

Micro-founded tax policy effects in a heterogeneous-agent macro-model

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Abstract

Microsimulation models are increasingly used to calibrate macro models for tax policy analysis. Yet, their potential remains underexploited, especially in order to represent the non-linearity of the tax and social benefit system and interactions between capital and labour incomes which play a key role to understand behavioural effects. Following DeBacker et al. (2018b) we use a microsimulation model to provide the output with which to estimate the parameters of bivariate non-linear tax functions in a macro model. In doing so we make marginal and average tax rates bivariate functions of capital income and labour income. We estimate the parameters of tax functions in order to capture the most important non-linearities of the actual tax schedule, together with interaction effects between labour and capital incomes. To illustrate the methodology, we simulate a reduction in marginal personal income tax rates in Italy with a microsimulation model, translating the microsimulation results into the shock for a dynamic overlapping generations model. Our results show that this policy change affects differently households distinguished by age and ability type.

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[†]The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They should not be attributed to the European Commission. Any mistake and all interpretations are their and their only.

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1 Introduction

Heterogeneous-agent macroeconomic models are increasingly used in empirical research. This class of models is a step forward in terms of adding more realism to macroeconomic models and allowing to capture distributional aspects of policies. However, heterogeneous-agent models are limited in the way they can account for important characteristics of the modern tax system like non-linearities and interactions between capital and labour incomes. Another limitation of heterogeneous-agent models is that their disaggregation level is insufficient for equity analysis. In this paper we show the ability of a heterogeneous-agent macroeconomic model to simulate an exemplary tax reform by incorporating non-linear tax functions estimated using microdata. A notable and novel feature of this methodology is the functional form for labour and capital income taxes, which was first introduced in DeBacker et al. (2018b). The income tax function combines labour income and capital income together into the same function, incorporating progressive tax rates, thereby enabling income from one source to impact on the marginal and average tax rates of the other. We argue that this is an important feature of the modern tax system. One rationale is that in entrepreneurial firms, shifting may occur between labour income and dividends or capital gains obtained as a shareholder, in order to reduce the overall tax burden (Gordon and MacKie-Mason (1995), Gordon and Slemrod (1998)).¹ A second reason is related to optimal tax theory. A classical result in optimal taxation theory is that capital taxes should be set to zero (Judd (1985), Chamley (1986)). One important assumption behind such result is that households optimize over an infinite time horizon, which equates to assume, quite unrealistically, that inter-temporal optimization choices hold for entire dynasties. More recent optimal tax research often assumes, in contrast, that the time horizon is finite. This is the case for overlapping generation models where by definition households live for a finite time. In such different settings, the optimal capital tax is usually found to be positive. Overall such research thread points to an optimal non-linear tax schedule made of both labour and capital taxation (Diamond and Saez (2011), Gordon and Kopczuk (2014)).

Considering that governments try and approximate an optimal tax design, and/or if the observation holds that often it is hard for governments to distinguish labour income from capital income thus opening the possibility for income shifting, the tax schedule needs to jointly take into account both types of income (e.g. Christiansen and Tuomala (2008)). Consequently, when trying to capture the general features of a tax system, it becomes important to associate the overall tax burden at a household level to an array of income sources as these are not independent tax-wise. In this paper we develop an overlapping generation (OLG) model incorporating tax functions estimated using a microsimulation model. We show that the functional form employed is capable of capturing important features of the tax system including its non-linearity and interactions between different tax categories, and illustrating our approach by considering the Italian case. We use the **EUROMOD** microsimulation model to provide the output with which to estimate the tax function for the baseline case and for simulating tax policy reforms. **EUROMOD** covers all EU countries with the data for Italy

¹The possibility for shifting is indeed the reason that prompted to include provisions for “income splitting” in Nordic Dual Income Tax systems: Sørensen (1994), Lindhe et al. (2004), Pirttilä and Selin (2011).

obtained from the Italian Survey of Income and Living conditions 2014 (IT-SILC).² Recent applications of **EUROMOD** include research on labour supply and income distribution, such as Figari and Narazani (2017) and Ayala and Paniagua (2018), pensions-related tax expenditures such as Barrios et al. (2018a) and the marginal cost of public funds as in Figari et al. (2018).³ Using **EUROMOD** we obtain the effective tax rate⁴ and the marginal tax rates as a function of combinations of labour income and capital income. This novel approach is in contrast to the tax functions commonly considered in macroeconomic models, where even the most advanced (such as Nishiyama (2015)) tend to constrain marginal income tax rates to be solely a function of that same income type or where the non-linearity of the income tax system is not considered (see Barrios et al. (2018b)). In this way we are able to incorporate several characteristics defining the complexity of the actual tax code into a general equilibrium model with overlapping generations.

The utility of OLG models for evaluating the dynamics of economic policies trace back to the work of Allais (1947), Samuelson (1958) and Diamond (1965). A major step forward in the use of applied, computable OLG models came with the publication of Dynamic Fiscal Policy (Auerbach and Kotlikoff (1987)), which made full use of the newly available computing power to solve more complex and detailed models. Most applied OLG models used today still recognise the Auerbach-Kotlikoff model as an important aspect of their heritage, despite the fact that such models have been extended and expanded across many dimensions since then (see Gorry and Hassett (2013) for an overview of the impact of the Auerbach-Kotlikoff model and Zodrow and Diamond (2013) for an overview of tax policy analysis using overlapping generations models). Another distinctive feature of our overlapping generations model is that it incorporates considerable heterogeneity in the way it represents households. Tesfatsion (2003) and Judd (2006) have argued in favour of computable models featuring heterogeneous agents, as opposed to more traditional macroeconomic models with representative agents. With regard to optimal tax theory, the inclusion of agents heterogeneity (not least, age differences) was often found to imply different results compared to models with homogeneous agents, see for example Conesa et al. (2009), Weinzierl (2011), Farhi and Werning (2013), Golosov et al. (2013). The reason why heterogeneity may be important is that first-best optimal taxation implied by models with heterogeneous households usually requires tax rates to be set as a function of some household endowment (e.g. ability, productivity) or innate preference (e.g. for leisure time, for different time allocations of consumption). Because initial endowments and preferences are usually unobserved by tax authorities, second-best optimal taxation often requires a combination of two or more taxes (e.g. on labour and on capital income) levied on tax bases that proxy for the unobservables in order to mimic a schedule that obtains, overall, an equilibrium solution closer to the first-best scenario.

The use of heterogeneous agents also allows us to account for the distributional effects of policies with great accuracy considering a broad range of income-age combinations. Already in Mirrlees (1971) it was recognized that, even assuming away heterogeneous preferences,

²More details on the data used and its validation are provided in Ceriani et al. (2017).

³More than 60 journal articles and more than 200 working papers using **EUROMOD** are listed on the official Web site: <https://www.euromod.ac.uk/publications>.

⁴The effective tax rate refers to the effective average tax rate.

the existence of equity concerns together with efficiency concerns and heterogeneous abilities may lead to a non-linear tax schedule at the optimum. The empirical evidence in Gruber and Saez (2002) shows that the elasticity of taxable income varies substantially with income. The latter would imply a concave optimal tax schedule even under a Rawlsian Welfare function. In our case, households are distinguished by age and ability type. We model every adult age from 20 to 99, using realistic demographics based on Eurostat projections. Households are split into seven earning-ability types so as to incorporate the inequality observed in the data. Each household chooses labour force participation, consumption and savings so as to maximise lifetime utility. In this way, we capture different behavioural responses of a large number of different household types to changes in fiscal policy. Having heterogeneous agents in our model who are not only classified by ability but also by age allows us to estimate more fine-grained distributional effects from simulated policies from a life-time perspective too.

Microsimulation models have become standard tools for fiscal policy analysis, offering a detailed representation of the options available to policy makers. Household microsimulation models are limited, though, in that they only report static effects (that is, without behavioural responses). In order to capture the whole impact of a policy on the economy thus including general equilibrium effects and a wider array of behavioural responses (e.g. related to savings and investment), it is necessary to link the microsimulation model to a macroeconomic model. A number of research papers have incorporated the detail of microsimulation models through the interaction with computable general equilibrium models (examples are: Peichl (2009), Maisonnave et al. (2015); Bourguignon et al. (2010) offers an overview). A number of recent papers have used microsimulation models to incorporate individuals' income, labour supply and tax heterogeneity into macroeconomic models, see Horvath et al. (2018), Barrios et al. (2018b), Benczur et al. (2018). However, the approaches used in these papers are limited in the way they incorporate characteristics of the modern tax system. They do not account for the non-linearity of the tax system and the level of disaggregation is insufficient for equity analysis.

Following DeBacker et al. (2018b) our micro-macro approach uses a microsimulation model to provide the output with which to estimate the tax function in the macro model for the base case and simulations. We take the following variables from the **EUROMOD** microsimulation model: marginal tax rates on labour income, marginal tax on capital income, total tax paid and total disposable income (the latter two variables are needed to calculate the average tax rates), ages and sample weights. Then the output from the microsimulation model is used to estimate the parameters that describe bivariate non-linear tax functions in order to approximate the Italian tax code in the OLG model. We estimate separately parameters for the marginal tax rate on labour income, marginal tax rate on capital income and the average tax rate on total income.⁵ Tax policy reforms are first simulated in **EUROMOD**. The simulation results are then used to re-estimate the parameters for each of the three tax functions in the macro model. With this approach important characteristics of the complex tax system

⁵One reason why we estimate marginal and average tax rate functions separately is to capture policy changes that have differential effects on marginal and average rates. For more discussion, see DeBacker et al. (2018b)

are automatically accounted for by means of the parameterised tax functions that enter the macro model. In this way we account for the non-linearity of the tax system in the OLG model and we also take into account the interactions between labour income and capital income. By running an output from the microsimulation model through the macro model we capture the behavioural responses, distributional effects and dynamic outcomes of tax policy changes based on micro-foundations.

We demonstrate this methodology through a hypothetical tax reform simulation. In EUROMOD, we simulate reducing the personal income tax rates by two percentage points, and this output is used to re-estimate the income tax functions. The results show the way in which personal income tax cuts incentivise employment, especially after age 50 and more so for those with a low earnings-ability. We show the extent to which different working age groups reduce their tax burden, with a minor impact on the retired population. The tax cuts boost consumption for most age groups, and also induce higher levels of savings for most ages, especially from around age 45, with the largest percentage increase for those with a low earnings-ability. The simulation demonstrates how the methodology integrates key complexities of the microsimulation model into the macro-model.

The rest of the paper is organised as follows. Section 2 outlines the key features of an overlapping generations (OLG) model we use. Section 3 explains the estimation of the income tax functions, describes the Italian income tax system and discusses the way we integrate micro tax data with the macroeconomic model. Section 4 presents a baseline solution from the OLG model. Section 5 demonstrates a simulation where we show the impact of a two percentage point reduction in the personal income tax rates, implemented across all tax brackets. Section 6 concludes and summarises directions for future work.

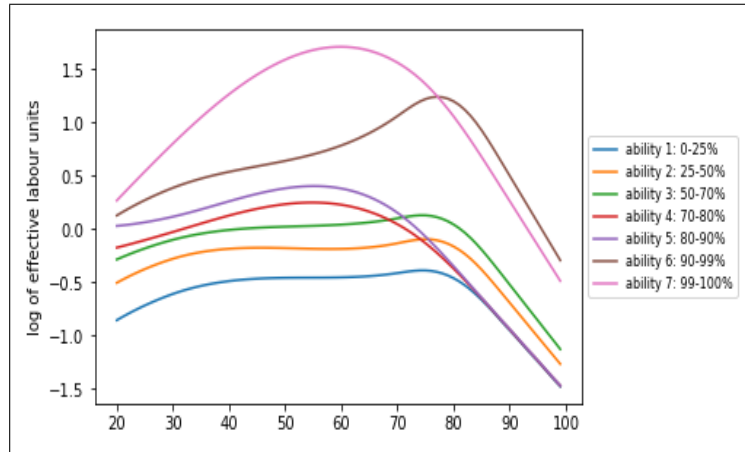
2 Description of the Model

We build an overlapping generations (OLG) model calibrated for Italy, which includes several dimensions of agent heterogeneity. Agents differ in ages and lifetime earnings-ability profiles. This allows the model to capture the richness of the cross-sectional and intergenerational distributions over income, wealth, labour supply and other endogenous variables. Here we briefly outline the dimensions of agent heterogeneity before explaining the model equations and focusing at length on the income tax functions.

There are seven earnings-ability groups in the model. These groups refer to a deterministic lifetime earnings-ability paths, which are shown in Figure 1. New cohorts of agents in the model are randomly assigned to each group and there is no mobility between groups. The groups are not of equal size: the first group represents the earnings-ability path for up to the 25th percentile, the next for the 25th to 50th percentile, then for 50th to 70th, 70th to 80th, 80th to 90th, 90th to 99th, and finally, the top group is for those workers with the highest one-percent of earnings-ability. Splitting earning-ability groups in this way allows us to focus on the highest earners, especially the top one percent. In order to estimate the earnings-ability paths, we construct a panel dataset by year and individual, where the age

and hourly wage of the person is observed. Separately for each ability group, we run panel fixed-effects regressions to derive the relation between age and hourly wage, according to the cubic regression model (more details on this procedure are given in d’Andria et al. (2019)). The resulting estimated earnings profiles are shown in Figure 1, which shows the calibrated life-cycle income ability paths in logarithms of the effective labour units. Effective labour units in our model correspond to hourly earnings in the underlying data. That is, all individuals have the same time endowment and receive the same wage per effective labour unit, but some are endowed with more effective labour units (see DeBacker et al. (2018a)). We can see that there is monotonicity across these paths during the main working ages, i.e. the lowest ability earners have lower earnings profile than the second ability group earners, and so on. Lines crossing is mostly due to extrapolation performed for ages over 80 that was needed due to scarce data for individuals aged 80 or more.

Figure 1: Exogenous life cycle income ability paths $\log(e_{j,s})$ with $S = 80$ and $J = 7$



Our OLG model includes recent Eurostat’s demographic projections on mortality rates, fertility rates, and immigration rates.⁶ Taken together, these imply a population distribution that evolves over time according to the law of motion implied by these rates. Model agents are economically active for as many as S years, facing mortality risk that is a function of their age, s . It means that they can die within one year with the probability of dying given by the one-period mortality rates. It is assumed that agents live no longer than 99 years. Further details are provided in Appendix C.

In the following equations, lifetime income groups are noted with the subscript j and the effective labour units (productivity) over the lifecycle for each type is given by $e_{j,s}$. The model year is denoted by the subscript t . All equations that follows in this paper are stationarised.⁷

⁶Eurostat database: Population and social conditions — Population projections (proj) — Population projections at national level (2015-2080) (proj_15n) <http://ec.europa.eu/eurostat/data/database>, access 30/08/2018.

⁷Since labour productivity is growing at rate g_y as can be seen in the firms’ production function (7) and

2.1 Households

The population of age- s individuals in period t is represented by $\omega_{s,t}$. Households are born each period, $\omega_{1,t}$, and become economically relevant at age $s = E + 1$ (if they survive to that age). They live for up to a maximum $E + S$ periods (S economically active periods).⁸. Let the age of a household be indexed by $s = \{1, 2, \dots, E + S\}$.

At birth, each household age $s = 1$ is randomly assigned to one of J ability groups, indexed by j . Let λ_j represent the fraction of individuals in each ability group, such that $\sum_j \lambda_j = 1$. Note that this implies that the distribution across ability types in each age is given by $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \dots, \lambda_J]$. Once a household is born and assigned to an ability type, it remains that ability type for its entire lifetime. This is deterministic ability heterogeneity as described in Section 2 above. Let $e_{j,s} > 0$ be a matrix of ability-levels such that an individual of ability type j will have lifetime abilities of $[e_{j,1}, e_{j,2}, \dots, e_{j,E+S}]$. The budget constraint for the age- s household in lifetime income group j at time t is the following:

$$c_{j,s,t} (1 + \tau_{s,t}^c) + e^{g_y} b_{j,s+1,t+1} = (1 + r_t) b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \zeta_{j,s} \frac{BQ_t}{\lambda_j \omega_{s,t}} + \eta_{j,s} \frac{TR_t}{\lambda_j \omega_{s,t}} - T_{j,s,t} \quad (1)$$

$$\text{with } c_{j,s,t} \geq 0, n_{j,s,t} \in [0, \tilde{l}], \text{ and } b_{j,1,t} = 0 \quad \forall j, t, \text{ and } E + 1 \leq s \leq E + S$$

where $c_{j,s,t}$ is consumption, $b_{j,s+1,t+1}$ is savings for the next period, r_t is the interest rate (return on savings), $b_{j,s,t}$ is current period wealth (savings from last period), w_t is the wage, $n_{j,s,t}$ is labour supply, $\tau_{s,t}^c$ is consumption tax rate, g_y is annual constant growth rate of labour-augmenting technological process, e is natural logarithm's base, and $e_{j,s}$ is individual's productivity (lifetime ability profile).

The third term on the right-hand-side of the budget constraint (1) represents the portion of total bequests BQ_t that go to the age- s , income-group- j household. Let $\zeta_{j,s}$ be the fraction of total bequests BQ_t that go to the age- s , income-group- j household, such that $\sum_{s=E+1}^{E+S} \sum_{j=1}^J \zeta_{j,s} = 1$. We must divide that amount by the population of (j, s) households $\lambda_j \omega_{s,t}$.

The penultimate term on the right-hand-side of the budget constraint (1) represents the portion of total government transfers TR_t that go to the age- s , income-group- j household. Let $\eta_{j,s}$ be the fraction of total transfers TR_t that go to the age- s , income-group- j household, such that $\sum_{s=E+1}^{E+S} \sum_{j=1}^J \eta_{j,s} = 1$. We divide that amount by the population of (j, s) households $\lambda_j \omega_{s,t}$, so that transfers are distributed uniformly per population groups.

the population is also growing (at rate $\tilde{g}_{n,t}$), the model variables are non-stationary. Different endogenous variables of the model are growing at different rates. Therefore to solve the model, non-stationary variables need to be transformed into the stationary ones. The details of this transformation is provided in DeBacker et al. (2018b) and d'Andria et al. (2019).

⁸We described the derivation and dynamics of the population distribution in Appendix C.

The last term on the right-hand-side of the budget constraint (1) represents total taxes paid by households, i.e. income tax, $T_{j,s,t}^I$ and consumption tax, $T_{j,s,t}^C$. Thus, $T_{j,s,t}$ is a sum of $T_{j,s,t}^I$ and $T_{j,s,t}^C$. In our model we model marginal income tax rates as non-linear functions of capital and labour income (see Section 2.6).

Households choose lifetime consumption $\{c_{j,s,t+s-1}\}_{s=1}^S$, labour supply $\{n_{j,s,t+s-1}\}_{s=1}^S$, and savings $\{b_{j,s+1,t+s}\}_{s=1}^S$ to maximize lifetime utility, subject to the budget constraints and non-negativity constraints. The household's period utility function is the following:

$$u(c_{j,s,t}, n_{j,s,t}, b_{j,s+1,t+1}) \equiv \frac{(c_{j,s,t})^{1-\sigma} - 1}{1-\sigma} + e^{g_y t(1-\sigma)} \chi_s^n \left(b \left[1 - \left(\frac{n_{j,s,t}}{\tilde{l}} \right)^v \right]^{\frac{1}{v}} \right) + \chi_j^b \rho_s \frac{(b_{j,s+1,t+1})^{1-\sigma} - 1}{1-\sigma} \quad \forall j, t \quad \text{and} \quad E+1 \leq s \leq E+S \quad (2)$$

The period utility function (2) is linearly separable in $c_{j,s,t}$, $n_{j,s,t}$, and $b_{j,s+1,t+1}$. The first right-hand-side term in this equation is a constant relative risk aversion (CRRA) utility of consumption where σ is relative risk aversion coefficient. The second term is the elliptical disutility of labour. We use elliptical functional form since, contrary to the many popularly used labour supply functional forms, it has Inada conditions on both the upper and lower bounds of labour supply. In addition, it can be fitted to approximate a linearly separable constant Frisch elasticity (CFE) functional form, one of the popularly used functional forms for labour disutility. In this specification $b > 0$ is a scale parameter and $v > 0$ is a curvature parameter (for more on the elliptical utility functional form see Evans and Phillips (2018)). Total time endowment $\tilde{l}$ is normalized to unity. The constant χ_s^n adjusts the disutility of labour supply relative to consumption and can vary by age s , which is helpful for calibrating the model to match hours worked in the data (see d'Andria et al. (2019) for a discussion of labour supply calibration). The intuition behind this is that an hour of work for an older person becomes more costly than an hour of work for a younger person. It is necessary to multiply the disutility of labour in equation (2) by $e^{g_y(1-\sigma)}$ because although labour supply $n_{j,s,t}$ is stationary, both consumption $c_{j,s,t}$ and savings $b_{j,s+1,t+1}$ are growing at the rate of technological progress, g_y . The $e^{g_y(1-\sigma)}$ term keeps the relative utility values of consumption, labour supply, and savings in the same units.

The final right-hand-side term in the period utility function (2) is the “warm glow” bequest motive. It is a CRRA utility of savings, discounted by the mortality rate ρ_s .⁹ Intuitively, it signifies the utility a household gets in the event that they don't live to the next period with probability ρ_s , or the utility of leaving unintentional bequests. It is a utility of savings beyond its usual benefit of allowing for more consumption in the next period. This utility of bequests also has constant χ_j^b which adjusts the utility of bequests relative to consumption and can vary by lifetime income group j . This is helpful for calibrating the model to match wealth distribution moments in the model with the corresponding moments in the data. The moments we aim at are average wealth for each of the seven income ability groups, a logarithm of wealth variance and the Gini coefficient. Note that any bequest before age $E+S$ is unintentional as it was bequeathed due to an event of death that was uncertain.

⁹See Annex C.2 for a detailed discussion of mortality rates we use here.

Intentional bequests are all bequests given in the final period of life in which death is certain $b_{j,E+S+1,t}$.

The household lifetime optimization problem is to choose consumption $c_{j,s,t}$, labour supply $n_{j,s,t}$, and savings $b_{j,s+1,t+1}$ in every period of life to maximize expected discounted lifetime utility as given by equation (3), subject to budget constraint 1 and upper-bound and lower-bound constraints.

$$\max_{\{(c_{j,s,t}), (n_{j,s,t}), (b_{j,s+1,t+1})\}_{s=E+1}^{E+S}} \sum_{s=1}^S \beta^{s-1} [\Pi_{u=E+1}^{E+s} (1 - \rho_u)] u(c_{j,s,t+s-1}, n_{j,s,t+s-1}, b_{j,s+1,t+s}) \quad (3)$$

where β^{s-1} is a one-year discount factor, or the reciprocal of a discount rate (the remaining variables as before).

The non-negativity constraint on consumption does not bind in equilibrium because of the Inada condition $\lim_{c \rightarrow 0} u_1(c, n, b') = \infty$, which implies consumption is always strictly positive in equilibrium, that is $c_{j,s,t} > 0$ for all j, s , and t . The warm glow bequest motive in (2) also has an Inada condition for savings at zero, so $b_{j,s,t} > 0$ for all j, s , and t . This is an implicit borrowing constraint.¹⁰ And finally, the elliptical functional form for the disutility of labour supply in (2) imposes Inada conditions on both the upper and lower bounds of labour supply such that labour supply is strictly interior in equilibrium $n_{j,s,t} \in (0, \tilde{l})$ for all j, s , and t .

The household maximization problem can be further reduced by substituting in the household budget constraint, which binds with equality. This simplifies the household's problem to choosing labour supply $n_{j,s,t}$ and savings $b_{j,s+1,t+1}$ every period to maximize lifetime discounted expected utility. The $2S$ first order conditions for every type- j household that characterize its S optimal labour supply decisions and S optimal savings decisions are the following:

$$\left(w_t e_{j,s} - \frac{\partial T_{j,s,t}^I}{\partial n_{j,s,t}} \right) \left(\frac{1}{1 + \tau_{j,s,t}^c} \right) (c_{j,s,t})^{-\sigma} = e^{g_y(1-\sigma)} \chi_s^n \left(\frac{b}{\tilde{l}} \right) \left(\frac{n_{j,s,t}}{\tilde{l}} \right)^{v-1} \left[1 - \left(\frac{n_{j,s,t}}{\tilde{l}} \right)^v \right]^{\frac{1-v}{v}} \quad \forall j, t, \quad \text{and} \quad E+1 \leq s \leq E+S \quad (4)$$

$$(c_{j,s,t})^{-\sigma} \left(\frac{1}{1 + \tau_{j,s,t}^c} \right) = e^{-g_y \sigma} \left(\chi_j^b \rho_s (b_{j,s+1,t+1})^{-\sigma} + \beta (1 - \rho_s) \left[1 + r_{t+1} - \frac{\partial T_{j,s+1,t+1}^I}{\partial b_{j,s+1,t+1}} \right] (c_{j,s+1,t+1})^{-\sigma} \left(\frac{1}{1 + \tau_{j,s+1,t+1}^c} \right) \right) \quad \forall j, t, \quad \text{and} \quad E+1 \leq s \leq E+S-1 \quad (5)$$

¹⁰It is important to note that savings also has an implicit upper bound $b_{j,s,t} \leq k$ above which consumption would be negative in current period. However, this upper bound on savings is taken care of by the Inada condition on consumption.

$$(c_{j,E+S,t})^{-\sigma} \left(\frac{1}{1 + \tau_{j,E+S,t}^c} \right) = \chi_j^b (b_{j,E+S+1,t+1})^{-\sigma} \quad \forall j, t \quad \text{and} \quad s = E + S \quad (6)$$

where the marginal tax rate with respect to labour supply $\frac{\partial T_{s,t}^I}{\partial n_{j,s,t}}$ is described in equation (22) and the marginal tax rate with respect to savings $\frac{\partial T_{s,t}^I}{\partial b_{j,s,t}}$ is described in equation (23), $\tau_{j,s,t}^c$ is the average consumption tax rate by age, s and ability, j .

2.2 Firms

Firms produce output Y_t using inputs of capital K_t and labour L_t according to the Cobb-Douglas production function:

$$Y_t = Z_t (K_t)^\gamma (e^{g_y t} L_t)^{1-\gamma} \quad \forall t \quad (7)$$

where Z_t is an exogenous scale parameter (total factor productivity) that can be time dependent, γ represents the capital share of income. We have included constant productivity growth g_y as the rate of labour augmenting technological progress. The profit function of the representative firm is the following.

$$PR_t = F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t \quad \forall t \quad (8)$$

Gross income for the firms is given by the production function $F(K, L)$ because we have normalized the price of the consumption good to 1. Labour costs to the firm are $w_t L_t$, and capital costs are $(r_t + \delta) K_t$. The per-period economic depreciation rate is given by δ .

Taking the derivative of the profit function (8) with respect to labour L_t and setting it equal to zero and taking the derivative of the profit function with respect to capital K_t and setting it equal to zero, respectively, characterizes the optimal labour and capital demands.

$$w_t = e^{g_y t} (Z_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[(1-\gamma) \frac{Y_t}{e^{g_y t} L_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t \quad (9)$$

$$r_t = (Z_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\gamma \frac{Y_t}{K_t} \right]^{\frac{1}{\varepsilon}} - \delta \quad \forall t \quad (10)$$

2.3 Government

The government is not an optimizing agent in our model. The government levies taxes on households and provides transfers to households. In the current version of the model there are two types of taxes: a consumption tax, $T_{j,s,t}^c$ and an income tax on a combination of labour and capital income that make $T_{j,s,t}^I$. There are two types of transfers: a general transfer that everybody receives, TR_t^u , and an old-age pension transfer, TR_t^p . The government budget is balanced every period. Thus, the following equation holds for every period:

$$\sum_{s=E+1}^{E+S} \sum_{j=1}^J T_{j,s,t}^I + \sum_{s=E+1}^{E+S} \sum_{j=1}^J T_{j,s,t}^c = TR_t^u + TR_t^p = TR_t \quad \forall t \quad (11)$$

The government sector influences households through three terms in the budget constraint ((1)): government transfers TR_t , income tax liability function $T_{s,t}^I$, which can be decomposed into the effective tax rate times total income (see 19), and a consumption tax liability function $T_{s,t}^C$. Total transfers to households by the government in a given period t is TR_t . The proportion of those transfers given to all households of age s and lifetime income group j is $\eta_{j,s}$ such that $\sum_{s=E+1}^{E+S} \sum_{j=1}^J \eta_{j,s,t} = 1$. In the current version of the model the transfer distribution function is set to distribute transfers uniformly among the population.

$$\eta_{j,s,t} = \frac{\lambda_j \omega_{s,t}}{\tilde{N}_t} \quad \forall j, t \quad \text{and} \quad E+1 \leq s \leq E+S \quad (12)$$

We discuss individual taxes and estimated tax functions for Italy below in more detail (see sections 2.5 and 2.6)

2.4 Market Clearing Conditions

Three markets must clear in the OLG model—the labour market, the capital market, and the product market. By Walras' Law, we only need to use two of those market clearing conditions because the third one is redundant. In the model, we choose to use the labour market clearing condition and the capital market clearing condition. The (redundant) products market clearing condition—sometimes referred to as the resource constraint—is used as a check on the solution method. We present the three market clearing conditions and the laws of motion for total bequests and transfers.

Labour market clearing (13) requires that aggregate labour demand L_t measured in efficiency units equal the sum of household efficiency labour supplied $e_{j,s}n_{j,s,t}$.

$$L_t = \sum_{s=E+1}^{E+S} \sum_{j=1}^J \omega_{s,t} \lambda_j e_{j,s} n_{j,s,t} \quad \forall t \quad (13)$$

Capital market clearing (14) requires that aggregate capital demand from firms K_t equal the sum of capital savings and investment by households $b_{j,s,t}$.

$$K_t = \frac{1}{1 + \tilde{g}_{n,t}} \sum_{s=E+2}^{E+S+1} \sum_{j=1}^J \left(\omega_{s-1,t-1} \lambda_j b_{j,s,t} + i_s \omega_{s,t-1} \lambda_j b_{j,s,t} \right) \quad \forall t \quad (14)$$

where $\tilde{g}_{n,t}$ is population growth rate. The second term inside the parentheses on the right hand side of equation (14) are the capital flows associated with immigration into or out of the country at time t , which is the only source of foreign capital inflow. It is assumed that immigrants have the same savings (and consumption) as natives of the same age.

Aggregate consumption C_t is defined as the sum of all household consumptions (including consumption of immigrants). Aggregate investment (under the closed economy assumption)

is defined by the resource constraint $Y_t = C_t + I_t$ as shown in (15).

$$Y_t = C_t + e^{gy}(1 + \tilde{g}_{n,t+1})K_{t+1} - e^{gy}\left(\sum_{s=E+2}^{E+S+1} \sum_{j=1}^J i_s \omega_{s,t} \lambda_j b_{j,s,t+1}\right) - (1 - \delta)K_t \quad \forall t$$

$$\text{where } C_t \equiv \sum_{s=E+1}^{E+S} \sum_{j=1}^J \omega_{s,t} \lambda_j c_{j,s,t}$$
(15)

Note that net exports in the economy are represented by the term incorporating immigration rates i_s , which describes the net inflow of capital brought into the country by immigrants. The term enters negatively into the equations, and therefore, if the term is positive, meaning positive capital inflows from immigrants, then net exports are negative.

Total bequests BQ_t are the collection of savings of household from the previous period who died at the end of the period. These savings are augmented by the interest rate because they are returned after being invested in the production process.

$$BQ_t = \left(\frac{1 + r_t}{1 + \tilde{g}_{n,t}}\right) \left(\sum_{s=E+2}^{E+S+1} \sum_{j=1}^J \rho_{s-1} \lambda_j \omega_{s-1,t-1} b_{j,s,t}\right) \quad \forall t$$
(16)

Because the form of the period utility function in (2) ensures that $b_{j,s,t} > 0$ for all j , s , and t , total bequests will always be positive $BQ_{j,t} > 0$ for all j and t . Note that this is not technically a market clearing condition, though it has similar properties as it can be as aggregate demand for bequests is equal to aggregate supply of savings of household from the previous period who died at the end of the period.

Total transfers to households TR_t are the collection of all taxes paid by households, i.e. income taxes, $T_{j,s,t}^I$ and consumption taxes, $T_{j,s,t}^C$. As shown in equation (17), income and consumption taxes are summed across all adult ages and across ability types. As for the bequest law of motion the transfers law of motion is not technically a market clearing condition, but nevertheless has similar properties.

$$TR_t = \sum_{s=E+2}^{E+S+1} \sum_{j=1}^J \lambda_j \omega_{s,t} T_{j,s,t}^I + \sum_{s=E+2}^{E+S+1} \sum_{j=1}^J \lambda_j \omega_{s,t} T_{j,s,t}^C \quad \forall t.$$
(17)

2.5 Tax transmission channels in the OLG model

The effect of individual income taxes on model agents' decisions is captured in three equations. First, total income tax paid by the model agent determines after-tax resources available for consumption and savings (budget constraint equation (1)). Second, individual income taxes influence households' decisions by introducing distortions into their optimization. Taxes affect the labour-leisure decision through the change in tax liability from a change in labour supply, $\frac{\partial \hat{T}_{s,t}^I}{\partial \hat{n}_{j,s,t}}$. Taxes affect savings through the partial derivative $\frac{\partial \hat{T}_{s+1,t+1}^I}{\partial \hat{b}_{j,s+1,t+1}}$, which reflects the additional taxes paid as a function of an additional euro of savings. The total

tax paid by the model agent determines after-tax resources available for consumption and savings. Consumption tax, mainly value added tax and excise tax, affects agents' decisions by determining after-tax resources available for consumption and savings as can be seen from the budget constraint equation (1). In what follows we focus on individual income taxes.

2.6 Income Tax Functions

In order to represent the personal tax system, we follow the novel approach described in DeBacker et al. (2018b), which feeds microsimulation output into an OLG model. The method enables not only the estimation of tax functions for current policy, but also for counterfactual tax policies, including tax policy levers that are difficult or impossible to model explicitly in a general equilibrium framework. The explanation of how we generate and use the **EUROMOD** microsimulation output follows in Section 3. Here we focus on the functional form and how it effectively translates the information from the microsimulation output into a function that can be entered into the OLG model.

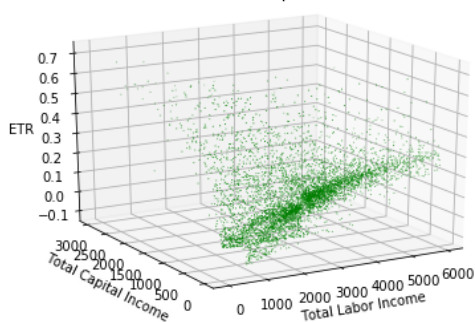
Figure 2 shows scatter plots of effective tax rates (ETR), marginal tax rates on labour income ($MTRx$) and capital income ($MTRY$) simulated using the **EUROMOD** model, each plotted as a function of labour income and capital income in the base year 2015. Labour and capital income are truncated at 6000 euros and 3000 euros per month in the plots for clarity in spite of the long tail of the income distribution. As three-dimensional plots are hard to appreciate when shown from only one angle, Appendix D shows the same plots from four angles.

Figure 2(a) shows the scatterplot of the effective tax rates for each labour and capital income combinations. Whilst noting some noise in the data, some key properties of the data can be clearly seen. First, those on low capital and labour incomes tend to face low $ETRs$. For those with low capital income, as labour income rises, a clear cluster of data is seen where the ETR rises but at a diminishing rate. From any level of labour income, higher capital income also raises one's ETR for most individuals. These anticipated properties of the data lead us towards the use of a functional form that captures these key features. Following DeBacker et al. (2018b) we fit to the data a Cobb-Douglas aggregator of two ratios of polynomials in labour and capital income (see eq. (18)) that we explain below. Important properties of the chosen functional form are that it produces the observed bivariate negative exponential shape and is monotonically increasing in both labour income and capital income, which are consistent with the observed data.

Before proceeding to explain the tax function in detail, we note that the data for the marginal tax rates on labour income ($MTRx$), Figure 2 (b), display similar properties. In particular, the data display a negative exponential shape and is monotonically increasing in labour income; however, the shape is less pronounced with respect to capital income. For the marginal tax rates on capital income, Figure 2 (c), the shape of the underlying function is less clear, but importantly does not appear to run contrary to the limits of the Cobb-Douglas aggregator. For these reasons, we estimate the same function for ETR , $MTRx$ and $MTRY$. As will be shown below, the function has sufficient flexibility to provide a good fit in each case.

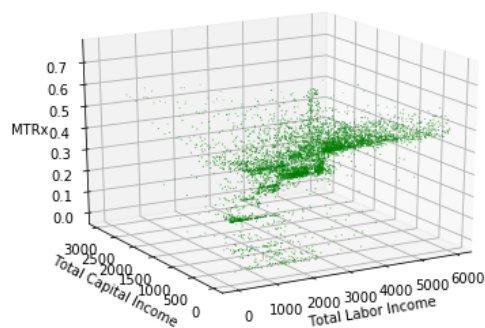
Figure 2: Scatter plot of ETR , MTR_x and MTR_y as functions of labour income and capital income (euros per month) from microsimulation model, year 2015

Truncated ETR , Lab. Inc., and Cap. Inc., Year=2015 baseline



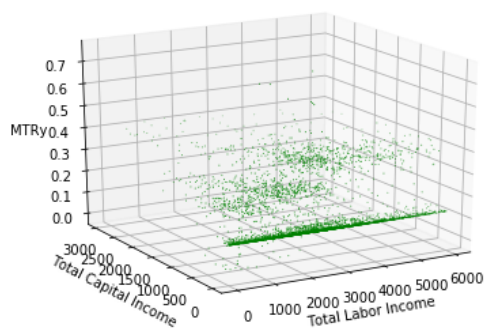
(a) Effective tax rates ETR

Truncated MTR_x , Lab. Inc., and Cap. Inc., Year=2015 baseline



(b) Marginal tax rates on labour income MTR_x

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) Marginal tax rates on capital income MTR_y

In order to explain the way our estimation algorithm works, let x be total labour income, $x \equiv w_t e_{j,s} \hat{n}_{j,s,t}$, and let y be total capital income, $y \equiv r_t \hat{b}_{j,s,t}$. Our tax rate function is a Cobb-Douglas aggregator of two ratios of polynomials in labour and capital income, and is expressed as follows:

$$\begin{aligned} \tau(x, y) &= [\tau(x) + shift_x]^\phi [\tau(y) + shift_y]^{1-\phi} + shift \\ \text{where } \tau(x) &\equiv (max_x - min_x) \left(\frac{Ax^2 + Bx}{Ax^2 + Bx + 1} \right) + min_x \\ \text{and } \tau(y) &\equiv (max_y - min_y) \left(\frac{Cy^2 + Dy}{Cy^2 + Dy + 1} \right) + min_y \\ \text{where } A, B, C, D, max_x, max_y, shift_x, shift_y &> 0 \quad \text{and} \quad \phi \in [0, 1] \\ \text{and } max_x > min_x \quad \text{and} \quad max_y > min_y \end{aligned} \tag{18}$$

The key building blocks of the functional form equation (18) as proposed by DeBacker et al. (2018b) are the $\tau(x)$ and $\tau(y)$ univariate functions. The ratio of polynomials in the $\tau(x)$ function $\frac{Ax^2+Bx}{Ax^2+Bx+1}$ with positive coefficients $A, B > 0$ and positive support for labour income $x > 0$ creates a negative-exponential-shaped function that is bounded between 0 and 1, and the curvature is governed by the ratio of quadratic polynomials. The multiplicative scalar term $(max_x - min_x)$ on the ratio of polynomials and the addition of min_x at the end of $\tau(x)$ expands the range of the univariate negative-exponential-shaped function to $\tau(x) \in [min_x, max_x]$. The $\tau(y)$ function is an analogous univariate negative-exponential-shaped function in capital income y , such that $\tau(y) \in [min_y, max_y]$. The respective $shift_x$ and $shift_y$ parameters in equation (18) are analogous to the additive constants in a Stone-Geary utility function. These constants ensure that the two sums $\tau(x) + shift_x$ and $\tau(y) + shift_y$ are both strictly positive. They allow for negative tax rates in the $\tau(\cdot)$ functions despite the requirement that the arguments inside the brackets be strictly positive. The general shift parameter outside of the Cobb-Douglas brackets can then shift the tax rate function so that it can accommodate different function values (including negative tax rates, if required). The Cobb-Douglas share parameter $\phi \in [0, 1]$ controls the shape of the function between the two univariate functions $\tau(x)$ and $\tau(y)$ (DeBacker et al. (2018b)). The precise shape of the function depends on these 12 parameters in (18), which are summarised in Table 1.

In the equation (18), we are allowing $\tau(x, y)$ to represent alternatively the effective and marginal tax rate functions $ETR(x, y)$, $MTRx(x, y)$ or $MTRY(x, y)$. We assume the same functional form for each of these functions. By assuming that each tax function takes the same form, we are breaking the analytical link between the effective tax rate function and the marginal tax rate functions. As DeBacker et al. (2018b) point out it is useful to separately estimate the marginal and average rate functions, in order to be able to capture policy changes that have differential effects on marginal and average rates. The total tax liability function is simply the effective tax rate function times total income $\tau(x, y)(x + y)$.

Table 1: Description of tax rate function $\tau(x, y)$ parameters

Symbol	Description
A	Coefficient on squared labour income term x^2 in $\tau(x)$
B	Coefficient on labour income term x in $\tau(x)$
C	Coefficient on squared capital income term y^2 in $\tau(y)$
D	Coefficient on capital income term y in $\tau(y)$
max_x	Maximum tax rate on labour income x given $y = 0$
min_x	Minimum tax rate on labour income x given $y = 0$
max_y	Maximum tax rate on capital income y given $x = 0$
min_y	Minimum tax rate on capital income y given $x = 0$
$shift_x$	$shifter > min_x $ ensures that $\tau(x) + shift_x > 0$ despite potentially negative values for $\tau(x)$
$shift_y$	$shifter > min_y $ ensures that $\tau(y) + shift_y > 0$ despite potentially negative values for $\tau(y)$
$shift$	shifter (can be negative) allows for support of $\tau(x, y)$ to include negative tax rates
ϕ	Cobb-Douglas share parameter between 0 and 1

Source: DeBacker et al. (2018b)

$$T_{s,t}^I(x, y) \equiv \tau_{s,t}^{etr}(x, y)(x + y) = \left([\tau_{s,t}(x) + shift_{x,s,t}]^{\phi_{s,t}} [\tau_{s,t}(y) + shift_{y,s,t}]^{1-\phi_{s,t}} + shift_{s,t} \right) (x + y) \quad (19)$$

A marginal tax rate (MTR) is defined as the change in total tax liability from a small change income. We differentiate between the marginal tax rate on labour income ($MTRx$) and the marginal tax rate on capital income ($MTRY$).

$$\tau^{mtrx} \equiv \frac{\partial T_{s,t}^I}{\partial w_t e_{j,s} n_{j,s,t}} = \frac{\partial T_{s,t}^I}{\partial x_{j,s,t}} \quad \forall j, t \quad \text{and} \quad E + 1 \leq s \leq E + S \quad (20)$$

$$\tau^{mtry} \equiv \frac{\partial T_{s,t}^I}{\partial r_t b_{j,s,t}} = \frac{\partial T_{s,t}^I}{\partial y_{j,s,t}} \quad \forall j, t \quad \text{and} \quad E + 1 \leq s \leq E + S \quad (21)$$

The derivative of total income tax liability with respect to labour supply $\frac{\partial T_{s,t}^I}{\partial n_{j,s,t}}$ and the derivative of total tax liability next period with respect to savings $\frac{\partial T_{s+1,t+1}^I}{\partial b_{j,s+1,t+1}}$ are present in the household Euler equations for labour supply (equation 4) and savings (equation 5), respectively. Though the data for these marginal tax rates are not directly available, they can both be decomposed into components for which data is available. Equation (22) shows the decomposition of the marginal tax rate influencing labour supply into the marginal tax rate on labour income times the household-specific wage. Equation (23) shows the decomposition

of the marginal tax rate influencing savings in the marginal tax rate of capital income times the interest rate.

$$\frac{\partial T_{s,t}^I}{\partial n_{j,s,t}} = \frac{\partial T_{s,t}^I}{\partial w_t e_{j,s} n_{j,s,t}} \frac{\partial w_t e_{j,s} n_{j,s,t}}{\partial n_{j,s,t}} = \frac{\partial T_{s,t}^I}{\partial w_t e_{j,s} n_{j,s,t}} w_t e_{j,s} = \tau_{s,t}^{mtrx} w_t e_{j,s} \quad (22)$$

$$\frac{\partial T_{s,t}^I}{\partial b_{j,s,t}} = \frac{\partial T_{s,t}^I}{\partial r_t b_{j,s,t}} \frac{\partial r_t b_{j,s,t}}{\partial b_{j,s,t}} = \frac{\partial T_{s,t}^I}{\partial r_t b_{j,s,t}} r_t = \tau_{s,t}^{mtry} r_t \quad (23)$$

This functional form for tax rates delivers flexible parametric functions that can fit the tax rate data shown in Figure 2 as well as wide variety of policy reforms. Further, these functional forms are monotonically increasing in both labour income x and capital income y . This characteristic of monotonicity does not appear to be a strong one when viewing the tax rate data. As DeBacker et al. (2018b) point out, while it does limit the potential tax systems to which one could apply our methodology, tax policies that do not satisfy this monotonicity assumption would result in non-convex budget sets, consequently they would require non-standard general equilibrium model solution methods and would not guarantee a unique equilibrium.

The estimations of the tax functions are performed on all observations, pooling all ages together.¹¹ Given the dispersion of the data, we require a method for solving that does not give a large weight to outliers. The chosen methodology is to solve for the least sum of weighted absolute distances between the estimated function and the data points (where the weights are the survey weights from EU SILC).¹² See Maronna et al. (2018) about robust regressors in general and the rationales behind their use.

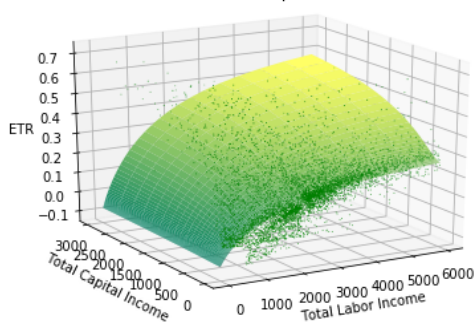
The estimated tax functions are shown in Figure 3. The curved planes are shown against the **EUROMOD** output for effective tax rates, marginal tax rates on labour and capital incomes, which were shown in Figure 2. It can be seen that the methodology captures salient features of the tax system. The effective tax rate, ETR , function is increasing in both labour and capital income. The marginal tax rate on labour income, $MTRx$, function captures the progressivity of the personal income tax system, while the curve for marginal tax rates on capital income, $MTRY$, is almost flat as the Italian system indeed levies special flat-rate taxes on several sources of capital income, most notably on financial income and some categories of real estate property. As only one angle is shown here, a more complete view of the tax functions is provided in Appendix D, which shows the same figures from four angles.

¹¹Future work will investigate estimating different functions according to age group. The justification would be that age could be used as a proxy for heterogeneity in the composition of assets and labour types that somewhat correlate with the age of taxpayers. For comparison, DeBacker et al. (2018b) estimated tax functions for each age separately in order to capture variation in taxes by filer age and model year, however they use a far larger data set than is available for Italy.

¹²Specifically, we use the SciPy optimise differential evolution solver. This is a global optimizer employing evolutionary Monte Carlo techniques. In our best-guessing estimates the optimizer is run for up to 2,000 periods with 20 candidate solutions each. A solution is achieved when the marginal improvement over the target function for additional periods is below a tolerance threshold, we we set at 10^{-15} .

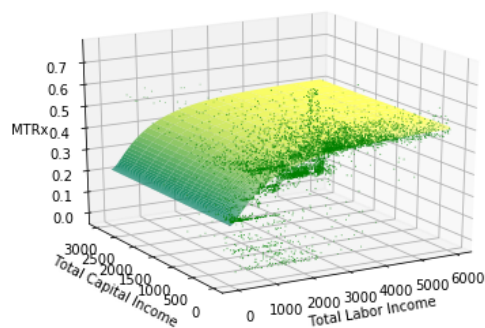
Figure 3: Estimated tax rate functions of ETR , MTR_x and MTR_y as functions of labour income and capital income from microsimulation model, year 2015

Truncated ETR , Lab. Inc., and Cap. Inc., Year=2015 baseline



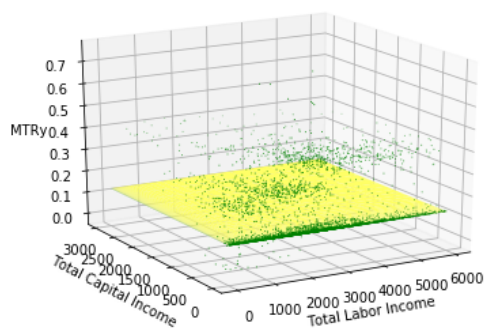
(a) Effective tax rates ETR

Truncated MTR_x , Lab. Inc., and Cap. Inc., Year=2015 baseline



(b) Marginal tax rates on labour income MTR_x

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) Marginal tax rates on capital income MTR_y

The functional forms not only provide a good approximation of the underlying microsimulation output, but also reflect well agent responses to the tax system. Whilst taxpayers may not be fully versed in all the intricacies of the tax system, they likely have a fair sense of how tax impacts their budget in general. We argue that the tax functions presented here capture this notion. Furthermore when considering a tax reform, the 12 parameters are re-estimated based on the microsimulation output for the reform. This allows a highly detailed reflection of the tax reform, from the microsimulation model, to be incorporated in the OLG model.

To show the importance of the assumption of tax rates being jointly functions of labour income and capital income, Table 2 gives a description of the estimated values of share parameter, ϕ . This parameter in the tax function (18) governs how important the interaction is between labour income and capital income for determining effective and marginal tax rates. The further interior is ϕ (away from 0 and 1), the more important it is to model tax rates as functions of both labour income and capital income. The closer ϕ is to 1, the more important is labour income for determining tax rates. What is apparent from Table 2 is that the interaction between labour income and capital income is quite important for determining effective tax rates, ETR and the marginal tax rates on labour income, MTR_x , while it is not important for the marginal tax rate on capital income, MTR_y .

**Table 2: Average values of ϕ
for ETR , MTR_x , and
 MTR_y , year 2015**

	Average for ages 20-99
ETR	0.849
MTR_x	0.211
MTR_y	0.999

Source: the authors

3 Integration with Microsimulation Model

This section describes how the income tax functions, described in Section 2.6, are incorporated into the model, so as to bridge the gap between the microsimulation model and the overlapping generations model. Each class of model has desirable properties. Microsimulation models are perfectly suited to calculate the total taxes paid, effective tax rates, and marginal tax rates for a population with richly defined demographic heterogeneity. They can incorporate much of the detail and interactions in the tax and benefit code, allowing for very specific policy levers to be adjusted and simulated. Overlapping generations models cannot accommodate this degree of policy detail and tax filer heterogeneity, however provide a full macroeconomic picture and life-cycle optimisation of agents, with some capacity to account for heterogeneity. By employing the methodology described in Section 2.6, we are able to capture a great deal of the complexities of the tax code in a manner that can be used in an

overlapping generations model framework.

This section first outlines the Italian income tax system in our base year of 2015. Second, some key features of the **EUROMOD** microsimulation model are provided, explaining the input data and the key output produced by the model. Third, we show how the **EUROMOD** output is mapped to the macro-model.

3.1 Tax on Labour and Tax on Capital in Italy as of 2015

The personal income tax ¹³ is progressive and consists of five personal income tax brackets with tax rates range from 23% to 43% (see Table 3 in section 5). In addition, most taxpayers are liable for social insurance contributions (in 2015, the largest share was a 9.19% pension contribution rate levied on most employment income), and top of these rates, which are set by the national government, additional rates are levied by regions (in 2015, between 1.23% and 3.33% with the possibility of progressive schedules) and municipalities (up to 0.8% with possibility to set a no-tax allowance) on the same tax base as the personal income tax. There are several exemptions, tax allowances and tax credits implemented. Also included under personal income tax are certain categories of capital income, such as most pension income and some property income (see Ceriani et al. (2017)). Therefore, there are clear interactions between labour and capital income in the income tax system, which further justifies the functional form explained in Section 2.6.

Capital income may be subject to separate taxation, though is sometimes subject to the personal income tax schedule as explained above. The separate capital taxation types are the following. In the 2011 a reform of the taxation of capital incomes changed the tax rates levied on interests from bank and postal accounts (from 27% to 20%) and on interests from long-term bonds and dividends (from 12.5% to 20%). The exception remains related to state bonds which are taxed at a lower 12.5% rate. From the 1st of July 2014 the standard rate increased to 26% (Ceriani et al. (2017)). In the **EUROMOD** model also property income is a part of capital income, that is why we give details on property taxation in what follows. Starting from the fiscal year 2012, the property tax has been redesigned. The new tax is called *Imposta Municipale Propria (IMU)*. The tax base for buildings registered at the cadastre is the cadastral value (for main residence and other buildings respectively) raised by 5% and multiplied by a coefficient equal to 160. Tax rates are different according to the type of building and municipalities can modify them. The baseline rates are: 0.4% for the main residence and 0.76% for other buildings. In the case of main residences there is a deduction of 200 EUR plus 50 EUR for each dependent children aged 26 or less living in the household. In 2013 the IMU on the main residence has been suspended. In 2014 the new tax (*TASI* in line with the Italian abbreviation) has been applied to the cadastral income of main residences raised by 5% and multiplied by a coefficient equal to 160. Tax rates are different according to the type of building and municipalities can modify them.

¹³In Italian: L'imposta sul reddito delle persone fisiche (IRPEF).

As an aside, note that a high marginal labour tax inevitably introduces negative incentives to work, especially among the lower paid whose labour market participation choices are typically highly responsive to marginal tax rates (see e.g. Meghir and Phillips (2009), Blundell (2016)).

3.2 The EUROMOD microsimulation model

EUROMOD is the European Union tax-benefit microsimulation model (Sutherland (2007), Sutherland and Figari (2013)). The model is a static tax and benefit calculator that makes use of representative microdata from the harmonised EU Statistics on Income and Living Conditions survey (EU-SILC) and from national statistics on income and living conditions surveys, to simulate individual tax liabilities and social benefit entitlements according to the rules in place in each Member State. Its main distinguishing feature is that it covers all EU countries within the same framework allowing for flexibility of the analysis and comparability of the results. Starting from gross incomes contained in the survey data, EUROMOD simulates most of the direct tax liabilities and benefit entitlements.¹⁴ EUROMOD represents all tax-relevant characteristics of individuals and households (including demographic and family characteristics, information on the type of labour activity, region of residence, and so on), it produces estimates of tax liabilities that take into account the full combined effects of all allowances, tax credits, exemptions, additional tax rates and differential tax treatments.

The effective tax rate on total income is calculated by simply taking income tax liabilities as a share of combined labour and capital income. These are obtained from the EUROMOD database, which is taken from the EU-SILC survey. Labour income is defined as earned income, which is the sum of wages, salaries and self-employment income. Capital income is defined as the sum of income from investment, pension and property.¹⁵ The microdata we use are at the individual level for main income earners in a household (we believe in this way we better capture the characteristics of the Italian tax system compared to household-level data) and come from the EUROMOD model.¹⁶ EUROMOD can also be used to compute marginal tax rates for each agent by assuming an increase of a fixed percentage of income and recalculating the labour or capital income tax liability.¹⁷

¹⁴For more information on EUROMOD check its official website: <https://www.euromod.ac.uk/>.

¹⁵We obtain labour income summing up EUROMOD's variables *yem* (wage employment income) and *yse* (self employment income), capital summing up *ypp* (private pension), *yiy* (investment income) and *ypr* (property income), labour taxes summing up *tinna_s* and *tinrg_s*, and capital taxes summing up *tinktcp_s*, *tinktdt_s*, *tinktdu_s*, *tinktbd_s*, *tinktgb_s*, *tprmb_s*, *tprob_s* and *tinrt_s* (all the latter variables starting with the letter *t* are for taxes and are endogenously computed by the EUROMOD model).

¹⁶We find that there are several observations with extreme values for their effective tax rate. Since effective (marginal and average) rates are calculated as ratios, unrealistically large values might be obtained, for example when the denominator is a measure of income and this is very small. We omit such outliers by imposing the following restrictions upon the raw output of the microsimulation model. First, we exclude observations with an effective tax rate greater than 70% and observations with a marginal tax rate greater than 75% or less than 0%. Second, we drop observations from the microsimulation model where adjusted total income is less than 5 EUR.

¹⁷We used a 3% increase for this purpose. Sensitivity tests were performed by also computing marginal tax rates with a 0.1% increase instead, and the resulting figures were identical after winsorizing the 1% lowest

3.3 Mapping income from micro to macro model

The link between EUROMOD and the OLG model is established based on income levels and age: a representative agent in the OLG has labour and capital incomes that are function of the general equilibrium market clearing and optimal choices of the agent with respect to labour supply and savings. In the OLG model, the tax rates associated with this agent are then determined solely by these endogenous income levels. The effective income tax rates enter into the agents' budget constraint and the marginal tax rates enter first-order conditions for the agents' optimization problem, thus impacting on agent behaviour (see Section 2.1 for more details). Any characteristic of an individual that is tax-relevant but not explicitly represented in the macro model is anyway indirectly captured by the estimated tax functions. Such modelling strategy allows one to account for the full range of average effects of the tax system from the EUROMOD microsimulation model, albeit implicitly.

Lastly, we note the mapping of the model to the data in terms of scale. In the model, we normalise the price of the consumption good to 1, while the tax rates are estimated as functions of income levels in the microdata. Therefore, we have to adjust the model income units to match the units of the microdata. To do this, we find the factor such that factor times average steady-state model income equals the mean income in the final year of the microdata. The tax rate functions are each functions of capital income and labour income $\tau(x, y)$. In order to make the tax functions return the correct tax rates for the associated levels of income, we multiply the model income x^m and y^m by a *factor*. This *factor* translates the model units into euros per month (i.e. tax data units).

$$factor \left[\sum_{s=E+1}^{E+S} \sum_{j=1}^J \lambda_j \omega_s (we_{j,s} n_{j,s} + rb_{j,s}) \right] = \text{average household income in tax data} \quad (24)$$

Since we do not know the steady-state wage, interest rate, household labour supply, and savings *ex ante*, the *factor* is an endogenous variable in the steady-state equilibrium computational solution. We hold the factor constant throughout the dynamic time-path equilibrium solution.

4 Baseline

The baseline steady-state shows how the model captures the behavioural choices over the life-cycle. Graphs in Figure 4 show baseline life-cycle patterns for labour supply, consumption, stock of assets (savings) and income taxes paid by the seven representative agents distinguished by income ability. Recall that the different earnings-ability groups have different sizes from ability group 1 representing 25 percent of the population to ability group 7 representing only the top 1 percent (see Section 2). Figure 4.a shows labour supply that rises to a high level around age 30, and continues to rise until the mid-50s before falling as

and largest values.

more households enter retirement. The figure shows how there is a tendency for more highly paid ability types to work less, which comes from the household optimisation problem, where households trade off utility from consumption and disutility from labour. Figure 4.b shows the consumption profile, which rises to a peak in the mid-to-late 40s before falling gradually in older age. As expected, higher ability types have higher levels of consumption. Figure 4.c shows the savings profile, which rises in all ability types until around age 60. After this age, most ability types stabilise or reduce their stock of savings, however the highest ability types continue to grow their savings. At older ages, savings behaviour is mostly driven by households trading off utility from consumption and utility from leaving bequests. Note that it is never optimal to end life without savings, because households gain a very large marginal utility from leaving small bequests. Figure 4.d shows income taxes paid, which rises to a peak for those of age mid-to-late 50s, corresponding to the peak in labour supply and earnings. The top one percent of earnings-ability, group seven, has much higher earnings and hence, much higher income taxes, reflecting the progressive nature of the income tax function.

Figure 4: Baseline steady-state values for labour supply, consumption, savings and income tax paid for seven ability types, ages 20-99

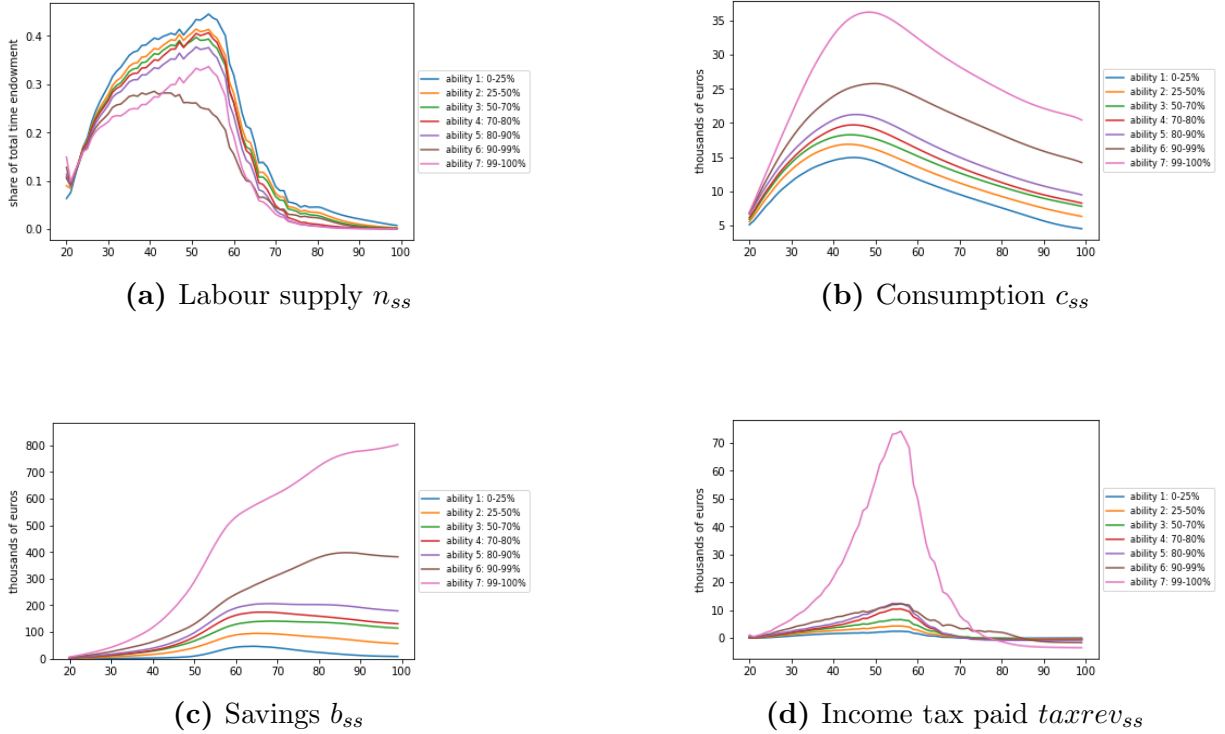
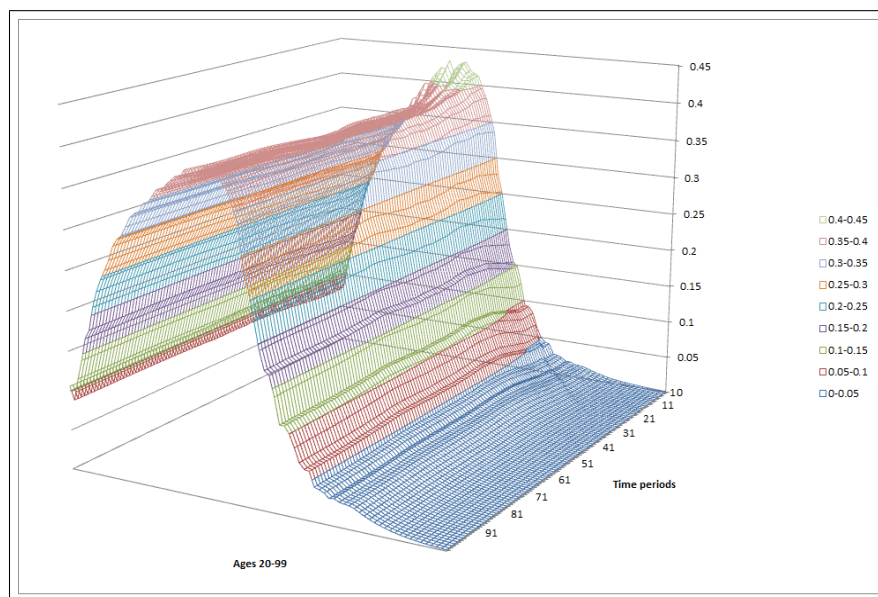


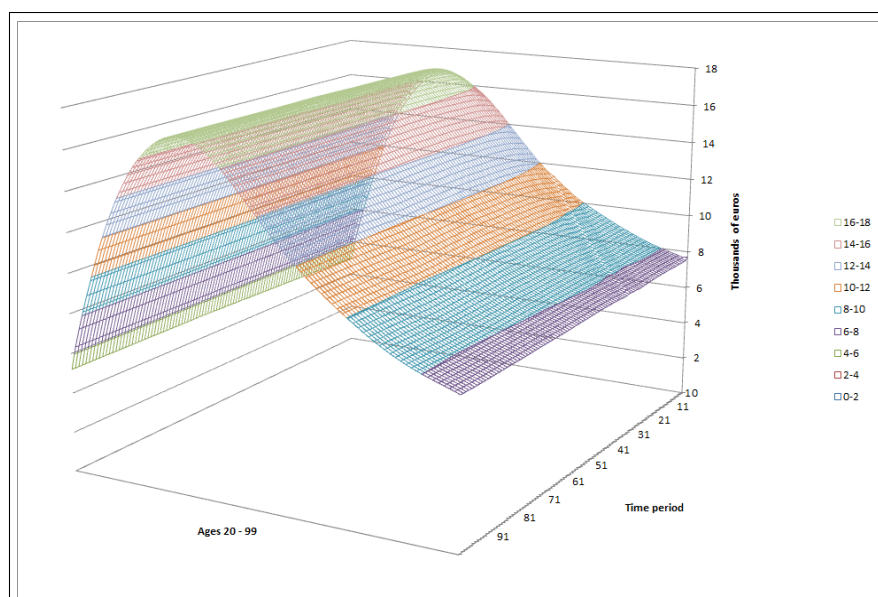
Table 5 in Appendix A details calibrated values and other exogenous parameters' values of the current OLG model version.

Figure 5: Labour supply in dynamic time path (3rd earnings-ability group; share of total time endowment): time 0 to 99, ages 20 to 99



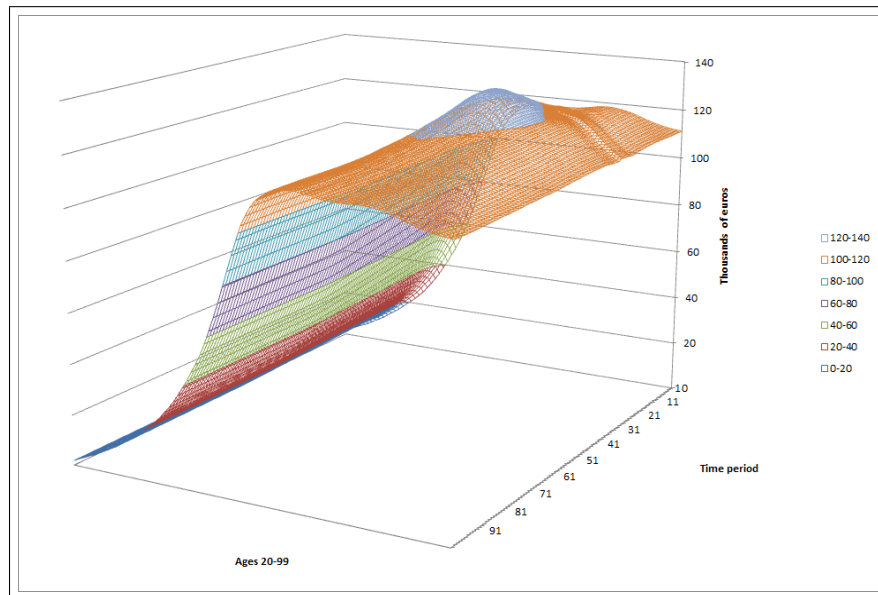
Source: Authors' calculations

Figure 6: Consumption in dynamic time path (3rd earnings-ability group; 1000s of euros per year): time 0 to 99, ages 20 to 99



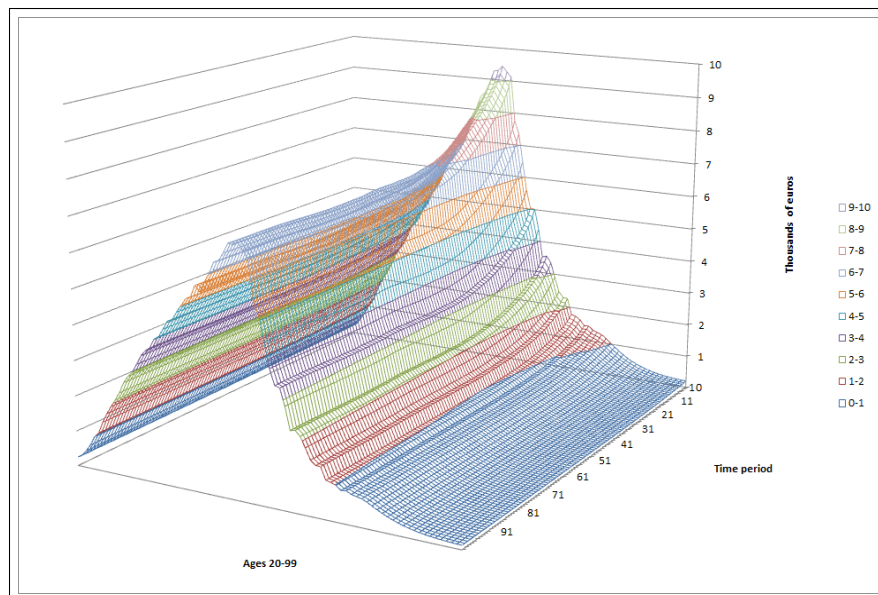
Source: Authors' calculations

Figure 7: Savings in dynamic time path (3rd earnings-ability group; 1000s of euros per year): time 0 to 99, ages 20 to 99



Source: Authors' calculations

Figure 8: Income tax paid in dynamic time path (3rd earnings-ability group; 1000s of euros per year): time 0 to 99, ages 20 to 99



Source: Authors' calculations

Figures 5 to 8 show how individual variables evolve over time. They show the time path for labour supply, consumption, savings and tax revenues paid for ability type 3 (the 50th to 70th percentile for earnings ability, as explained in Section 2). These are level variables expressed in thousands of euros apart from labour supply that is expressed as ratio of hours worked to the total time endowment. Similar broad shapes can be seen to those in the steady state. However the impact of demographic change can be seen. For example, today’s peak of population between approximately age 40 to 52 causes *inter alia* a peak of output. As this group ages, they hit their peak earnings, after which they begin to retire. (See d’Andria et al. (2019) for details on the population structure.) The influence of this demographic feature can be seen, for example in the savings path, Figure 7. The labour supply path, Figure 5, begins at a higher level and over a few years falls to nearly the steady state level. This fall causes a clear fall in the income tax, as shown in Figure 8.

5 A Hypothetical Tax Reform Simulation

We present here an exemplary policy simulation to demonstrate the model capabilities. The aim of this paper is not to discuss any actual tax reform that is on the political agenda. Rather the aim is to show the ability of the model to simulate an exemplary tax reform in order to exhibit the advantages of applying DeBacker et al. (2018b)’s approach of incorporating non-linear tax functions into a macroeconomic model. Our tax reform assumes a cut in the marginal tax rates for personal income tax by 2 percentage points, for all tax brackets at the national level. The baseline for 2015 and the assumed policy for the simulation are shown in Table 3.

Table 3: Statutory marginal tax rates for personal income tax by bracket: baseline vs. reform

PIT bracket (annual income in EUR)	Baseline	Reform
up to 15,000	0.23	0.21
15,000 to 28,000	0.27	0.25
28,000 to 55,000	0.38	0.36
55,000 to 75,000	0.41	0.39
75,000 and above	0.43	0.41

Source: **EUROMOD** and authors

The direct impact of the simulated policy is to reduce the rates for all personal income taxpayers. Such a reform would also interact with other taxes and benefits. Both the direct effects and the interactions with other policies are captured in the **EUROMOD** simulation. The simulation outcome then provides the basis for re-estimating tax functions, which are defined by the 12 parameters presented in Table 1 above. Table 4 compares the values for the tax rate function parameters, for ETR , $MTRx$ and $MTRy$ separately for the baseline and the

hypothetical reform scenario.

Table 4: Estimated tax function parameter values: baseline vs. reform

Parameter	Baseline			Reform		
	ETR	MTR _x	MTR _y	ETR	MTR _x	MTR _y
<i>A</i>	2.06E-08	2.76E-07	6.48E-04	2.23E-08	2.44E-07	6.48E-04
<i>B</i>	5.32E-04	6.36E-16	3.50E+00	4.91E-04	6.36E-16	3.50E+00
<i>C</i>	1.57E-07	4.08E-05	9.93E-03	1.52E-07	5.05E-05	8.28E-07
<i>D</i>	4.45E-06	8.95E+00	5.32E+01	5.34E-06	7.30E+00	5.25E+01
<i>max_x</i>	8.00E-01	9.75E-02	1.25E-01	8.00E-01	8.03E-02	1.25E-01
<i>min_x</i>	-7.19E-02	5.00E-04	1.25E-01	-8.00E-02	5.00E-04	1.25E-01
<i>max_y</i>	8.00E-01	8.00E-01	7.00E-04	8.00E-01	8.00E-01	7.00E-04
<i>min_y</i>	-9.67E-02	6.00E-04	6.00E-04	-9.67E-02	6.00E-04	6.00E-04
<i>shift_x</i>	8.06E-02	9.70E-04	1.00E-06	8.88E-02	7.98E-04	1.00E-06
<i>shift_y</i>	1.06E-01	7.99E-03	1.00E-06	1.06E-01	7.99E-03	1.00E-06
<i>shift</i>	-9.67E-02	5.00E-04	6.00E-04	-9.73E-02	5.00E-04	6.00E-04
<i>φ</i>	8.49E-01	2.11E-01	9.99E-01	8.37E-01	2.07E-01	9.99E-01

Source: Authors' calculations

The consequences for the shape of the effective tax rate function are shown in Figure 9. The figure shows that those with a high labour income and a low capital income receive the greatest reduction in *ETR*, close to the 2 percentage points in the reform. Those with more capital income receive less of a change in their *ETR*, as their capital income is mostly unaffected by the reform. Those with low labour income receive less benefit, as they were not paying income tax on large portions of their labour income.

In a similar fashion, the consequences for the marginal tax rate on labour income are shown in Figure 10. The figure shows how little capital income impacts the change, with nearly all the variation in the labour income dimension. Those with low labour income receive little reduction. As labour income rises, fall in the *MTR_x* from the reform becomes larger (in absolute terms), such that all those earning in excess of 1000 euros face an *MTR_x* between 1.5 and 2 percentage points lower. Lastly, the consequences for the marginal tax rate on capital income were insignificant, which was anticipated; the largest absolute difference between the reform and baseline *MTR_y* functions was less than 10^{-7} pp.

A reduction across the board of tax rates, like the one here simulated, decreases marginal tax rates on labour income. This alone implies a substitution effect such that labour supply will increase. At the same time, income effects would imply less labour supply. Also due to the general equilibrium design of the model, gross wages and capital formation will be affected and this in turn will impact ability groups differently. The different composition of labour versus capital incomes also imply different effects by age. Overall, as will be shown, substitution effects dominate income effects, but with different intensities across ability groups and ages. Our model is particularly suitable as it is able to individually simulate

Figure 9: Change in effective tax rate (ETR) function depending on labour and capital income (in 1000s of euros per month), reform less baseline

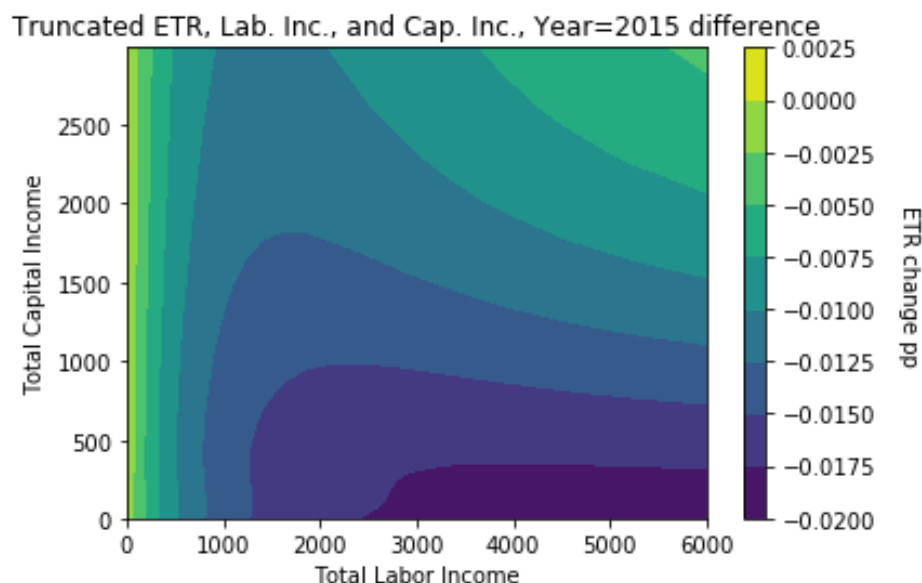
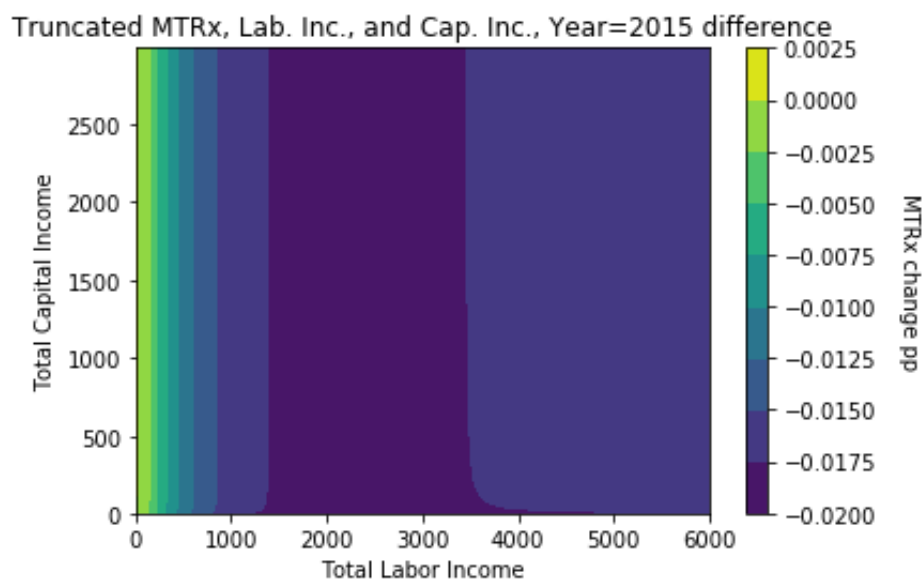


Figure 10: Change in marginal tax rate on labour income ($MTRx$) function depending on labour and capital income (in 1000s of euros per month), reform less baseline



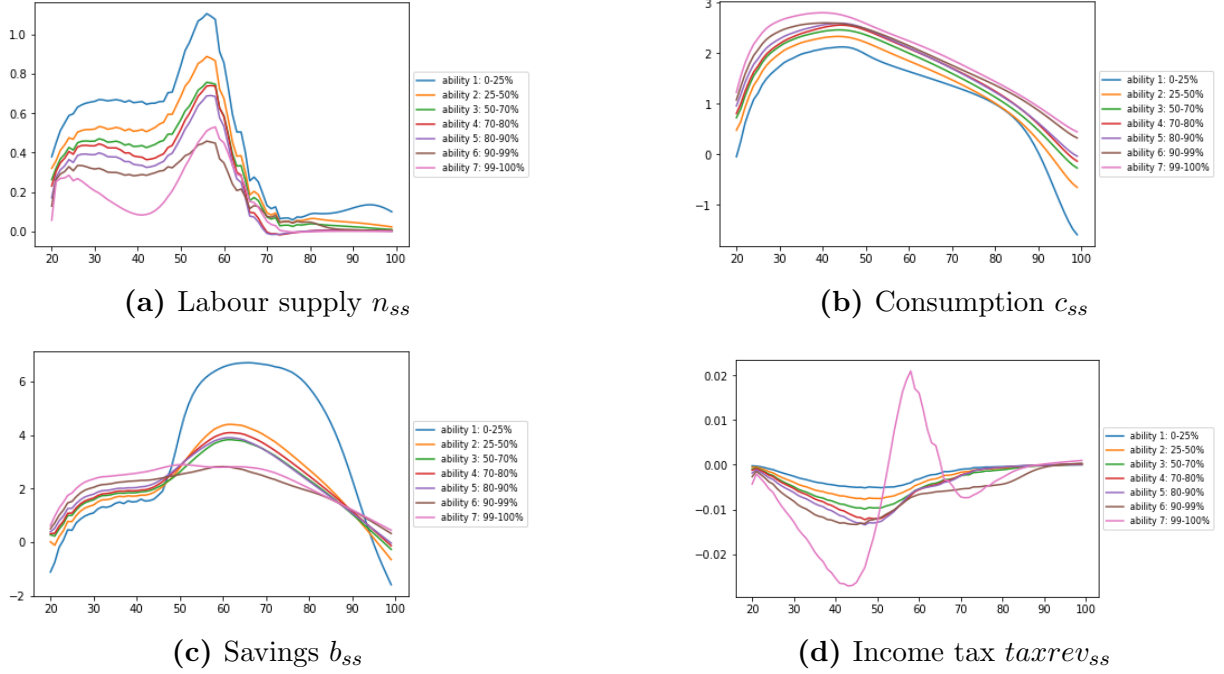
agents by ability and age, to derive macroeconomic as well as distributional effects and to disentangle their differential impacts across the population.

Figure 11 reports the changes in labour supply, consumption, savings (percent deviations from the baseline solution) and income tax paid (absolute change: value under the reform minus value under the baseline)¹⁸ produced by the simulated tax change in comparison to baseline values. Note that, differently from Figure 4, the graphs in Figure 11 display changes of the steady-state equilibrium values compared to the baseline's steady state, rather than the actual values. As anticipated (given the chosen functional forms for agents' utility), the reduction in marginal tax rates on labour income causes an increase in the labour supply. Aggregate consumption in the long run increases by 1.85% versus the baseline, aggregate saving increases by 1.18% and aggregate output goes up by 2.14% versus the baseline with higher marginal tax rates. A positive feature of the model is that the policy effects can be further analysed from the generational and redistributive perspectives.

As can be seen from graphs on Figure 11.a to 11.d the effects of the tax rate cut at the steady-state equilibrium differ substantially based on age and ability type. The observed changes for labour supply are substantial for all working ages, especially in the later working ages. The changes for saving are generally positive and especially so between the ages of 45 and 80 (approximately, depending on the ability type). As the marginal utility of savings also differs across ability groups (it decreases with income), increases in savings are found larger for low-ability agents for these ages. Figure 11.d shows that income tax revenues decrease for the whole population. Younger and higher-ability agents are who mostly benefit from the tax cut, as one would expect being these the highest income earners in the population. An outstanding pattern for the highest earners can be explained as follows. As the richest fraction of the population, the highest ability earners when young get the greatest reduction in the amount of income tax paid. As they further increase their labour supply at the age of 50, they start to generate positive change (as compared with negative change for other ability types) in income taxes they pay to the state budget. Consumption tax revenues increase in the long run after the reform due to the increase in the overall consumption tax paid.

¹⁸The reaction of income tax revenues is presented as the absolute change instead of percent change due to a very low denominator value that leads to enormous percent change.

Figure 11: Change in steady-state values between baseline and the simulated tax reform for labour supply (absolute deviation), consumption (% deviation), savings (% deviation) and income tax (absolute deviation), ages 20-99



The results stemming from this exemplifying simulation show the potential benefits of our model for evaluation of actual policies. Heterogeneous responses and effects across ages and abilities, after taking into account general equilibrium effects, are reported thus providing additional insight compared to models such as, for example, standard overlapping generations models. Moreover thanks to the integration with the **EUROMOD** microsimulation model, a wide array of characteristics of the tax and benefits system can be captured, albeit in a stylized way. Of special interest are age-dependant heterogeneous effects which likely affect the scoring of different policy options related to tax reforms.

In Figures 22, 23, 24, and 25 in the Appendix F we show effects of the marginal tax rates cut reform over time for individual variables. For better readability we present the first 100 years of the solution for the third ability group. Except for income tax revenues, which are reported as absolute difference (see footnote 19), other variables are expressed as percentage deviations from the baseline.

In addition, in Appendix E we also present the full dynamic path (i.e. for 479 years) of the aggregate variables: capital, labour, output and consumption.

6 Conclusions and Future Work

In this paper we incorporate micro-founded tax policy shock into the heterogeneous-agent macroeconomic model using the output from the **EUROMOD** microsimulation model. The specific version of the model used here was calibrated on Italian data for the year 2015. We show that the approach used for modelling tax functions is powerful enough to be able to capture the most important non-linearities of the actual tax code, together with the interaction effects between labour and capital incomes on both average and marginal tax rates.

To illustrate our approach we ran an exemplary policy simulation of cutting the marginal personal income tax rates across all tax brackets by two percentage points. Using the model with 560 categories of heterogeneous agents (80 ages times 7 ability types), we were able to study dynamic effects of the tax policy over the life-cycle and its re-distributional effects. We showed that this policy affects differently households distinguished by age and ability type. We show the incentive effect of reducing labour taxes, which has the strongest impact on those in their later working years and those with lower earnings-ability. The higher after-tax income also translates into higher consumption and savings (for most ages and earnings-ability types). As the tax cuts benefit those of working age, the immediate direct impact on the older generations was small, however in the long run, these age groups benefit from higher savings levels. The hypothetical simulation gives a clear example of how the heterogeneity regarding age and earnings-ability can be retained in the macro-model founded on the microsimulation output.

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Appendices

A Exogenous Parameters and Calibrated Values

All exogenous parameters that are inputs to the model are listed in Table 5.

Table 5: List of exogenous parameters and baseline calibration values

Symbol	Description	Value
S	Maximum periods in economically active household life	80
E	Number of periods of youth economically outside the model	$\text{round}\left(\frac{S}{4}\right) = 20$
T_1	Number of periods to steady state for initial time path guesses	320
T_2	Maximum number of periods to steady state for nonsteady-state equilibrium	400
R	Eligible age for pension transfers	62
$\{\{\omega_{s,0}\}_{s=1}^{E+S}\}_{t=0}^{T_2+S-1}$	Initial population distribution by age	(see Appendix C)
$\{f_s\}_{s=1}^{E+S}$	Fertility rates by age	(see Appendix C.1)
$\{i_s\}_{s=1}^{E+S}$	Immigration rates by age	(see Appendix C.3)
$\{\rho_s\}_{s=0}^{E+S}$	Mortality rates by age	(see Appendix C.2)
$\{e_{j,s}\}_{j,s=1}^{J,S}$	Deterministic ability process	(see 1)
$\{\lambda_j\}_{j=1}^J$	Lifetime income group percentages	$[0.25, 0.25, 0.20, 0.10, 0.10, 0.09, 0.01]$
J	Number of lifetime income groups	7
$\tilde{l}$	Maximum labour supply	1
β	Discount factor	$(0.975)^{\frac{80}{S}}$
σ	Coefficient of constant relative risk aversion	2.2
b	Scale parameter in utility of leisure	0.527
v	Shape parameter in utility of leisure	1.497
	Frisch elasticity	0.9
χ_s^n	Disutility of labour level parameters	(see d’Andria et al. (2019))
χ_j^b	Utility of bequests level parameters	(see d’Andria et al. (2019))
ζ	Share of bequests received by households	(see d’Andria et al. (2019))
η	Share of transfers received by households	(see d’Andria et al. (2019))
τ_s^c	Marginal tax rate on consumption by age	(see d’Andria et al. (2019))
Z	Level parameter in production function	1.0
γ	Capital share of income	0.4
δ	Capital depreciation rate	$1 - (1 - 0.044)^{\frac{80}{S}} = 0.044$
g_y	Growth rate of labour augmenting technological progress	$(1 + 0.03)^{\frac{80}{S}} - 1 = 0.03$

B Endogenous Variables

Endogenous variables of the OLG model are listed in Table 6.

Table 6: List of endogenous variables

Symbol	Description
$n_{j,s}$	Labour supply by age and ability
$b_{j,s}$	Savings by age and ability
$c_{j,s}$	Consumption by age and ability
$T_{j,s}^I$	Total income tax by age and ability
$T_{j,s}^C$	Total consumption tax by age and ability
r	Real interest rate
w	Real wage rate
$\tau_{j,s}^{mtrx}$	Marginal tax rate on labour income by age
$\tau_{j,s}^{mtry}$	Marginal tax rate on capital income by age
$\tau_{j,s}^{etr}$	Average tax rate on income by age
L	Aggregate labour supply
BQ	Aggregate bequests
K	Aggregate capital stock
C	Aggregate consumption
I	Aggregate investment
Y	Aggregate output
TR	Aggregate transfers
$factor_t$	Factor transforming model units into the data units

C Demographics

In this appendix, we characterize the equations and parameters that govern the transition dynamics of the population distribution by age. Mortality rates, fertility rates and net immigration rates projections are taken from Eurostat.

We define $\omega_{s,t}$ as the number of households of age s alive at time t . A measure $\omega_{1,t}$ of households is born in each period t and live for up to $E+S$ periods, with $S \geq 4$.¹⁹ Households are termed “youth”, and do not participate in market activity during ages $1 \leq s \leq E$. The households enter the workforce and economy in period $E+1$ and remain in the workforce until they unexpectedly die or live until age $s = E+S$. We model the population with households age $s \leq E$ outside of the workforce and economy in order most closely match the empirical population dynamics.

The population of agents of each age in each period $\omega_{s,t}$ evolves according to the following function,

$$\begin{aligned}
 \omega_{1,t+1} &= \sum_{s=1}^{E+S} f_s \omega_{s,t} + i_1 \omega_{1,t} \quad \forall t \\
 \omega_{s+1,t+1} &= (1 - \rho_s) \omega_{s,t} + i_{s+1} \omega_{s+1,t} \quad \forall t \quad \text{and} \quad 1 \leq s \leq E+S-1
 \end{aligned} \tag{25}$$

¹⁹Theoretically, the model works without loss of generality for $S \geq 3$. However, because we are calibrating the ages outside of the economy to be one-fourth of S (e.g., ages 21 to 100 in the economy, and ages 1 to 20 outside of the economy), it is convenient for S to be at least 4.

where $f_s \geq 0$ is an age-specific fertility rate, i_s is an age-specific net immigration rate, ρ_s is an age-specific mortality hazard rate.²⁰ The total population in the economy N_t at any period is simply the sum of households in the economy, the population growth rate in any period t from the previous period $t - 1$ is $g_{n,t}$, $\tilde{N}_t$ is the working age population, and $\tilde{g}_{n,t}$ is the working age population growth rate in any period t from the previous period $t - 1$.

$$N_t \equiv \sum_{s=1}^{E+S} \omega_{s,t} \quad \forall t \quad (26)$$

$$g_{n,t+1} \equiv \frac{N_{t+1}}{N_t} - 1 \quad \forall t \quad (27)$$

$$\tilde{N}_t \equiv \sum_{s=E+1}^{E+S} \omega_{s,t} \quad \forall t \quad (28)$$

$$\tilde{g}_{n,t+1} \equiv \frac{\tilde{N}_{t+1}}{\tilde{N}_t} - 1 \quad \forall t \quad (29)$$

We discuss the approach to estimating fertility rates f_s , mortality rates ρ_s , and immigration rates i_s in Sections C.1, C.2, and C.3.

C.1 Fertility rates

In our model, we use Eurostat baseline projections for fertility rates.²¹ Annual data are used until 2070, after which the fertility rates are assumed constant at the 2070 rates. Figure 12 shows the fertility-rate data and the estimated average fertility rates for $E + S = 100$ for selected years.

The blue line in Figure 12 shows the 2015 Italian fertility rate per person by age (showing the peak in fertility at age 33). Eurostat baseline projections show modest increases in fertility rates over time, with the values for 2040 and 2070 shown in Figure 12.

C.2 Mortality rates

The mortality rates in our model ρ_s are a one-period hazard rate and represent the probability of dying within one year, given that a household is alive at the beginning of period s . We assume that the mortality rates for each age cohort ρ_s are constant across time.

We use Eurostat baseline projections for Italian mortality rates by age.²² Annual data are used until 2070, after which the mortality rates are assumed constant at the 2070 rates. Figure 13 shows the mortality rate data and the corresponding model-period mortality rates for $E + S = 100$. We constrain the mortality rate to be 1.0 or 100 percent at the maximum age of 100.

²⁰The parameter ρ_s is the probability that a household of age s dies before age $s + 1$.

²¹Eurostat database proj_15naasfr. We convert Eurostat fertility per woman data to fertility per person using Eurostat baseline projections for female population compared with total population - Eurostat database proj_15npms. Note also that Eurostat fertility data are for live births.

²²Eurostat database proj_15naasmr. As the mortality data is provided separately for male and female, we calculate the mortality per person using Eurostat baseline projections for male and female population compared with total population - Eurostat database proj_15npms.

Figure 12: Fertility rates by age (f_s) for $E + S = 100$ selected years

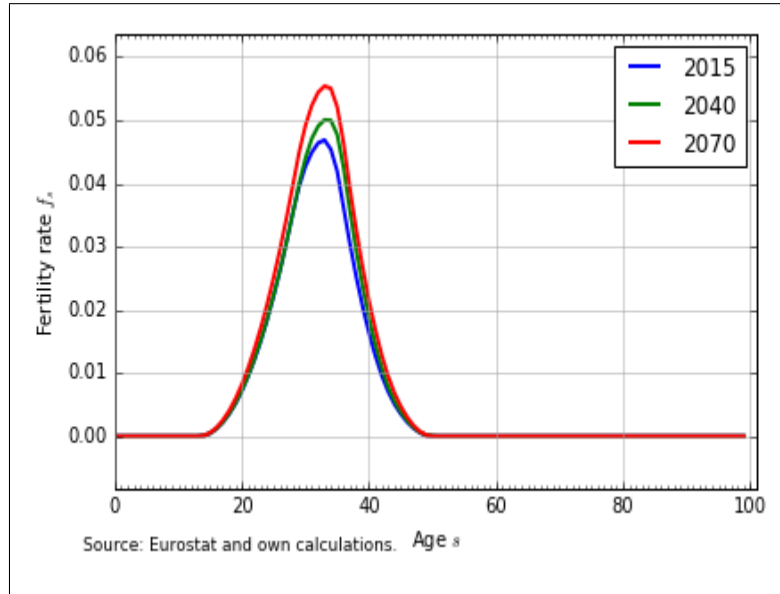
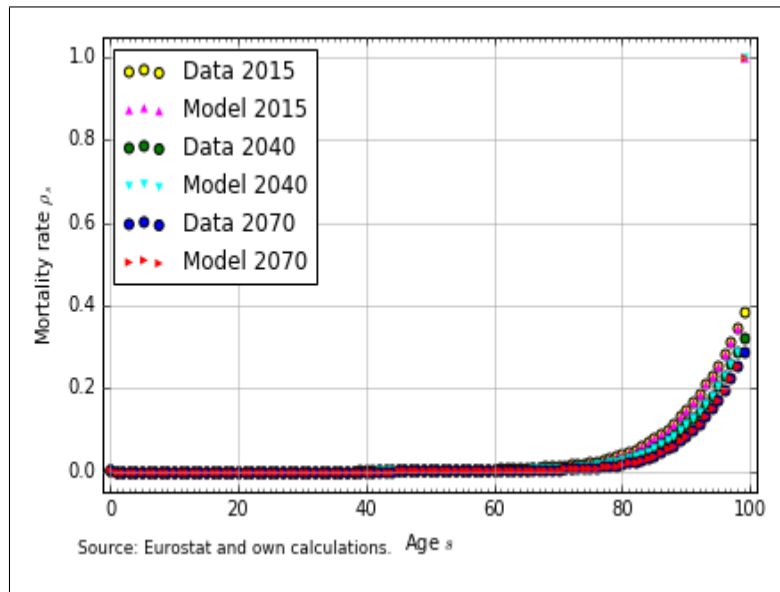
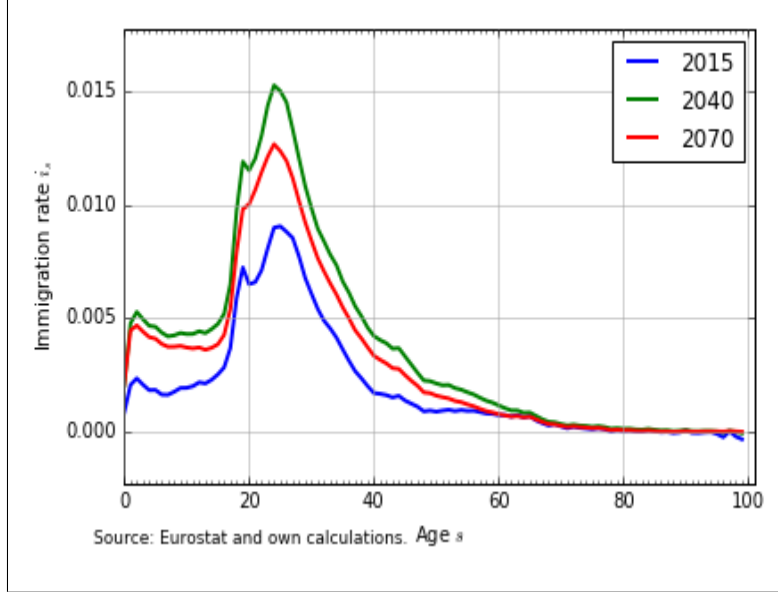


Figure 13: Mortality rates by age (ρ_s) for $E + S = 100$ selected years



C.3 Immigration rates

Figure 14: Immigration rates by age (i_s) for $E + S = 100$ selected years



Our model uses net immigration rates from Eurostat.²³ Annual data are used until 2070, after which the net immigration rates are assumed constant at the 2070 rates. Figure 14 shows the net immigration rates for selected years, showing that the general pattern of immigration by age is projected to continue, with the rates rising from 2015 to 2040 (the peak year is 2039), before falling gradually until 2070.

We make a small adjustment to the immigration rates after a large number of periods in order to make computation of the transition path equilibrium of the model compute more robustly. d'Andria et al. (2019) gives more details on it and also discusses population steady-state and transition path in our model.

D Income Tax Data and Estimated Functions

The 3D figures showing the effective tax rates (ETR), the marginal tax rates on labour income ($MTRx$) and on capital income ($MTRY$), Figure 2, are only shown from one angle, which can only give a partial view of the data. For completeness, the same figures are repeated below from multiple angles: ETR in Figure 15, $MTRx$ in Figure 16 and $MTRY$ in Figure 17. In each case, the top-left figure shows angle 240 degrees, which is identical to Figure 2, and then rotations to 300 degrees, 60 degrees and 120 degrees are shown.

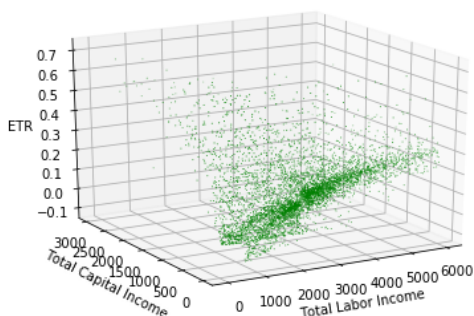
The same is also done for the estimated tax functions for ETR , $MTRx$ and $MTRY$, so they can also be viewed from multiple angles: ETR in Figure 18, $MTRx$ in Figure 19

²³Eurostat database proj_15nanmig. As the data are in levels of net immigration, we calculate the rates using the Eurostat baseline projections for total population - Eurostat database proj_15npms.

and MTR_y in Figure 20. In each case, the top-left figure shows angle 240 degrees, which is identical to Figure 3, and then rotations to 300 degrees, 60 degrees and 120 degrees are shown.

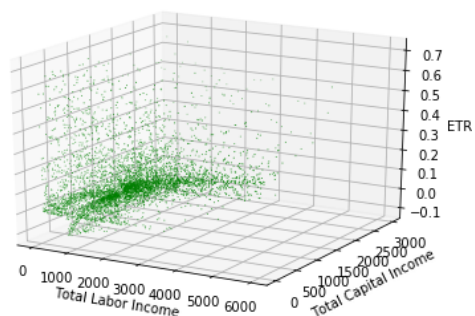
Figure 15: Scatter plot of ETR as functions of labour income and capital income from microsimulation model, year 2015, viewed from different angles

Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



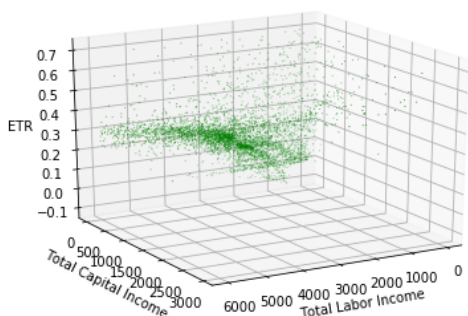
(a) ETR , from angle 240 degrees

Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



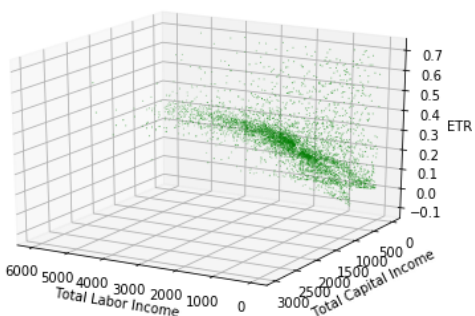
(b) ETR , from angle 300 degrees

Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) ETR , from angle 60 degrees

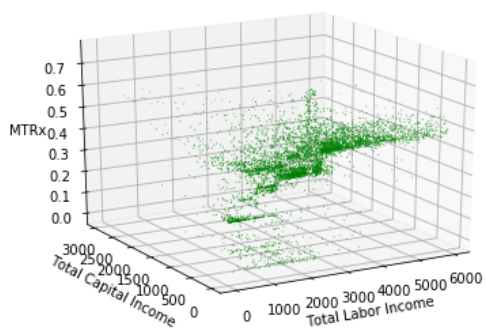
Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



(d) ETR , from angle 120 degrees

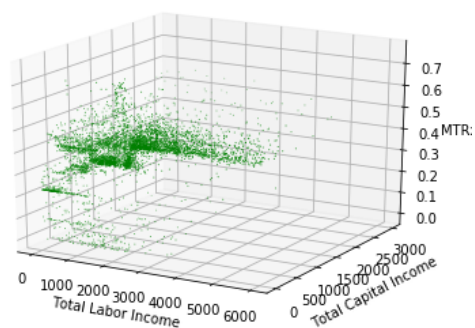
Figure 16: Scatter plot of $MTRx$ as functions of labour income and capital income from microsimulation model, year 2015, viewed from different angles

Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



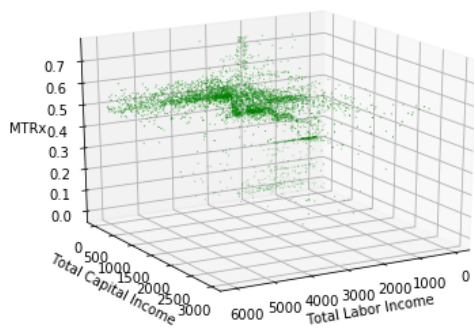
(a) $MTRx$, from angle 240 degrees

Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



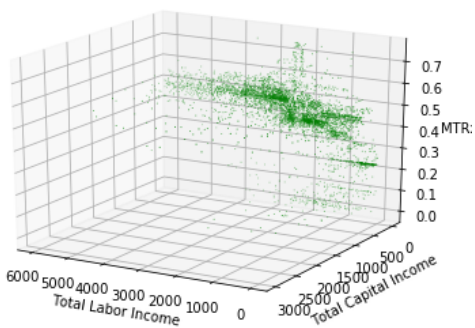
(b) $MTRx$, from angle 300 degrees

Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) $MTRx$, from angle 60 degrees

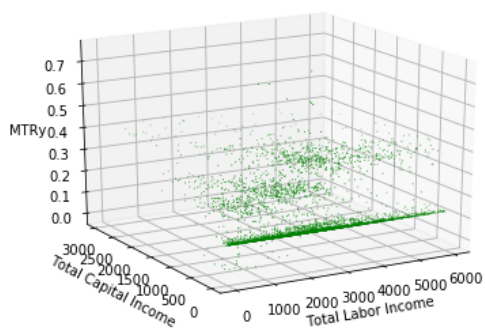
Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



(d) $MTRx$, from angle 120 degrees

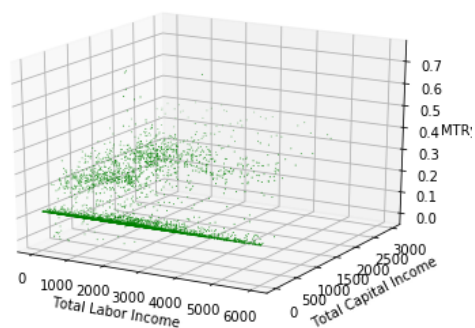
Figure 17: Scatter plot of MTR_y as functions of labour income and capital income from microsimulation model, year 2015, viewed from different angles

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



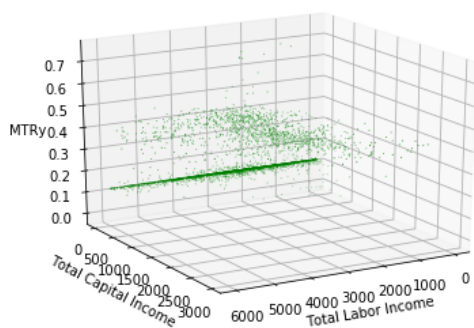
(a) MTR_y , from angle 240 degrees

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



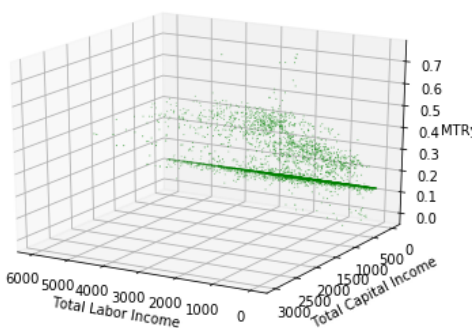
(b) MTR_y , from angle 300 degrees

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) MTR_y , from angle 60 degrees

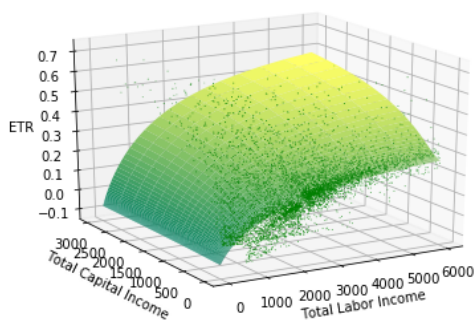
Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



(d) MTR_y , from angle 120 degrees

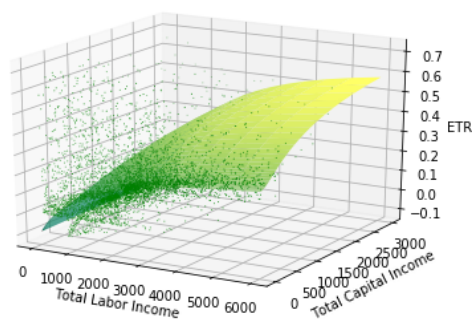
Figure 18: Estimated tax rate functions of ETR functions of labour income and capital income from microsimulation model, year 2015, viewed from different angles

Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



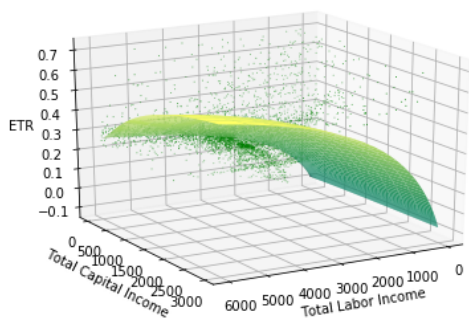
(a) ETR , from angle 240 degrees

Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



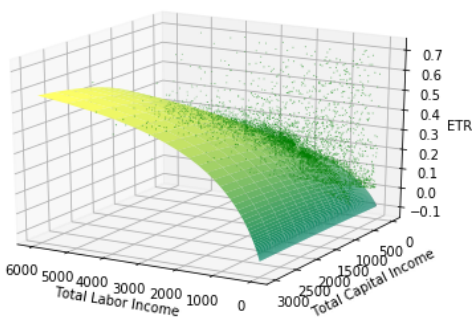
(b) ETR , from angle 300 degrees

Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) ETR , from angle 60 degrees

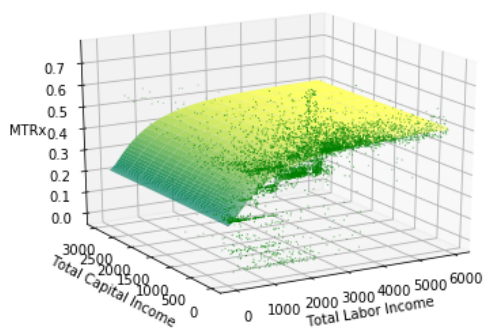
Truncated ETR, Lab. Inc., and Cap. Inc., Year=2015 baseline



(d) ETR , from angle 120 degrees

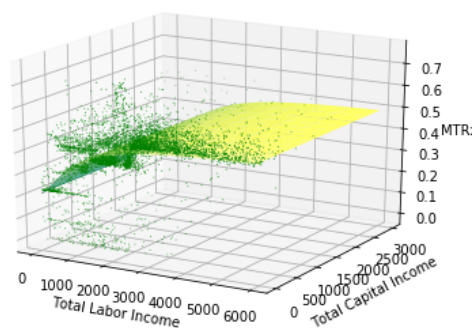
Figure 19: Estimated tax rate functions of $MTRx$ functions of labour income and capital income from microsimulation model, year 2015, viewed from different angles

Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



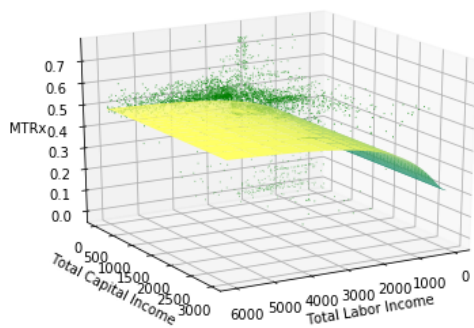
(a) $MTRx$, from angle 240 degrees

Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



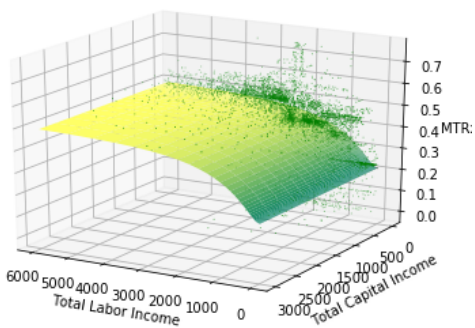
(b) $MTRx$, from angle 300 degrees

Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) $MTRx$, from angle 60 degrees

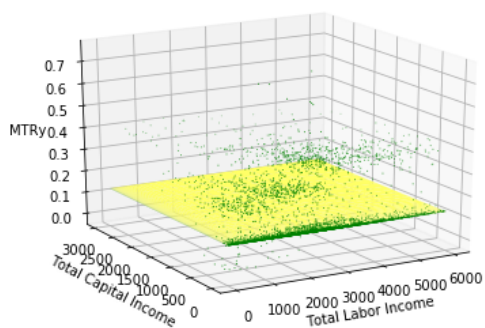
Truncated $MTRx$, Lab. Inc., and Cap. Inc., Year=2015 baseline



(d) $MTRx$, from angle 120 degrees

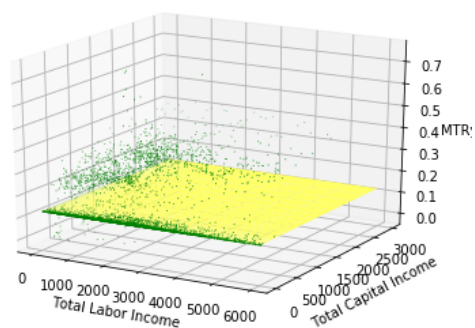
Figure 20: Estimated tax rate functions of MTR_y functions of labour income and capital income from microsimulation model, year 2015, viewed from different angles

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



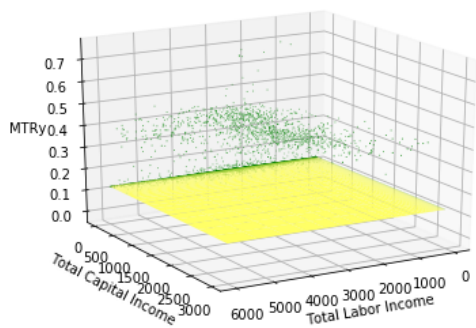
(a) MTR_y , from angle 240 degrees

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



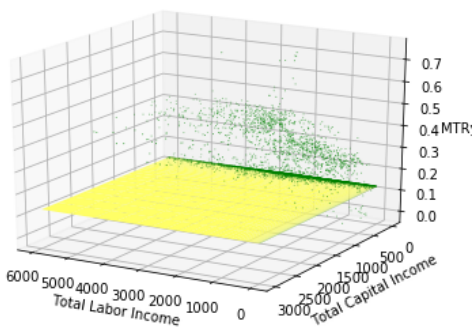
(b) MTR_y , from angle 300 degrees

Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



(c) MTR_y , from angle 60 degrees

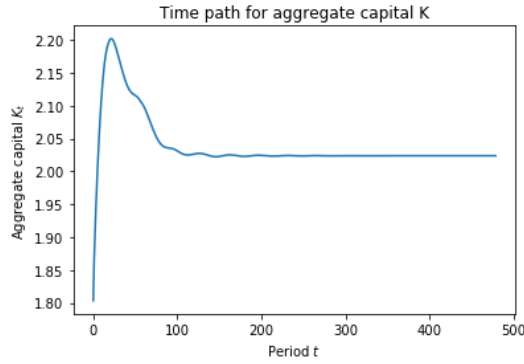
Truncated MTR_y , Lab. Inc., and Cap. Inc., Year=2015 baseline



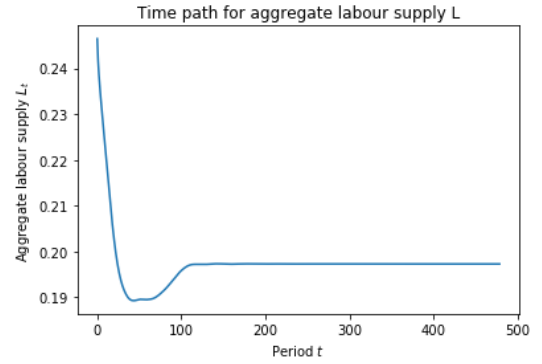
(d) MTR_y , from angle 120 degrees

E Time path solution for aggregate variables

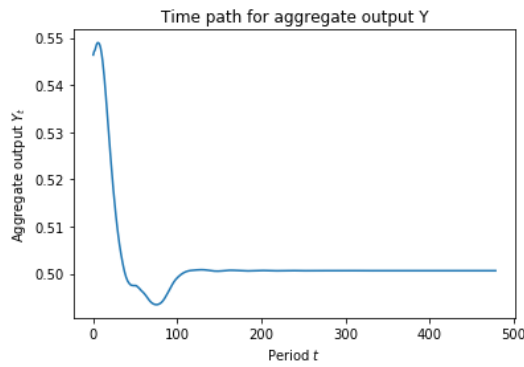
Figure 21: Time path solution for aggregate variables: period 0 to 479



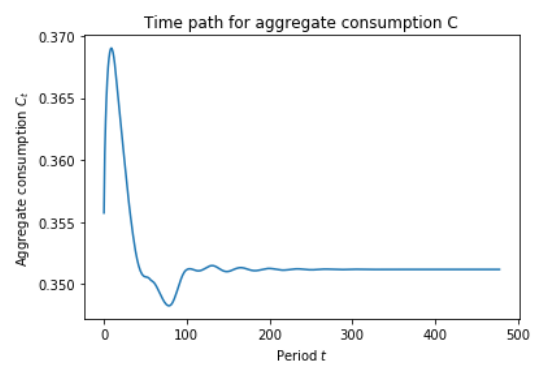
(a) Aggregate capital K_t



(b) Aggregate labour supply L_t



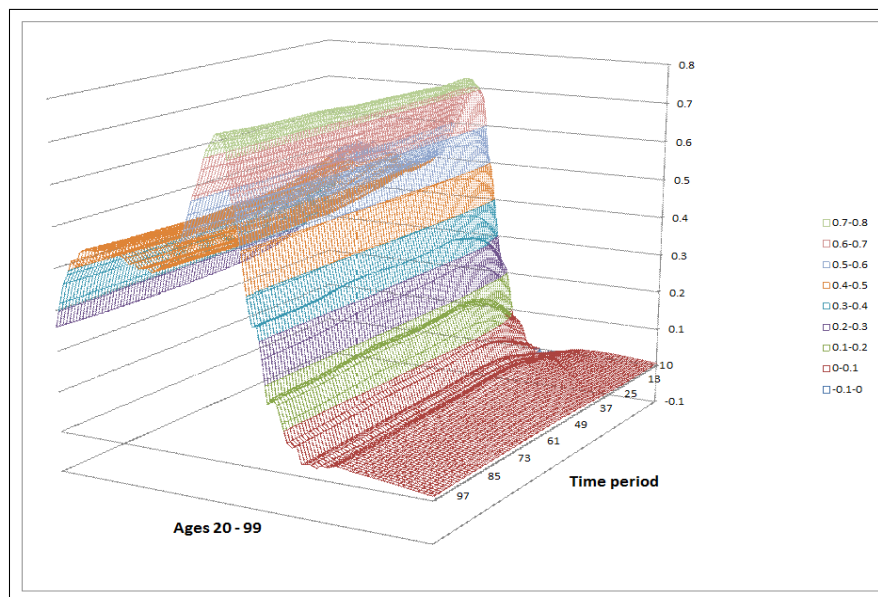
(c) Aggregate output Y_t



(d) Aggregate consumption C_t

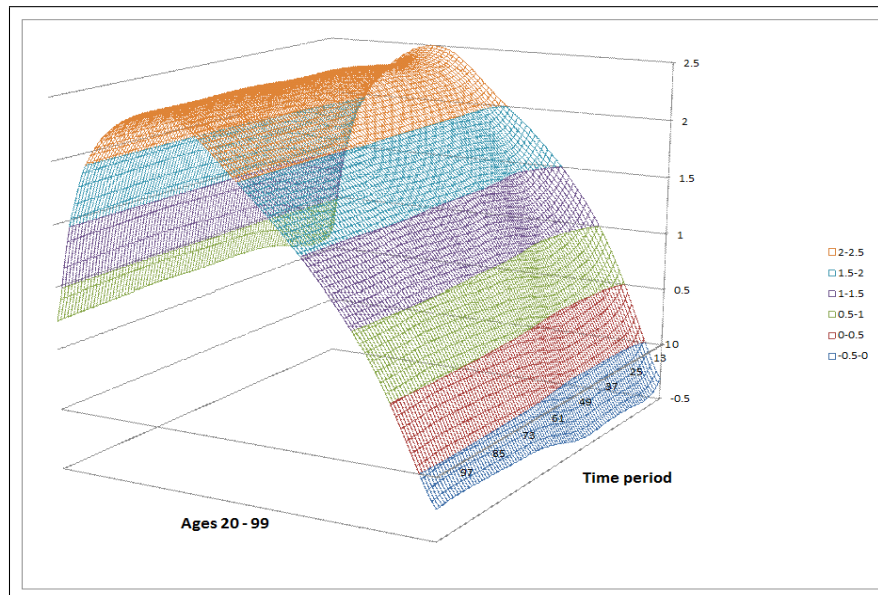
F Time path solution - reform vs baseline

Figure 22: Labour supply over time: 1st to 100th period, ages 20 to 99, 3rd ability group (absolute change in share of total time endowment, reform vs baseline)



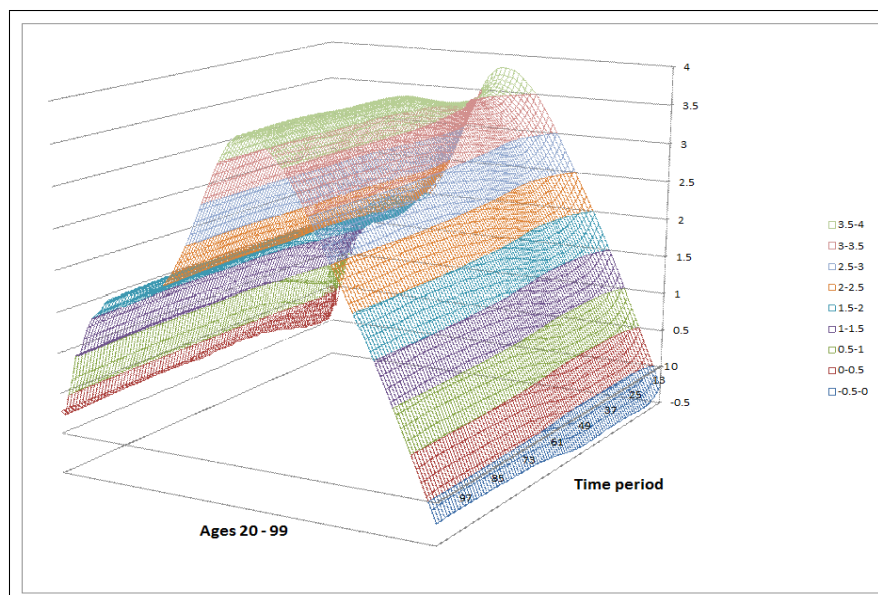
Source: Authors' calculations

Figure 23: Consumption over time: 1st to 100th period, ages 20 to 99, 3rd ability group (percentage change, reform vs baseline)



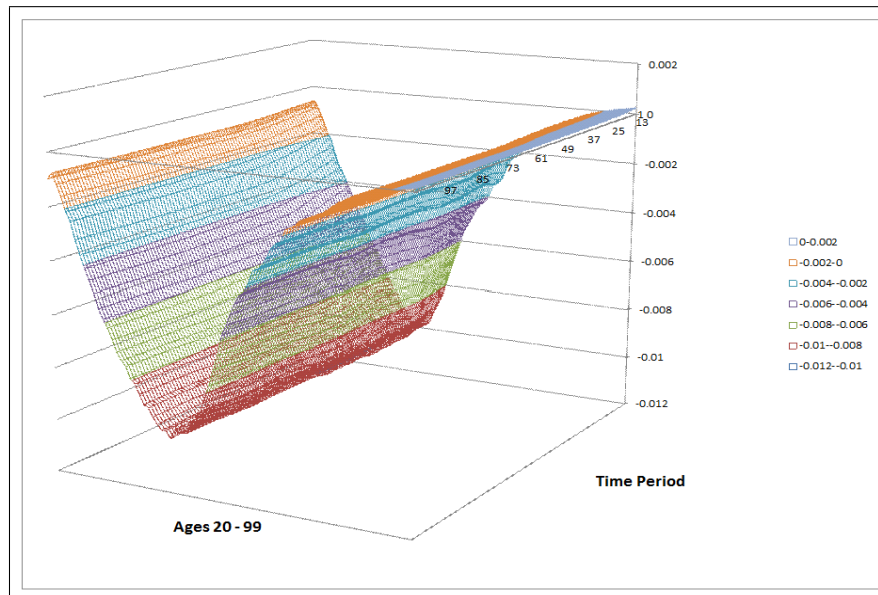
Source: Authors' calculations

Figure 24: Savings over time: 1st to 100th period, ages 20 to 99, 3rd ability group (percentage change, reform vs baseline)



Source: Authors' calculations

Figure 25: Income tax over time: 1st to 100th period, ages 20 to 99, 3rd ability group (absolute change, reform vs baseline)



Source: Authors' calculations