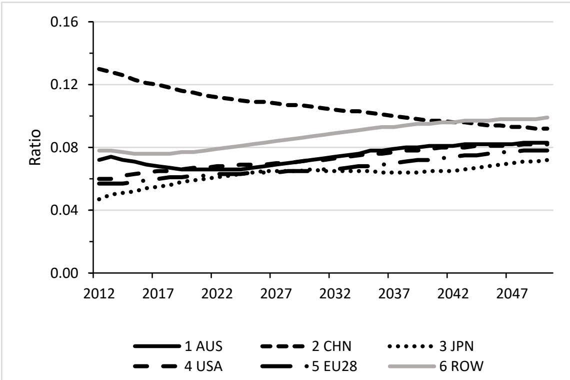
Figure 11: **What do projections of $sqkworld$ and the gross investment to capital look like over the base-line projection period**

Percent, ratio

<table>
<tr><th>GDyn 2012</th><th>GDyn-F</th></tr>
<tr><td>Capital adjustment costs (sqkworld) (percent)</td><td>Capital adjustment costs (sqkworld) tends to around minus 3 percent</td></tr>
<tr><td>


</td><td>


</td></tr>
<tr><td>IKRATIO</td><td>IKRATIO tends towards region-specific rates</td></tr>
<tr><td>


</td><td>


</td></tr>
</table>

Notes: GDyn-F parameters: CET 0.05; CES 0.10. Based on figure 2.8 Ianchovichina and McDougall (2012).

Source: Author estimates based on GDyn-2012 and Gdyn-F.

Wealth, ownership and income shares

For completeness, this section reports (in figures 12 and 13) baseline projections of the wealth, ownership and income shares comparable to those reported in figures 7 and 8 (above) for the case where only time is shocked with the elimination of regional differences in risk.

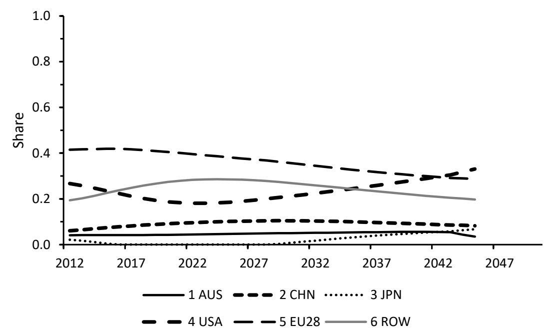
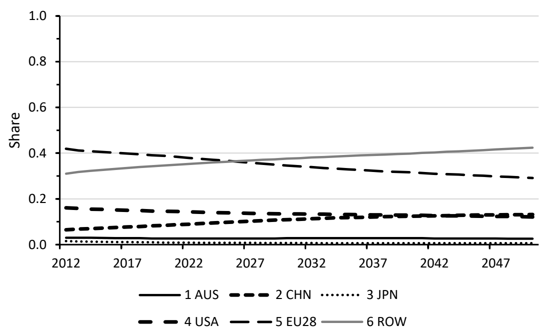
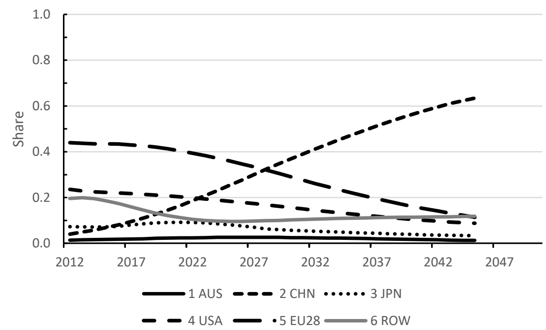
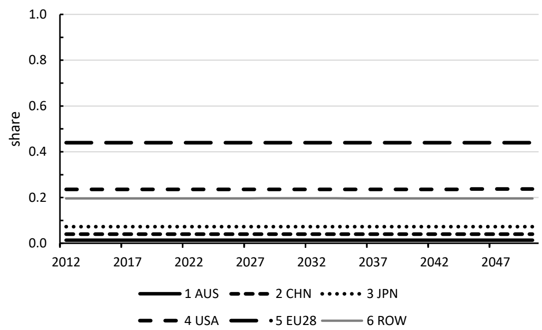
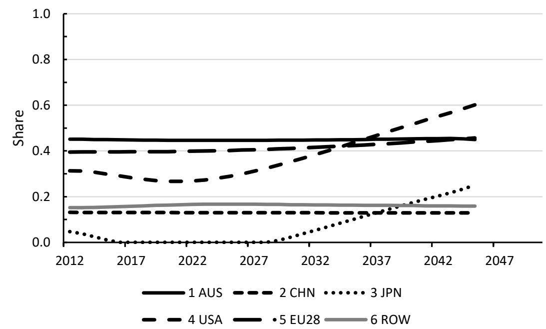
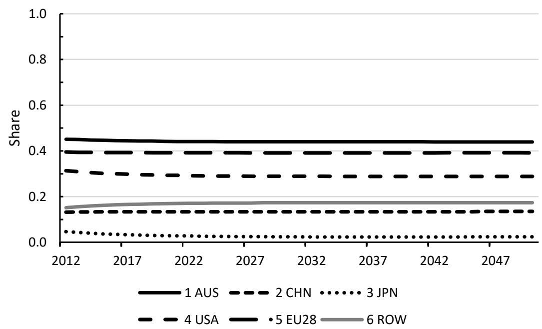
The projections suggest that with the above average GDP growth in the Rest-of-the-World region (figure 9) and associated higher fixed capital growth, the share of global investment funds invested in this region will increase (figure 12, top right panel). The relatively high growth in China (around a declining trend) is also projected to be associated with a rise in the share of global funds going to that region. These projected increases are associated with a projected decline in the share of global funds invested in other regions, particularly the EU28. These changes are associated with a (slight) rise in the foreign ownership share of local capital in the Rest-of-the-World region with slight declines in the foreign ownership shares in Australia and the United States relative to the current situation (figure 12, bottom right panel).

Projections by the GDyn-2012 model suggested more stark shifts particularly so towards the termination of the baseline simulation with error.

Baseline projections using the modified model — GDyn-F — suggest only slight and gradual changes in the benchmark wealth to income, foreign asset to GDP and foreign borrowings to GDP ratios (figure 13, right hand panels). The original model — GDyn-2012 — projects larger variations, particularly in the projection years leading to the end of the baseline simulation with error.

Figure 12: **Model base-line projections of global trust and regional household wealth shares, 2012 to 2050**

Shares

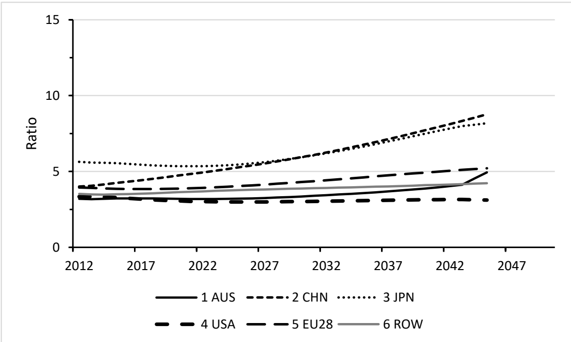
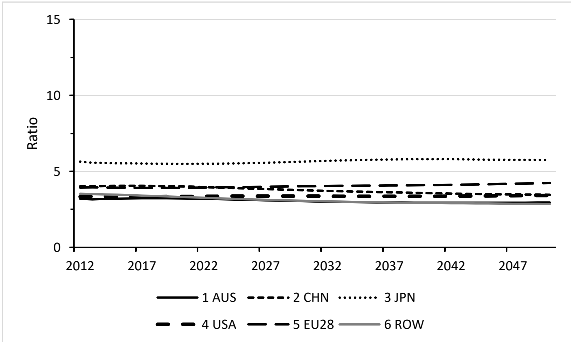
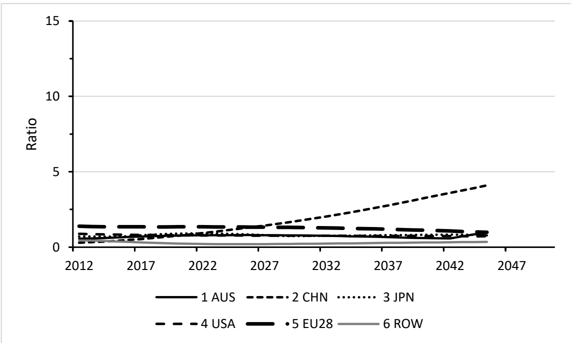
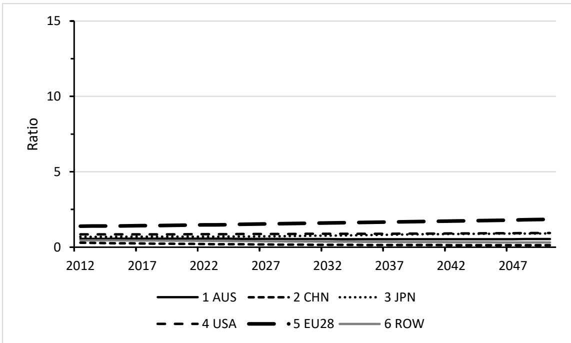
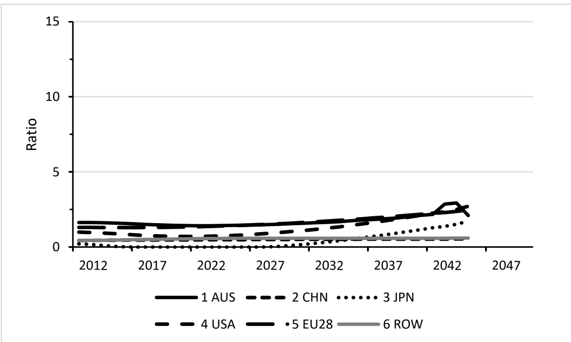
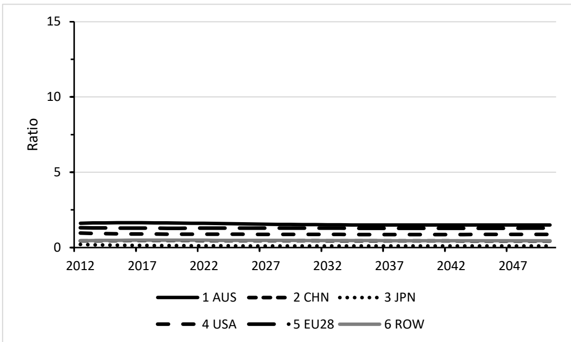
<table>
<tr><th>GDyn-2012</th><th>GDyn-F</th></tr>
<tr><td>Share of the global trust in regional firms (Regional shares add to 1)</td><td>WQ_TFIRMSHR(r)</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Regional household share in global trust (Regional shares add to 1)</td><td>WQ_THHLDSHR(r)</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Foreign ownership share of local capital (Region specific shares)</td><td>WQ_FTRUSTSHR(r)</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Note: GDyn-F parameters: CET 0.05; CES 0.10.

Source: Author estimates based on GDyn-2012 and GDyn-F.

Figure 13: **Model baseline projections of national wealth as a ratio of income and foreign assets/ borrowings as a share of GDP, 2012 to 2050**

Ratios

<table>
<tr><th>GDyn-2012</th><th>GDyn-F</th></tr>
<tr><td colspan="2">Wealth to income ratio</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Foreign assets to GDP ratio</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Foreign borrowings GDP ratio</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Notes: GDyn-F parameters: CET 0.05; CES 0.10. Based on figures 14.9, 14.12, 14.13 and 14.15 in Golub and McDougall (2012). The reference figures show net foreign assets as a share of GDP. This figure shows the inflow and outflow components.

Source: Author estimates based on GDyn-2012 and Gdyn-F.

5 Simulation of an illustrative policy scenario

The analysis of the long-run equilibrium indicated how regional economies may respond to changes in country institutions and risk — simulated as the convergence of the six regional economies considered to a common target rate. This simulation provides a benchmark within the GDyn framework for assessing the impact of national policies or changes in general economic conditions that may affect regional institutions and risk. This benchmark simulation was conducted with only time shocked. It could be extended to include baseline estimates of GDP, population and labour force growth such as those presented above as well as more nuanced assumptions about the evolution of regional institutional and risk profiles.

In this section, the additional illustrative scenario considered is a one-off 1 percent primary factor productivity improving technological change in a medium-sized open economy. Australia is selected to represent such an economy. As many government policies (including trade policies) affect productivity, the simulation is of general relevance. In the simulation, the impacts are reported as percentage deviations from the baseline. Because the baseline embodies changes in the structure of the global economy, the projected impacts of a policy change affecting one region (in this case Australia) will also be influenced by those changes, that is, the policy projections will be 'path dependent'. For comparison, the projections are made using the original GDyn model (labelled GDyn-2012 as above) and the modified model (GDyn-F). The GDyn 2012 simulations extend to 2039 the year when the model terminated with error. The GDyn-F simulations extend for the full baseline projection period ending in the year 2050. The productivity improving technical change is assumed to occur in the first year of the simulation period, that is, 2012. GDyn-F projections are shown by solid lines in figure 14; GDyn-2012 projections by dotted lines.

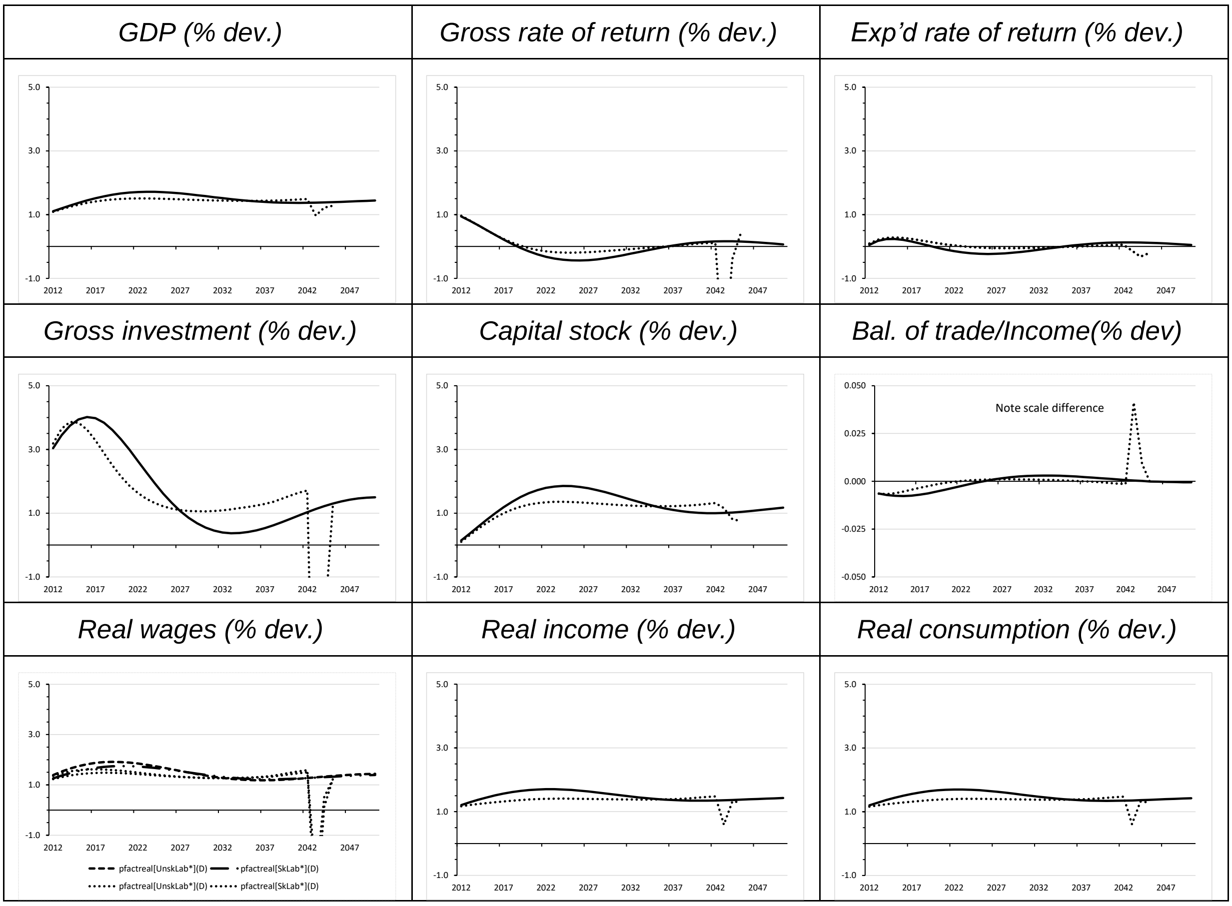
A 1 percent improvement in primary factor productivity is projected to raise real value-added output by around 1 percent relative to the baseline, in the year that it occurs (figure 14, top left panel). The estimated impact (not graphed) on the other five regions modelled is negligible.

In the first year, with given labour and capital inputs, gross returns to capital and real wages are projected to rise as is consumption in line with the scale of economic activity. With the gross return to capital rising, gross investment is also projected to rise and with it the ratio of the balance of trade to GDP as imports make up the shortfall in supply of investment goods (figure 14, centre right panel). The expected rate of return is then projected to rise with the increase in actual returns to capital and planned capital growth (figure 14, top right panel), adding further to gross investment activity. New investment progressively augments capital stocks (figure 14, centre panel). The process of adjustment is projected to continue towards a long-run equilibrium around a regular oscillating pattern with the GDyn-F model.

The GDyn-2012 projections (shown, as noted, by dotted lines in figure 14), initially follow a pattern similar to the GDyn-F projections, then deviate sharply as the series approaches the period when that model terminates with error. The oscillations on GDP and gross investment are projected to be more compressed with GDyn-2012 than with GDyn-F.

Figure 14: **Economy-wide adjusts to a 1 percent increase in primary factor technological change for a medium-sized open economy, the case of Australia, 2011 to 2050**

Percentage deviation from base line



Notes: GDyn-F parameters: CET 0.05; CES 0.10. Projections with GDyn-F reported by solid lines, GDyn-2012 by dotted lines.

Source: Author estimates based on GDyn-2012 and Gdyn-F.

6 What achieved and some other issues

This paper presents an improvement in the economic behaviour in GDyn that satisfies the long-run neo-classical equilibrium conditions set for the model. The improvement is directed at overcoming a weakness in the original model whereby returns to capital decline without apparent bounds. The improvement also introduces optimization behaviour of regional firms in financing capital investment and regional households in the allocation of household wealth between domestic firms and the model's global investment trust.

The innovation is intended to contribute to the micro-foundations of the model and to model stability over long time periods, the latter being confirmed through steady state model simulation periods out to 200 years, with results reported to the 100-year mark.

The stronger theoretical foundations and model stability are intended to help extend the general use of global CGE modelling techniques to analyse issues of interest within a standard, well documented and readily available framework. The modified model is also intended to provide a firmer platform for the development of a more nuanced baseline, and other innovations such as the modelling of regional household saving behaviour and firm behaviour in response to changes in the terms under which finance is provided to regional firms for capital accumulation. The innovation also provides a firmer platform for the modelling of longer-run issues where model stability over long periods is important, including generational and educational issues as well as environmental issues.

There are some further matters that are suggested for consideration and possible further development, including:

- the modelling of adjustment costs — what scope is there to expand and refine the modelling of adjustment costs to improve the articulation of the links between new investment, growth in capital stocks and the provision of capital services in production from those stocks?
- the modelling of regional household saving — what scope is there to introduce a more flexible treatment of regional household optimization allowing saving rates to vary along the lines of the modelling of firm financing decisions and household investment decisions introduced in this paper?
- the modelling of labour supply and labour demand by occupation — would there be merit for longer term modelling to allow for mobility between occupational groups within the standard framework to augment the setting of values for occupational employment exogenously through the modelling baseline?
- the values for primary factor substitution elasticities (ESUBVA) — are the default values of between 1.12 and 1.63 (for non-primary industries) carried forward from comparative static modelling in GTAP the most appropriate values for year-to-year dynamic modelling?
- the values for the elasticity of transformation of primary factors between industries (ETREA) — similarly, are the default values of 1 for land, 2 for labour and capital from standard GTAP the most appropriate values for year-to-year modelling?

With the availability of a series of GDyn databases and an extensive range of country and region development indicators from the World Bank, the OECD, Conference Board and others, there appears to be a substantial global statistical base that could support an historical validation of the GDyn model. Such a validation exercise could inform productivity and adjustment assumptions made in modelling baselines and the evaluation of projected responses to policy and other economic change. It could also inform choice of model behavioural parameter values.

It is hoped that the innovations introduced in this paper through the GDyn-F model will help encourage the wider use of dynamic modelling in a global framework and lead to further developments to the Global Trade Analysis Project frameworks. Some suggestions are made for directions of further development and other matters to consider.

Appendix

Table A1: **Country/region mapping**

Regions in database	*G20*	*Code*	*Country(s) in database region*
1 Australia	G20	AUS	Australia
2 China	G20	CHN	China
3 Japan	G20	JPN	Japan
4 United States	G20	USA	The United States
5 European Union (28)	G20	FRA	France
	G20	DEU	Germany
	G20	ITA	Italy
	G20	GBR	United Kingdom
	G20	REU	Austria; Belgium; Bulgaria; Cyprus; Czech Republic; Denmark; Estonia; Finland; Greece; Hungary; Ireland; Latvia; Lithuania; Luxembourg; Malta; Netherlands; Poland; Portugal; Romania; Slovakia; Slovenia; Spain; Sweden
6 Rest-of-the-World		NZL	New Zealand
		HKG	Hong Kong
	G20	KOR	Korea
		TWN	Taiwan
	G20	IDN	Indonesia
		MYS	Malaysia
		PHL	Philippines
		SGP	Singapore
		THA	Thailand
		VNM	Vietnam
	G20	IND	India
		ROA	Cambodia; Iran; Kazakhstan; Kyrgyzstan; Laos; Myanmar; Pakistan; Sri Lanka; Bangladesh; Rest of East Asia; Rest of Oceania; Rest of South Asia; Rest of Southeast Asia; Rest of Western Asia
	G20	CAN	Canada
	G20	MEX	Mexico
	G20	BRA	Brazil
	G20	ARG	Argentina
		ROM	Bolivia; Caribbean; Chile; Colombia; Costa Rica; Ecuador; Guatemala; Nicaragua; Panama; Paraguay; Peru; Uruguay; Venezuela; Rest of Central America; Rest of North America; Rest of South America

Continued next page

Table A1: **Country/region mapping (continued)**

Regions in database	*G20*	*Code*	*Country(s) in database region*
27 Turkey	G20	TUR	Turkey
28 Russia	G20	RUS	Russian Federation
29 Rest of Europe		ROE	Albania; Armenia; Azerbaijan; Belarus; Croatia; Georgia; Norway; Switzerland; Ukraine; Rest of EFTA; Rest of Eastern Europe; Rest of Europe; Rest of Former Soviet Union
30 Saudi Arabia	G20	SAU	Saudi Arabia
31 South Africa	G20	ZAF	South Africa
32 Rest of World		ROW	Rest of Africa and the Middle East, and other countries not separately identified.

Source: Author's GTAP version 9a data base aggregation.

Table A2: **GTAP industries and industry sectoring adopted**

No.	GTAP code	Industries	No.	Industry sectors adopted
1	pdr	Paddy rice	1	Grains, Crops, Forestry
2	wht	Wheat	1	Grains, Crops, Forestry
3	gro	Cereal grains nec	1	Grains, Crops, Forestry
4	v_f	Vegetables, fruit, nuts	1	Grains, Crops, Forestry
5	osd	Oil seeds	1	Grains, Crops, Forestry
6	c_b	Sugar cane, sugar beet	1	Grains, Crops, Forestry
7	pfb	Plant-based fibres	1	Grains, Crops, Forestry
8	ocr	Crops nec	1	Grains, Crops, Forestry
9	ctl	Cattle, sheep, goats, horses	2	Livestock, fishing
10	oap	Animal products nec	2	Livestock, fishing
11	rmk	Raw milk	2	Livestock, fishing
12	wol	Wool, silk-worm cocoons	2	Livestock, fishing
13	frs	Forestry	1	Grains, Crops, Forestry
14	fsh	Fishing	2	Livestock, fishing
15	coa	Coal	3	Mining
16	oil	Oil	3	Mining
17	gas	Gas	3	Mining
18	omn	Minerals nec	3	Mining
19	cmt	Meat: cattle, sheep, goats, horse	4	Processed food
20	omt	Meat products nec	4	Processed food
21	vol	Vegetable oils and fats	4	Processed food
22	mil	Dairy products	4	Processed food
23	pcr	Processed rice	4	Processed food
24	sgr	Sugar	4	Processed food
25	ofd	Food products nec	4	Processed food
26	b_t	Beverages and tobacco products	4	Processed food
27	tex	Textiles	5	Textiles and clothing
28	wap	Wearing apparel	5	Textiles and clothing
29	lea	Leather products	6	Light manufacturing
30	lum	Wood products	6	Light manufacturing
31	ppp	Paper products, publishing	6	Light manufacturing
32	p_c	Petroleum, coal products	7	Heavy manufacturing
33	crp	Chemical, rubber, plastic products	7	Heavy manufacturing
34	nmm	Mineral products nec	7	Heavy manufacturing
35	i_s	Ferrous metals	7	Heavy manufacturing
36	nfm	Metals nec	7	Heavy manufacturing
37	fmp	Metal products	7	Heavy manufacturing
38	mvh	Motor vehicles and parts	7	Heavy manufacturing
39	otn	Transport equipment nec	7	Heavy manufacturing
40	ele	Electronic equipment	7	Heavy manufacturing
41	ome	Machinery and equipment nec	7	Heavy manufacturing

Continued next page

Table A2: **GTAP industries and industry sectoring adopted (continued)**

No.	GTAP code	Industries	No.	Industry sectors adopted
42	omf	Manufactures nec	6	Light manufacturing
43	ely	Electricity	8	Utilities
44	gdt	Gas manufacture, distribution	8	Utilities
45	wtr	Water	8	Utilities
46	cns	Construction	9	Construction
47	trd	Trade	10	Transport and communication
48	otp	Transport nec	10	Transport and communication
49	wtp	Sea transport	10	Transport and communication
50	atp	Air transport	10	Transport and communication
51	cmn	Communication	10	Transport and communication
52	ofi	Financial services nec	11	Financial services nec
53	isr	Insurance	11	Financial services nec
54	obs	Business services nec	11	Financial services nec
55	ros	Recreation and other services	12	Other services
56	osg	Public administration, defence, education, health	12	Other services
57	dwe	Dwellings	13	Dwellings

Source: Author's GTAP version 9a data base aggregation.

Table A3: **Primary factors**

Description	GTAP code	GTAP database description
Unskilled labour	UnSkLab	Agricultural and Unskilled
	UnSkLab	Clerks
	UnSkLab	Service/Shop workers
Skilled labour	SkLab	Technicians/AssocProfessional
	SkLab	Officials and Managers
Land	Land	Land
Natural resources	NatlRes	Natural Resources
Capital	Capital	Capital

Source: Author's GTAP version 9a data base aggregation.

References

Barro, R. and Sal-i-Martin, X. 1995, *Economic Growth*, McGraw-Hill, New York.

Carballero, R. 'A Fallacy of Composition', *The American Economic Review*, Vol. 82, No. 5, December, pp. 1279-1292.

Gabbitas, O. and Salma, U. 2016, *VUMR Modelling Reference Case, 2009-10 to 2059-60*, Staff Working Paper, Productivity Commission, Canberra.

Golub, A. and McDougall, R. 2012, 'Household Saving Behavior in the Dynamic GTAP Model: Evaluation and Revision', Chapter 14 in Ianchovichina, I. and Walmsley, T., *Dynamic Modelling and Applications for Global Economic Analysis*, Cambridge.

Decreux, Y. and Valin, H. 2007, 'MIRAGE, Updated Version of the Model for Trade Policy Analysis', CEPII, Working Paper No 2007-15, October.

Dixon, P. and Rimmer, M. 2002, *Dynamic General Equilibrium Modelling for Forecasting and Policy: A Practical Guide and Documentation of MONASH*, North Holland.

Fontagne, L., Foure, J., and Ramos, M.P. 2013, 'MIRAGE-e: A General Equilibrium Long-term Path of the World Economy', CEPII Working Paper, N°2013-39, December.

Gretton P. and Fisher, B. 1997, *Productivity Growth and Australian Manufacturing Industry*, Industry Commission Staff Research Paper, AGPS, Canberra.

Hamermesh, D. and Pfann, G 1996, 'Adjustment Costs in Factor Demands', *Journal of Economic Literature*, Vol. 34, No. 3, Sept, pp. 1264-1292.

Ianchovichina, I. and McDougall, R. 2012, 'Theoretical Structure of Dynamic GTAP', Chapter 2 in Ianchovichina, I. and Walmsley, T., *Dynamic Modelling and Applications for Global Economic Analysis*, Cambridge.

Ianchovichina, I. and Walmsley, T. 2012, *Dynamic Modelling and Applications for Global Economic Analysis*, Cambridge.

Leamer, E. 1988, 'The sensitivity of international comparisons of capital stock measures to different real exchange rates', *The American Economic Review: Papers and Proceedings*, No. 78, pp. 479-483.

Lucas, R. 1967, 'Adjustment Costs and the Theory of Supply', *Journal of Political Economy*, Vol. 75, No. 4, Part 1, August, pp. 321-354.

McKibbin, W. and Vines, D. 2000, 'Modelling Reality: The Need for Both Inter-Temporal Optimization and Stickiness in Models for Policy Making', *Oxford Review of Economic Policy*, Vol. 16, No. 4.

MacKinnon, R. 1973, *Money and Capital*, The Brookings Institution, Washington.

Productivity Commission 2010a, 'A CGE Analysis of Some Economic Effects of Trade Agreements', Supplement to *Bilateral and Regional Trade Agreements*, Productivity Commission Research Report, Canberra.

Productivity Commission 2010b, *Impacts and Benefits of COAG Reforms: Reporting Framework*, Research Report, Canberra, December.

Schumpeter J.1934, *The Theory of Economic Development*, Sixteenth Printing, Cambridge, Mass. Harvard University.

Shaw, E. 1973, Financial Deepening in Economic Development, New York, Oxford University Press.

Walmsley, T., Dimaranan, B. and McDougall, R. 2012, 'A Baseline Scenario for the Dynamic GTAP Model', Chapter 5 in Ianchovichina, I. and Walmsley, T., *Dynamic Modelling and Applications for Global Economic Analysis*, Cambridge.

Walmsley, T. and McDougall, R. 2012, 'Negative Investment: Incorporating a Complementarity into the Dynamic GTAP Model', Appendix in Ianchovichina, I. and Walmsley, T., *Dynamic Modelling and Applications for Global Economic Analysis*, Cambridge.