

Estimation of Growth Elasticities in the General Equilibrium Framework under Model Uncertainty

Askar Mukashov^a, Ding Jin^b, Christian Henning^b and Johannes Hedtrich^b

¹Kiel Institute for the World Economy, Germany

²Institute of Agricultural Economics, University of Kiel, Germany

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Abstract

In recent years the concept of uncertainty analysis is being employed more frequently in order to test the validity and robustness of results of deterministic Computable General Equilibrium (CGE) models. This paper suggests the formal method to evaluate the Total Factor Productivity (TFP) growth elasticities under the uncertainty of all exogenous parameters of the CGE model developed by the International Food Policy Research Institute (IFPRI). First, we use the historical data and our assumptions to parametrize the uncertainty of the model parameters and present it as a covariance matrix. Then we employ the Latin Hypercube Sampling method to draw a sample of 1000 simulation scenarios from the multivariate normal distribution characterized by this covariance matrix and the vector of the baseline parameters. Then for each generated scenario we run the IFPRI CGE model and get the endogenous response variables. Finally, we employ a metamodeling concept to formalize and evaluate the relationship between the policy-induced sector-specific TFP growth and variables of interest given the uncertainty of other model parameters. The approach is applied empirically to a highly aggregated CGE model of the Senegalese economy.

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List of Acronyms

CGE	Computable General Equilibrium
TFP	Total Factor Productivity
SDGs	Sustainable Development Goals
IFPRI	International Food Policy Research Institute
SAM	Social Accounting Matrix
CES	Constant Elasticity of Substitution
CET	Constant Elasticity of Transformation
LES	Linear Expenditure System
GTAP	Global Trade Analysis Project
RMSE	Root Mean Squared Error
MARE	Maximum Absolute Relative Error

1 Motivation

Over the last decades, the focus of the global development agenda has gradually shifted from the Washington Consensus with its concentration on a macroeconomic performance to a much broader and multidimensional concept of the Sustainable Development Goals (SDGs) (Grimm et al., 2015; UNDP, 2012). For the domain of potential policy instruments, this conceptual change determined the shift of the focus from macro-level policies to sector-specific interventions.

Sector-level policy interventions can have diverse effects on indicators of sustainable development. For example, the public investment of the same amount either in an agriculture or industry sector can have the similar effect on income growth or poverty reduction, yet the investment in an industry sector is likely to deliver poorer environmental performance. All in all, the evolution of the development agenda formed the demand for the instruments that can quantitatively represent the impact of policy intervention options along the dimensions of the SDGs (Henning et al., 2018).

One of the instruments that can represent potential policy impact of available investments is CGE models¹. Comparing to traditional econometric-based policy evaluation studies (see e. g. Fan and Zhang 2008), this class of deterministic models has certain advantages that made them famous in the policy analysis. First, by the construction CGE models can capture both direct and indirect economy-wide effects of sector-specific policy intervention, while traditional impact evaluation studies typically do not go beyond a sector of an investigation. Second, unlike the ex-post nature of econometric-based methods, the ex-ante nature of CGE models allows considering measures that did not have historical precedents. Third, CGE models do not have high data requirements. This issue is particularly important for developing countries like Senegal, where reliable spatial and time-series data, that is essential for econometric models, is often not available.

At the same time, while CGE models are internally consistent and allow unambiguous identification of shock transmission mechanisms, their application requires potentially influential assumptions. First, functional forms of equations used in a CGE model need to be assumed or justified (see e. g. McKittrick 1998; Bouët et al. 2014). Then, for the specified functional forms the values of model parameters and exogenous variables² need to be assigned. The assignment is usually done based on own estimates (performed outside of CGE models), literature research or ad hoc assumptions (see e. g. Annabi et al. 2006). Furthermore, the validity and robustness of CGE simulation results can be tested by conducting so-called sensitivity analysis. This method implies re-parameterization of specific baseline

¹For an introduction to CGE models see Löfgren (2003)

²Hereinafter we use the terms ‘model parameter’ and ‘exogenous variable’ interchangeably.

parameters and consequent comparative analysis of corresponding response variables. This approach has been criticized for the inherited arbitrariness and lack of formal objective methodology (e. g. Schürenberg-Frosch 2015). A researcher has to decide what model parameters can change and how, and given the nature of a General Equilibrium, where any parameter affects all response variables, the number of potential re-parameterizations soon becomes infeasible. For example, three parameters each with just two distinct potential values already imply eight different simulations. Therefore, sensitivity analysis is usually conducted for arbitrarily chosen combinations of model parameters.

In recent years, however, the sensitivity analysis is being replaced by uncertainty analysis, which can be considered as a generalization of former. The uncertainty analysis implies multiple re-parameterizations of model parameters within defined ranges and consequent multiple runs of a CGE model in order to obtain possible variations of endogenous variables of interest (see e. g. Webster et al. 2008; Phimister and Roberts 2017). Following the recent developments in the literature, this paper is addressing the problem of uncertainty of CGE model parameters and suggests a new method to evaluate the expected effectiveness of policy intervention options using metamodeling tools³. We select Senegal as a case-study country and narrow the domain of policy intervention options and target performance indicators. In particular, we narrow the set of the SDGs to a GDP per-capita and per-capita emission of a CO₂ equivalent (from the combustion of fossil fuels). Former indicator serves us as a proxy for both income growth and poverty reduction, while the latter variable represents the environmental performance. As a domain of policy instruments we consider the growth of the TFP induced by the public investment in the following sectors of Senegalese economy:

- crop production;
- production of other agricultural products (forestry, fishing, and livestock);
- agribusiness sector (processing of agricultural products);
- other industrial production;
- public goods and services;
- private-sector services.

Accordingly, we present a method to evaluate sector-specific TFP growth elasticities with respect to income and environmental performance under the uncertainty of all other exogenous parameters of Senegalese CGE model. Section 2

³For an introduction to metamodeling concept see Barton (2015)

briefly outlines the most important specifics of the employed CGE model and performs the standard deterministic approach to calculate sectoral TFP growth elasticities. Section 3 quantifies the model uncertainty, introduces the most important concepts of the metamodeling and empirically evaluates the sectoral TFP elasticities. Section 4 highlights the potential implications for policy analysis and concludes.

2 Growth Elasticities: the standard approach

2.1 Structure of the Senegalese CGE model

We employ the standard Recursive Dynamic CGE model developed by the IFPRI (Löfgren et al., 2002; Diao and Thurlow, 2012) and corresponding Senegalese Social Accounting Matrix (SAM)⁴ for 2015 (Randriamamonjy and Thurlow, 2019). The use of the IFPRI-type CGE model implies that we make specific assumptions about the adjustment possibilities of the Senegalese economy.

Production block. We assume the 2-staged nested production function with a Constant Elasticity of Substitution (CES) for primary factors and Leontief function for the intermediate inputs at the bottom technology nests; at the top technology nest, we assume a Leontief function for aggregate primary and aggregate intermediates. In other words, we reserve the technological possibility for substitution between primary factors, but the intermediate inputs are assumed to be complementary to each other and aggregate primary inputs.

Domestic market. Senegalese SAM constructed by the IFPRI has a regionalized structure, meaning that several region-specific activities can produce similar, but not necessarily identical commodities (for example, region-specific food crops). At the same time, the CGE model assumes the single national market for commodities at which aggregate domestic demand meets aggregate domestic supply. Therefore, we reflect the degree of homogeneity of region-specific outputs using a CES function and model the aggregation of the products at the national market level.

Trade. Senegalese national products, in turn, are also not identical to products produced in other countries. We use a CES for import and a Constant Elasticity of Transformation (CET) for export products to model the degree of homogeneity of Senegalese products versus similar goods produced by its trading partners.

Consumption. We model the consumption side in the CGE model using the Linear Expenditure System (LES) function. Most importantly, this functional form highlights the importance of a subsistence consumption and, therefore, restricts the demand adjustment possibilities of the households. This factor is es-

⁴For a detailed explanation of a SAM concept, see Breisinger et al. (2009).

pecially crucial for Senegal, where according to the latest estimate, 46.7 % of the population is below the national poverty line (WB, 2018).

Closure rules. The definition of the model closure rules is based on the closure rules defined in Löfgren et al. (2002) and Diao and Thurlow (2012) and is tailored to adequately reflect the current adjustment possibilities of Senegalese economy.

Trade. Senegal is a member of the West African Economic and Monetary Union, and the exchange rate of CFA franc⁵ was historically pegged first to the French franc and later to the euro. The nominal exchange rate of CFA franc changed only twice - in 1948 and 1994 (Boogaerde and Tsangarides, 2005). Accordingly, to approximate current Senegalese trade adjustment possibilities, we assume the following:

- the trade currency is euro, and the nominal exchange rate of CFA franc is fixed (model numeraire);
- the current account balance is also fixed and given exogenously;
- consequently, the real exchange rate adjusts via endogenous domestic prices.

Primary factors. We employ a Recursive Dynamic version of a Standard IFPRI CGE model described in Diao and Thurlow (2012). This sub-type of IFPRI CGE models is designed to simulate medium-term developments, and in our analysis, we select a 10-year simulation period (2016-2025). Following Diao and Thurlow (2012), we assume that

- capital is fully-employed and immobile across sectors within the years;
- newly generated capital is mobile across sectors, and its distribution depends on capital rents paid by each sector ('putty-clay' assumption);
- labor and land are fully employed and mobile across sectors.

Government budget. Given (relative) public debt sustainability (IMF, 2017a), we assume rigidity of government taxation policy and endogenize government savings/investment variable. At the same time, we give special attention to government demand expenditures. In particular, we assume that the productivity growth that is exogenous in our CGE model has a direct correspondence to government demand. We assume that in addition to recurrent expenditures on public goods, government demand account in the SAM also includes public Research and Development (R&D). Then we use the estimation of Gumus and Celikay (2015) and

⁵The CFA (*fr. Communauté Financière d'Afrique*, "Financial Community of Africa") franc is the name of two currencies, the West African CFA franc and the Central African CFA franc. Both currencies are guaranteed by the French treasury and have been historically at parity and, therefore, are effectively interchangeable.

assume a one-to-one mapping between an economy-wide TFP growth and growth of the public R&D recurrent expenditures. This adjustment mechanism allows us to reflect the cost mechanism of the exogenous TFP growth: from one side, higher economy-wide productivity would require more government expenditures that in turn means fewer government savings/investment; from another side, more government expenditures would mean increased production of the public goods, and, therefore, consequent diversion of the resources from other sectors.

Savings-Investment. We use and modify so-called ‘balanced’ saving-investment closure rule described in Löfgren et al. (2002). The original ‘SI-4’ and ‘SI-5’ closures in Löfgren et al. (2002) are presented as a middle ground between classic savings-driven and investment-driven closures because they allow to evenly distribute the adjustment in absorption across all components and preserve eventual dominance of the investment demand (in case of savings-driven closure) or household demand (in case of investment-driven closure). In our model, we assume that

- the investment demand is endogenous, but its proportion in the total absorption is fixed;
- households’ propensity to save is rigid, and consequently, the saving rate of the enterprises is chosen as a balancing variable for the total investment;
- given the government budget closure, the proportions of government and private consumption in the total absorption are not fixed.

It is important to emphasize that chosen functional forms and closures determine the set of the model parameters that need to be assigned (the summary is given in Table 6 of the Appendix). Consequently, the degree of uncertainty of the specified CGE model depends on a degree of SAM aggregation. More disaggregated SAM implies a broader set of uncertain CGE model parameters. Therefore, we try to reasonably reduce the dimensions of the model uncertainty and aggregate the original SAM compiled by the IFPRI.

The regional breakdown done by the IFPRI in the original SAM (Figure 1 and Table 1) is the first factor that we take into account when designing the aggregation scheme. We follow the IFPRI’s breakdown for all SAM accounts, except for capital, which is assumed to be nationwide. For our simulation horizon, we acknowledge general regional immobility of all primary factors, but at the same time, we assume that newly generated capital should be mobile at the national level. Furthermore, we aggregate the production sectors and corresponding commodities. We begin with the typical breakdown for primary, secondary and tertiary economic sectors and take into account the specifics of Senegalese economy. In particular, given the focus of the policymakers on the agricultural sector (e. g. CAADP 2015), we distinguish crops, other agricultural commodities, and agribusiness sectors. Consequently, we separate the industrial sector not related to agribusiness.



Figure 1: Actual administrative regions (retrieved from the Wikimedia)

SAM	Administrative regions
r1	Dakar
r2	Thies & Diourbel
r3	Saint-Louis & Matam
r4	Louga, Kaolack, Fatick & Kaffrine
r5	Ziguinchor, Sédhiou, Kolda, Kédougou & Tambacounda

Table 1: Mapping between the SAM regions and actual administrative regions

Furthermore, given the importance of the public sector, we separate public goods and services, and, more importantly, we amend the original SAM such that public goods are consumed only by the government. This amendment allows us to emphasize that public goods and services are outside of consumers LES and that the price of this specific product depends on the production side. Finally, we also separate private services as a distinct sector.

The final structure of the SAM employed in our CGE model is presented in Table 7 of the Appendix.

2.2 Economic structure and baseline parameters

Now that we have set the functional forms and closure rules for our CGE model, we can use the SAM to sketch the starting point of Senegalese economy in the base year of 2015 (Tables 2 and 3).

Regional structure. Most of the country's economic activity is concentrated along the Atlantic coast, with Dakar (r-1) being the most significant region (55.5 % of GDP). Coastal regions are dominated mostly by services and industry, while remote inland areas specialize in agriculture. Accordingly, most of the urban population is concentrated in the coastal regions, while inland regions are mostly populated with rural residents. Furthermore, the shares of agricultural and public sectors are typical for a developing country (15.5% and 10.4% respectively), yet economy-wide the largest sectors are services and industry.

Trade. As in many developing countries, the Senegalese economy is more import-dependent than export-oriented. The industry is the most trade intensive sector, while public goods and services are non-tradable at all. Ceteris paribus this

implies that the industrial sector is the most prone to the shocks coming from the trading partners, while public goods mostly depend on domestic developments.

Factor payments. As expected, the most capital-intensive is the industrial sector, and services and agriculture more labor-dependent. The sector of other agriculture production is a special case because it consists of livestock, forestry, and fishery. These sub-sectors heavily depend on agricultural capital stock (**fcap-a**) that is deliberately separated from other non-agricultural capital (**fcap-n**). Furthermore, residential differences mostly characterize the income distribution from the households side. Rural households get most of their income from land, labor and agricultural capital rents, while urban households rely more on rents from non-agricultural capital. The most notable pattern from the intermediate inputs side is the high dependence of all sectors on industrial products.

	Sectors, % of the total GDP							Population, % of total		
	crop	oagr	agrib	oind	pub	prserv	all	urb	rur	all
r1	0.4	0.2	6.5	8.8	7.4	32.2	55.5	23.7	-	23.7
r2	0.9	0.9	2.6	2.9	1.1	8.5	16.9	7.0	17.7	24.7
r3	2.1	1.9	2.0	1.2	1.0	2.1	10.3	3.9	6.9	10.8
r4	3.5	1.9	0.7	0.5	0.3	2.2	9.0	4.1	17.3	21.4
r5	2.3	1.3	0.9	0.5	0.5	2.7	8.3	5.2	14.3	19.5
Total	9.3	6.2	12.6	14.0	10.4	47.6	-	43.8	56.2	-
Export/ output	8.2	2.7	14.4	21.6	-	8.4	10.4	-	-	-
Import/ cons.	16.2	0.3	8.8	31.1	-	9.2	15.1	-	-	-

Table 2: Structure of Senegalese economy, SAM 2015

As the next step of our analysis, we need to specify the values of the model parameters to simulate the baseline scenario for the years 2016-2025. We use different sources and estimates to define the baseline parameters. Most importantly, we prefer the forecasts over the historical observations and estimates over assumptions and rules of thumb. The summary of the baseline model parameters is given in Table 8 of the Appendix, and a few notable factors worth special attention. First, we distinguish between export and import commodities because the composition of these goods can be different. For example, Senegalese export crops consist mostly of nuts, vegetables, and fruits, while the major import crop is rice. This implies that both trade elasticities and the composite world market prices of import and export commodity accounts are not necessarily equal. Second, we

		crop	oagr	agrib	oind	pub	prserv	hhd-u	hhd-r
Primary factors-shares across	fnd	100	-	-	-	-	-	6.8	93.2
	fcap-a ^a	34.0	66.0	-	-	-	-	7.7	92.3
	fcap-n	-	-	16.4	18.3	7.0	58.2	86.7	13.3
	flab	8.8	5.5	9.9	10.7	20.6	44.5	81.4	18.6
Primary factors-shares within	fnd	45.9	-	-	-	-	-	0.5	18.3
	fcap-a	25.0	72.6	-	-	-	-	0.8	29.0
	fcap-n	-	-	75.9	76.6	39.3	71.3	58.7	26.1
	flab	29.1	27.4	24.1	23.4	60.7	28.7	40.0	26.6
Interm. factors-shares within	crop	49.7	9.4	28.1	0.1	1.5	3.0	7.7	10.5
	oagr	1.0	20.9	13.3	0.8	0.9	1.3	6.4	4.9
	agrib	1.9	29.3	24.6	1.2	5.9	10.2	43.0	60.6
	oind	34.7	30.0	24.0	79.2	30.2	23.2	8.9	5.0
	pub	0.2	0.0	0.0	0.0	0.0	0.0	-	-
	prserv	12.6	10.4	9.9	18.7	61.4	62.2	34.0	19.0

Table 3: Factor payments structure (SAM 2015), %

^aCapital payments to the households include transfers from the enterprises

have to use approximations and assumptions when the necessary estimates are not available. For example, we use the latest Global Trade Analysis Project (GTAP) estimates by Aguiar et al. (2016) and calculate the weighted average values for our sectoral production and trade elasticities. Finally, we use our assumption about the productivity growth of 1 % per year for all sectors in the baseline scenario. We prefer this modest estimate over the observed productivity decline in 2006-2015 and forecasted very high productivity increase by the IMF (2017b).

2.3 Standard approach

Now that we have set all baseline parameters, first we run the CGE model and define the baseline path of our endogenous variables of interest⁶. In order to calculate the sectoral growth elasticities under the standard approach, we define six alternative scenarios each of which is different from the baseline by the values of sectoral TFP growth. As mentioned in Section 2.1, in the baseline we assume 1% yearly TFP growth of all sectors; consequently, for each alternative scenarios, we assume a 2% yearly TFP growth of a respective sector. 1% increase of the TFP

⁶We follow Heerden et al. (2006) and use the GTAP 9 database to associate the quantities of the consumed fossil fuels and CO2 emissions

versus the baseline is convenient because it allows us to interpret the obtained results as growth elasticities (e. g. by how much GDP and CO2 per capita would change if we raise the TFP of a particular sector by 1 %?). In this regard, it is important to mention that under our baseline scenario Senegal is experiencing an economic boom (3% of the yearly average growth rate of the GDP per capita, see Figure 2). This development is in line with the boom expectations of the IMF (2017b) that expects 3.9% of yearly per-capita GDP growth throughout 2016-2021. Accordingly, our alternative simulations are counterfactuals against the high baseline.

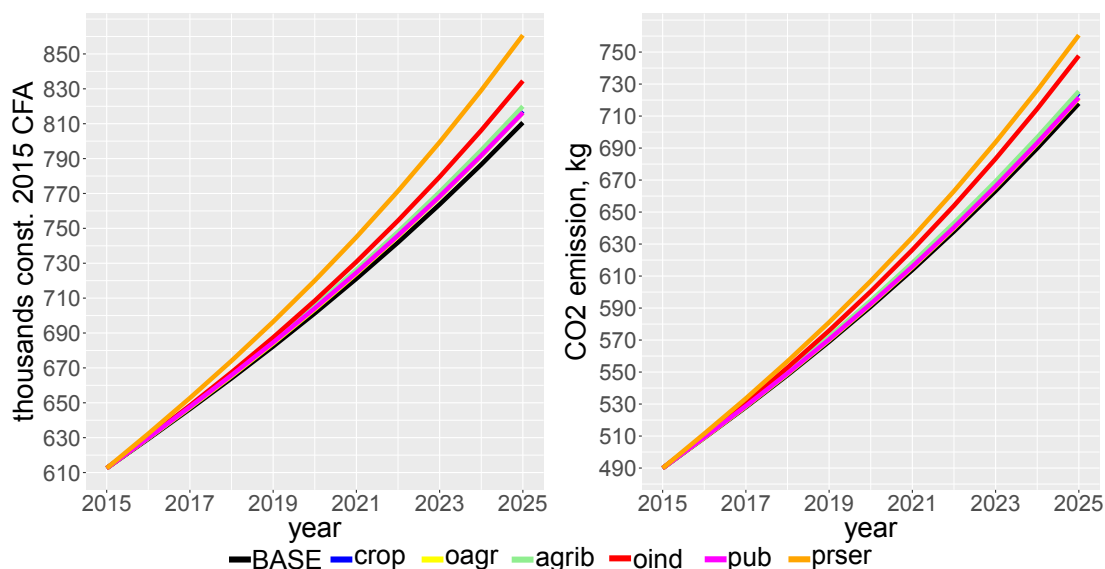


Figure 2: Per-capita GDP and CO2 emissions under different TFP scenarios

Given the specification of our CGE model, we observe that higher productivity is associated with higher CO2 emissions for all scenarios/sectors. The two-way transmission mechanism can be considered. From the production side, higher productivity of any sector under the assumption of full employment leads to redistribution of resources and eventual growth of the consumption of the most important intermediate - industrial products that in turn is associated with the CO2 emission. From the consumption side, higher productivity implies higher wages and, therefore, higher incomes of the households. Together with the LES specification with its positive income elasticities, more income implies higher consumption of the industrial products, and therefore, higher CO2 emission.

The magnitude of the differences versus the baseline⁷ to a large extent depends

⁷ Average of the yearly differences versus the baseline for the period of 2016-2025

on the initial sectoral sizes, and the increased productivity of important sectors has a more pronounced effect (Table 4). Consequently, the ranking of the growth elasticities for each indicator individually can be misleading. For example, the growth elasticity of the largest sector of private services is highest for both GDP and CO2 emission. Therefore, when ranking the efficiency of sectoral interventions, we prefer to use the ratio between the two elasticities. This ratio can be interpreted as an additional income (measured in constant thousand 2015 CFA per capita) per 1 kg of CO2 per-capita emission. Accordingly, the best sectoral intervention is the one that delivers the highest increase in income and yet the least increase in CO2 emission. Consequently, according to this ‘balanced’ performance score, the sector of other agricultural products is the most efficient, while the industrial sector is the worst.

Sector/ scenario	GDP share (2015)	GDP p/c		CO2 p/c		GDP per CO2	
		Elasticity	Rank	Elasticity	Rank	Ratio	Rank
crop	9.3	3.5	4	3.4	4	1.0	5
oagr	6.2	2.8	6	1.5	6	1.8	1
agrib	12.6	4.8	3	4.1	3	1.2	4
oind	14.0	10.0	2	12.8	2	0.8	6
pub	10.4	3.2	5	2.0	5	1.6	2
prserv	47.6	23.4	1	20.0	1	1.2	3

Table 4: Growth elasticities calculated under the standard approach

3 Growth Elasticities: the metamodeling approach

3.1 Model parameters

The deterministic nature of the defined CGE model has real implications for the growth elasticities calculated under the standard approach. In terms of the mathematical statistics, all model parameters can be viewed as random variables that have been fixed. In other words, we calculated sector-specific TFP elasticities conditioning that all other model parameters are fixed to the baseline. However, due to various reasons, it is not plausible to assume that any of the model parameters is genuinely fixed. All baseline parameters specified in Section 2.2, have some degree of uncertainty associated with them. The sources of this uncertainty are

different, ranging from the innocuous forecast errors and ending with the various ad-hoc rules of thumb used to calculate the parameter values. The approach often met in the literature, is to address the problem by re-parametrization of some of the baseline parameters and to re-run a CGE model to test the robustness of results (this approach is often named ‘sensitivity analysis’). However, what is more important, is that model parametrization is often not transparent, and it can be assumed that confirmatory intentions or publication bias can drive a definition or a re-definition of CGE model parameters. For example, Schürenberg-Frosch (2015) conducts a meta-sensitivity analysis of the influence of Armington (CES) elasticity misspecification on CGE simulation results and concludes that a change in the trade elasticities within reasonable boundaries has significant quantitative and qualitative effects in more than half of the investigated studies.

We follow the recent developments in the literature and depart from the standard sensitivity analysis and deploy so-called uncertainty analysis. This method can be viewed as a generalization of sensitivity analysis, and it implies multiple reparameterizations of baseline parameters within the defined uncertainty ranges and consequent multiple runs of a CGE model in order to obtain variation ranges of endogenous variables of interest.

We follow Webster et al. (2008) and select the Gaussian distribution to specify the uncertainty of the model parameters. This distribution is convenient as it requires only the first two moments to describe it explicitly. Therefore, in order to comprehensibly describe the parameter uncertainty of our CGE model, we need to specify the vector of means and the variance-covariance matrix of the multivariate normal distribution.

While it is plausible to assume that the vector of means corresponds to the baseline parameters, the specification of the variance-covariance matrix is more complex and requires additional assumptions. We start with segregation of model parameters to a group that has historical observations and a group for which historical observations are not available.

Parameters with available historical observations. This set of model parameters corresponds to a dynamic component of a standard IFPRI model described by Diao and Thurlow (2012), and therefore, the standard units that we use are the yearly growth rates. For this group of variables, we use country-specific historical observations and empirically estimate the variance-covariance matrix. We do not use cross-country or panel data because we want to define the uncertainty ranges based on the patterns that have been historically observed in Senegal and we want to avoid the noise that can be introduced to our data by the specifics of the other countries. Another critical question is the relevance and depth of Senegalese historical sample for a future period in our simulations. We address this question by employing two rules of thumb. First, we switch from yearly observations to

10-year moving averages. This manipulation allows us to get rid of the short-term noises and cast the long-term dependencies among the model parameters. Second, we define the depth of the historical data to a period of 1990-2015 where the value of each year corresponds to a moving average of 10 preceding years. We believe that this definition of the historical sample is appropriate for our simulation horizon as it allows us to capture the country's long-lasting fundamental transformation from a primary to a secondary stage of economic development.

Other variables for which observations are not available. All variables in this set, except for TFP growth, correspond to a static component of a standard IFPRI model described by Löfgren et al. (2002) and consist of various elasticity parameters. Given the data shortage, for this subset, we have to rely on our ad-hoc assumptions to specify the variance-covariance matrix.

TFP growth rates. Because we are focusing on the growth elasticities, this group of model parameters is distinct. We assume that the standard deviations of all sectoral TFP growth rates equal the baseline forecast of 1 %. Consequently, by using the rule of 3σ , the range of the possible values of yearly TFP growth rates can vary from 4% (max) to -1% (min). Furthermore, we assume that each of the sectoral TFP growth rates is an independent random variable because we assume that the policymakers can control sectoral productivity.

Elasticity parameters. For this set of model parameters, we assume that individual standard deviations equal quarter of the respective baseline values. Then the rule of 3σ suggests that the range of the possible values is $[0.25 \cdot \text{mean}; 1.75 \cdot \text{mean}]$, and we assume that it is sufficient to cover the uncertainty ranges of all elasticity parameters in our model. Finally, we assume the correlation of 0.5 for the following sub-groups:

- LES income elasticities within same products among the different household types;
- CES production elasticities within industries;
- CES and CET regional and trade elasticities within products.

By imposing the moderate positive correlation, we intend to reserve a mechanism of possible nationwide structural changes that can alter the adjustment possibilities of the Senegalese economy (e. g. legislation changes).

3.2 Metamodeling approach

3.2.1 Endogenous variables

As the next step of our analysis, we map the described uncertainty of the input CGE parameters with the consequent uncertainty of GDP and CO2 per capita.

We start with a sampling of the model parameters from a specified multivariate normal distribution. In order to technically implement the sampling, we employ the R-package developed by Millard (2013) and follow Webster et al. (2008) and use the method of Latin Hypercube Sampling. Furthermore, we define the size of the sample as $n = 1000$ simulations, and we consider it sufficient for our 90 model parameters. Finally, for each of the n simulations, we run the CGE model for the period of 2016-2025 and get the responses of our endogenous variables.

Figures 3 and 4 represent the obtained possible variation of the GDP and CO2 per capita variables. The red line on the left plot represents the baseline dynamics of a variable, and the grey area represents its possible variation over the years (light grey: all sample; dark grey: $[2.5; 97.5]$ percentiles). The right plots represent the estimated density of yearly averaged values (red) and corresponding normal distributions (dashed black).

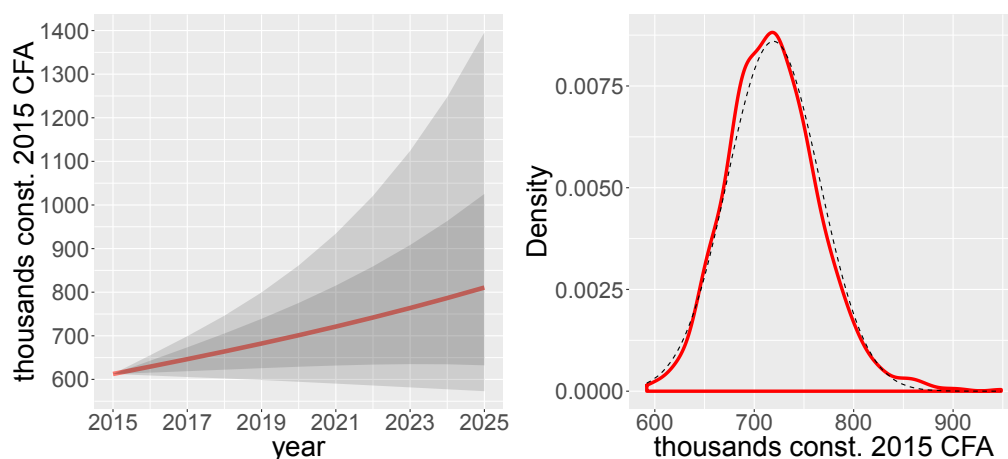


Figure 3: GDP per capita: uncertainty (left) and density of yearly avg (right)

We can already make essential conclusions regarding the variation of our endogenous variables. First, the distributions of both GDP and CO2 per capita are close to Gaussian with some outliers in the right tale. It is also important to emphasize that within each year the mean values have a negligible difference with the corresponding baseline. Furthermore, if removing the outliers from both left and right, and consider a 95% interval, the median values also follow the baseline. Last, but not least, a possible variation of both GDP and CO2 per capita is increasing over the years. This implies that the specified uncertainty of the model parameters has an amplifying impact on a variation of the output variables.

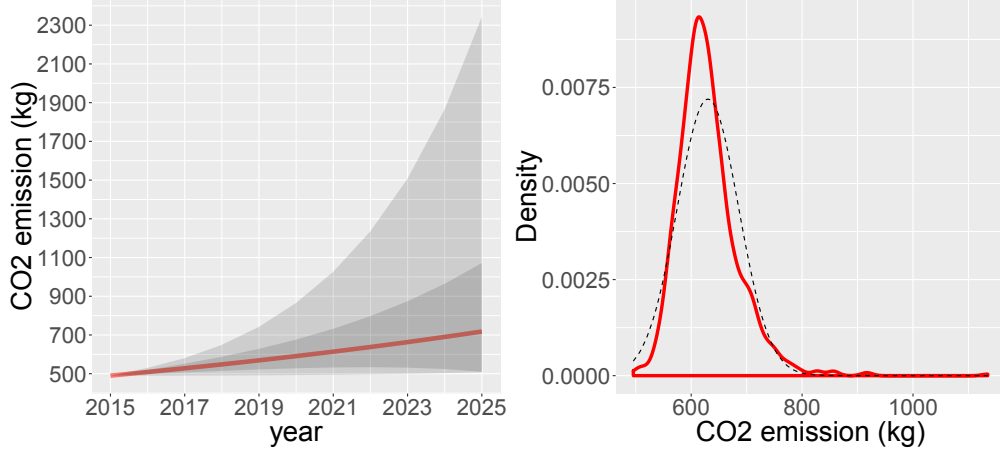


Figure 4: CO2 per capita: uncertainty (left) and density of yearly avg (right)

3.2.2 Metamodeling: key concepts

To our knowledge, so far the uncertainty analysis has been used in the literature to map the uncertainty ranges of selected model parameters and the resulting uncertainty ranges of the endogenous variables of interest (e. g. Webster et al. 2008; Phimister and Roberts 2017). Our approach contributes to the existing literature strands as we not only acknowledge and define the model uncertainty, but we also use it to estimate the growth elasticities. In terms of the mathematical statistics, we want to depart from the conditional sectoral growth elasticities described in Section 2.3 (e. g. $TFP_{sector}^{elastGDP}$ | parameters) and evaluate their expected values unconditional of all model parameters (e. g. $E[TFP_{sector}^{elastGDP}]$) utilizing metamodeling tools.

The idea and term ‘metamodeling’ was originated by Blanning (1975) and popularized by Kleijnen (1975). In its core, the concept aims at approximating complex simulation models in order to either simplify the understanding of particular input-output relationships or to increase the simulation speed in case of computationally intensive models (for more details see Barton 2015). Our goal is to analytically specify the relationship between sectoral TFP growth and GDP and CO2 per capita given the variation of other model parameters.

We assume the most straightforward linear relationship between all model parameters x_i (including sectoral TFP growth) and endogenous GDP and CO2 per capita. We use the OLS estimator to estimate the corresponding coefficients β_i for each of our output variables. In the standard OLS setup, our metamodel can be viewed as the approximation of the linear relationship between sectoral TFP growth and GDP/CO2 per capita while controlling for the variation of the other

variables. In particular, we are estimating the following functional form:

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{j=2017}^{2025} \gamma_j D_j + \epsilon, \quad (1)$$

where D_j are the yearly dummies and ϵ is the error term.

To evaluate the performance of the specified metamodels, in addition to the adjusted R^2 , we use Root Mean Squared Error (RMSE) and the Maximum Absolute Relative Error (MARE) criteria:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (2)$$

$$MARE = \max \left| \frac{y_i - \hat{y}_i}{y_i} \right|, \quad (3)$$

where y_i refers to the real simulation output and $\hat{y}_i$ refers to the predicted output from the metamodel. RMSE then provides a good evaluation of the global error over the region of interest and MARE offers a good evaluation of the local error by assessing the worst error within the region of interest (Simpson et al., 2001).

Furthermore, we split the dataset into two sub-samples to which we refer as “training” and “test”. We use the “training” sample to estimate the metamodel and make predictions for the “test” sample. Then for each sample, we compute the defined statistics and use the bootstrap to evaluate the averages of the three statistics.

3.2.3 Results

Table 5 presents the estimated coefficients of sectoral TFP growth variables under the pooled OLS setup described in Equation 1. Figures 5, 6, 7, 8 in the Appendix present RMSE and the MARE criteria defined in Equations 2 and 3. Figures 9 and 10 display the adjusted R^2 of the regression. Most importantly, we need to emphasize the reliability of the obtained estimates. First, although it is not common in a metamodeling framework, we can mention that the specified variation of our sectoral productivity shocks allowed us to make the reliable inference on all TFP coefficients that are significant at 1% using HC3 estimator developed by MacKinnon and White (1985). Second, the analysis of RMSE and the MARE criteria, as well as the analysis of the performance of train and test samples, allows us to conclude that the overall performance of our metamodels is satisfactory for the specified simulation period. In particular, we can observe the general deterioration of the prediction performance which is naturally explained by the

growing model uncertainty of our response variables (Figures 3 and 4). However, if we check the ratio of the largest RMSE versus the respective values of our dependent variables, we notice that the errors are not more than 10% at the maximum (for CO2 emission).

All in all, following the standard OLS notation, we can conclude that the obtained estimates represent the average partial impact of sectoral TFP growth rate on GDP and CO2 per capita given the variation of other CGE model parameters. Moreover, because of the considerations presented in Section 3.2.1 (e. g. mean $\equiv$ baseline), we can assume that the obtained coefficients can be interpreted as growth elasticities (e. g. by how much on average would GDP or CO2 per capita change if we increase a specific sectoral TFP growth by 1 % relative to the baseline). This means that our estimates can be directly compared to the growth elasticities calculated under the standard approach described in Section 2.3.

Sector	Metamodeling						Standard method	
	GDP p/c		CO2 p/c		GDP/CO2			
	Coef.	Rank	Coef.	Rank	Ratio	Rank	Ratio	Rank
crop	3.4	5	3.6	4	0.94	5	1.03	5
oagr	3.2	6	2.7	5	1.19	3	1.80	1
agrib	4.9	3	4.0	3	1.20	2	1.16	4
oind	10.8	2	16.3	2	0.66	6	0.78	6
pub	3.5	4	2.6	6	1.39	1	1.61	2
prserv	23.2	1	20.8	1	1.12	4	1.17	3

Table 5: Estimation results

If we compare the results in Tables 4 and 5, we can see that the standard approach in most cases underestimates the growth elasticities for income and overestimates the elasticities for the CO2 emissions. What is more important is that the ratios of the marginal income to the marginal CO2 emission have also changed under the metamodeling approach. The most dramatic change is observed in the sector of other agriculture. The ranking of the efficiency of this sector has fallen from the 1st place to the 3rd. Furthermore, the ranking of the other sectors, except crops and industrial products, has also changed, with the public services now being the 1st, and the agribusiness being the 2nd most efficient sectors.

Several general implications of the obtained results can be outlined. First, the suggested approach has demonstrated that the sectoral growth elasticities calculated under the standard approach are not robust. The impact of the sectoral TFP increase on endogenous variables is not independent of other CGE model

parameters. If using the approach and formulation of Schürenberg-Frosch (2015), we can conclude that the re-parametrization of the baseline parameters within the reasonable boundaries has significant quantitative and qualitative effects on the sectoral TFP growth elasticities. Moreover, the suggested application of the metamodeling concept allows to emulate the stochastic data generating process and formally estimate the particular dependencies, and yet take into account the variation of other model parameters. For practical policy making, this means that the presentation of CGE simulations results can be simplified and, more importantly, become more convincing. CGE models themselves are criticized for being too complicated for the outsiders, and often practical policy-makers treat them as a black box (Böhringer et al., 2003). In this regard, the presentation of CGE modeling results in an OLS metamodeling framework can serve as an additional tool that can streamline the understanding of the model dependencies.

4 Conclusion

The metamodeling approach described in our paper has demonstrated the alternative way of analyzing the effectiveness of the policy intervention options in a CGE modeling framework. For methodological purposes, we selected Senegal as a case study country, chose sectoral TFP growth options as policy instruments and restricted the domain of the performance indicators to GDP and CO2 per capita. We started with the specification of the Senegalese CGE model and made specific assumptions to define the general adjustments possibilities of the Senegalese economy. Then we outlined the starting structural characteristics of the Senegalese economy and defined its baseline path. On the next stage, we acknowledged and carefully defined the uncertainty of both input and output variables in the defined CGE model. Then we emulated the stochastic data generating process that allowed us to formally estimate the effectiveness of sectoral TFP interventions under the uncertainty of other model parameters.

Most importantly, comparing to a standard deterministic CGE modeling method, the application of the metamodeling resulted in significantly different growth elasticities that gave rise to a different ranking of sectoral efficiency. Although our target variables and considered policy intervention options might not adequately represent the potential trade-offs that are currently actual for Senegal, the suggested methodology paves the way for future extensions. Besides the TFP growth, our approach can be applied to estimate the effectiveness of sectoral taxation or trade policy, transaction costs or any other model parameters depending on a country context. Furthermore, it might be plausible to consider more disaggregated sectors in a SAM and compare the effectiveness of competing options within one sector (e. g. rice production versus maize production). From the perfor-

mance side, the set of the response variables can also be expanded to a variety of social and environmental indicators that will reflect the multidimensional interests of various stakeholders (e. g. welfare of rural/urban households, domestic producers/consumers, various environmental indicators, and others).

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Appendices

Block	Category	Form/endog.	Exog. parameters to specify
Production	Value-added	CES	TFP & Elasticities
	Intermediate	Leontief	-
	Top nest	Leontief	-
Dom. market	-	CES	Elasticities
Trade	Import	CES	Elasticities & World Prices
	Export	CET	Elasticities & World Prices
Consumption	-	LES	Elasticities & Population
Closures	Trade	Dom. prices	Current account
	Capital	‘Putty-clay’	Economy-wide stock
	Labor & Land	Full empl. & mobile	Stock
	Gov. budget	GSAV	-
	Sav/Inv	Balanced	-

Table 6: Functional forms and closures used in Senegalese CGE model

Category	Description	SAM	Dimension
Activities	Crops, other agriculture, agribusiness, other industry,	acrop, aoagr, aagrib, aoind, apub, aprser	Region
Commodities	public, private services	ccrop, coagr, cagrib, coind, cpub, cprser	National
Lab. & land	All types of labor and land	flab & flnd	Region
Capital	Agruc. & non-agric.	fcap-a & fcap-n	National
Enterp.	All enterprises	ent	National
H-holds	Urban & Rural, all deciles	hurb & hrur	Region

Table 7: Aggregated SAM accounts

Category	Used sources
TFP growth	Own assumption
Factor supplies	ILO for labor, FAO for land and Penn World Tables for economy-wide capital
Population	UN
World market prices	World Bank Commodities Price Forecast and FRED data
Current account	IMF WEO
Income elasticities	Own estimates based on the household survey and King and Byerlee (1978)
Frisch parameter	Own estimates based on the household survey, WB WDI and Lluch et al. (1977)
Production CES elasticities	Own calculations based on GTAP 9 database
Trade CET and CES elasticities	
Regional CES agg. elasticities	

Table 8: Sources used to calculate the baseline parameters

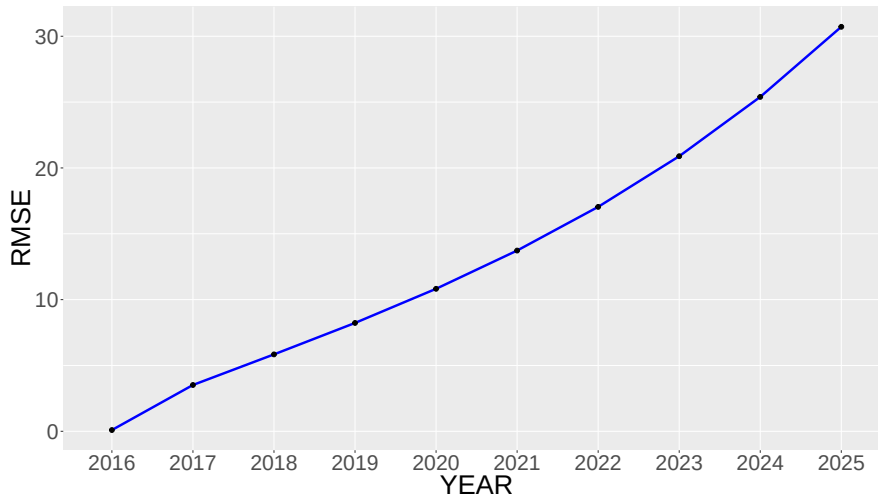


Figure 5: RMSE - GDP per capita

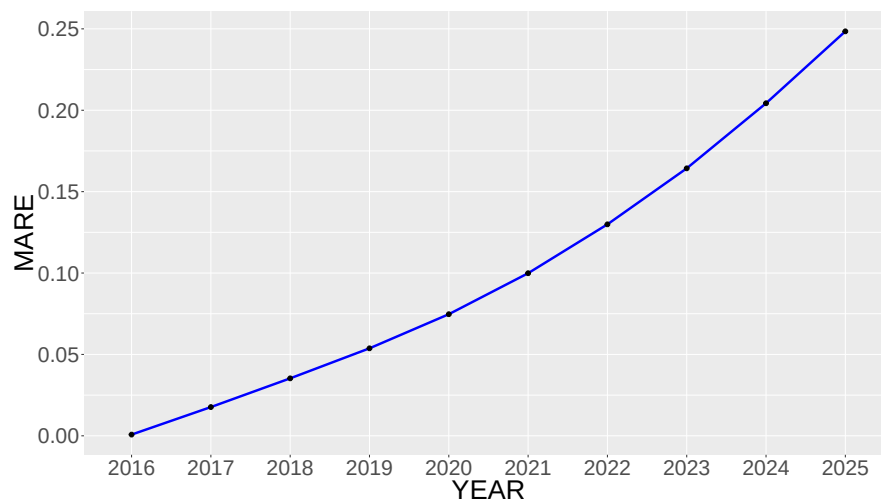


Figure 6: MARE - GDP per capita

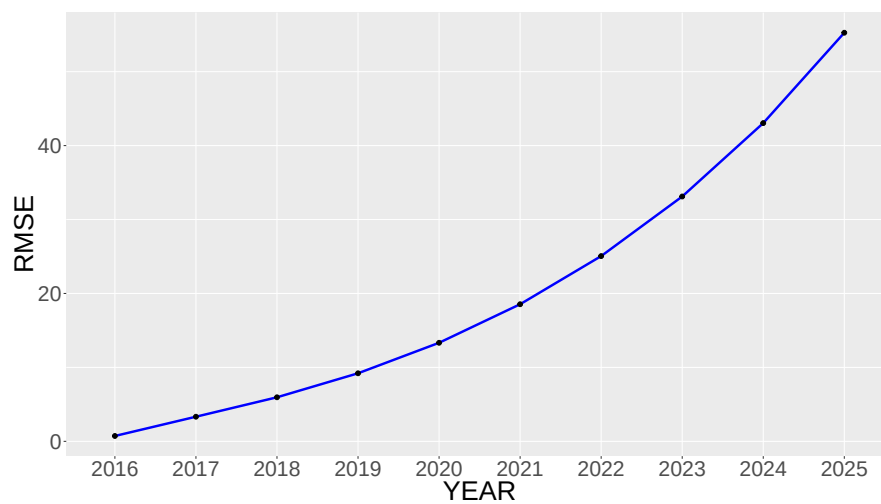


Figure 7: RMSE - CO2 per capita

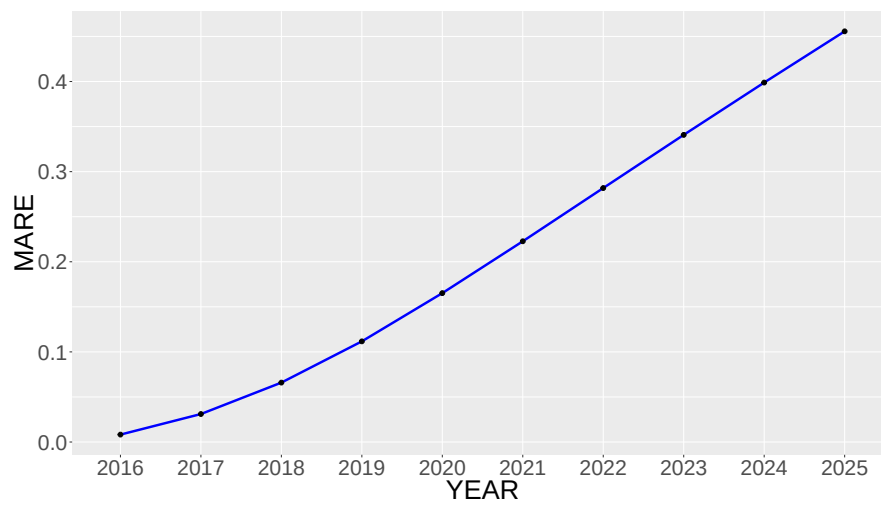


Figure 8: MARE - CO2 per capita

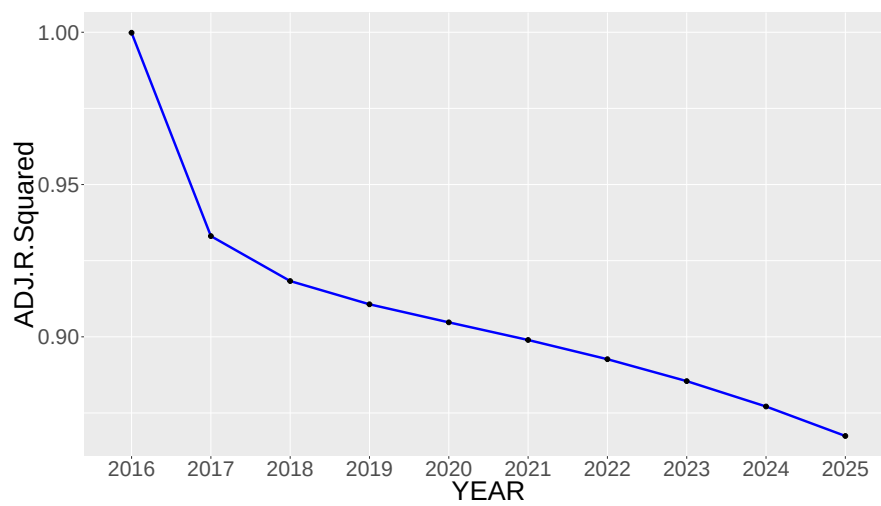


Figure 9: Adjusted R^2 - GDP per capita

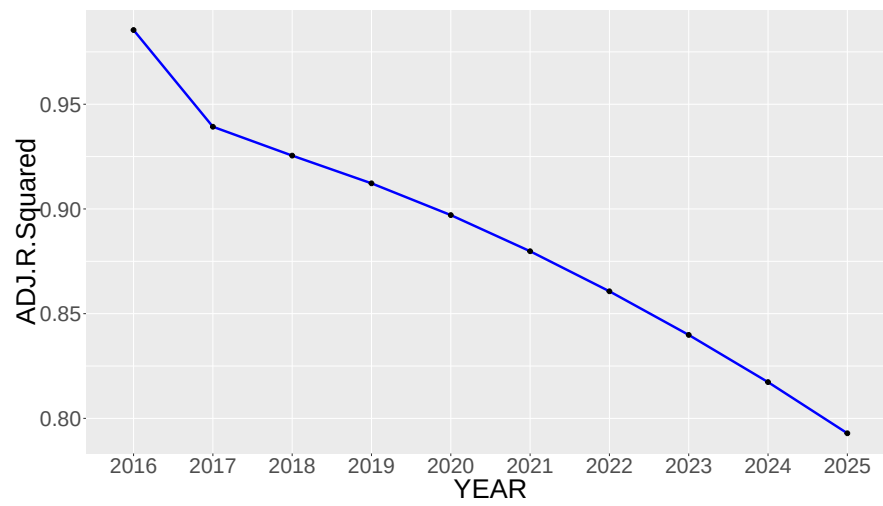


Figure 10: Adjusted R^2 - CO2 per capita