# Comparing three penalty functions – Cross Entropy, Quadratic and Linear Loss – in SAM balancing and splitting applications

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# Why?

- Old interest in comparing cross-entropy and quadratic loss
  - differences with regard to estimates
  - solution behavior: accuracy, speed, suitable solvers
- CGEBox comprises routine to split larger data bases (e.g. more details for agri-food, 57x40 => 80x40), requires stable and performant solution:
  - what penalty function?
  - what solvers work best?

# Two parts

1. Small didactic SAM balancing example with controlled data generation: are cross entropy (CE) and quadratic loss (QL) different? Compare to linear loss (LL, minimizing absolute differences)
2. Split exercises of larger GTAP data sets: compare penalty functions and solvers, with regard to accuracy and solution time

=> Recommendations for similar exercises

# The SAM balancing example

- Monte-Carlo exercise with 100 Draws:
  - 30x30 SAM, entries from n~(0,100), cutoff at 1
  - use RAS remove imbalances
  - add weight noise *n*~$(0,\sigma^2)$, $\sigma = \{0.1,0.5,1,2,5\}$
  - use QL, CE (with different assumed normal errors) and LL to balance the distorted SAM
  - w/wo non-negativity of estimates imposed
- Coded in GAMS, 100x5x3x2 = 3.000 problems solved with CONOPT4

# SAM balancing example, key findings

- CE and QL give identical results
- QL ~100 times faster than CE, LL slightly faster than QL
- Assuming different (but uniform) variances a priori for CE has no impact
- QL and CE give better estimates ⇔ assume correct distribution of error terms
- QL maximizes the posterior log likelihood under normally distributed errors (Highest Posterior Density), seems not really different from entropy criterion here
- Sign preserving approach does not change much

# The split example

- Empirical example splitting differently detailed GTAP data sets
  - Split „Other Food Processing“ to „concentrate feed“ and „rest“ based on data from FABIO MRIO
  - Data: 35x10, 47x10, 57x10, 57x24, filtered to 0.01%,0.001%,0.0001% => 12 test cases
  - True distribution of error terms unknown: QL and CE not by construction “right” choice
  - Same standard error (QL,CE) or same relative absolute error (LL) assumed, i.e. no weights for split factors

# The split example

- Constraints of problem all linear:
  - Various exhaustion and market balancing conditions
  - Includes bounds, not only non-negativity conditions on estimates
  - Includes also inequalities, e.g. ensure that factor subsidies do not exceed value at market prices
- Three penalty functions: CE, QL and LL, tested under three solvers: CONOPT4, GUROBI, CPLEX
- RAS no candidate (bounds, inequalities …)

# The solvers

- CPLEX and GUROBI:
  - specialized solvers for MIP and LP/QP problems
  - free academic licenses, otherwise quite expensive
  - tuned for massive parallelism
- CONOPT4:
  - general purpose gradient based solver
  - moderate license fees, not free for academic use
  - workhorse in CGEBox to solve CGE model and various other problems, sole candidate for CE
  - some parallelism

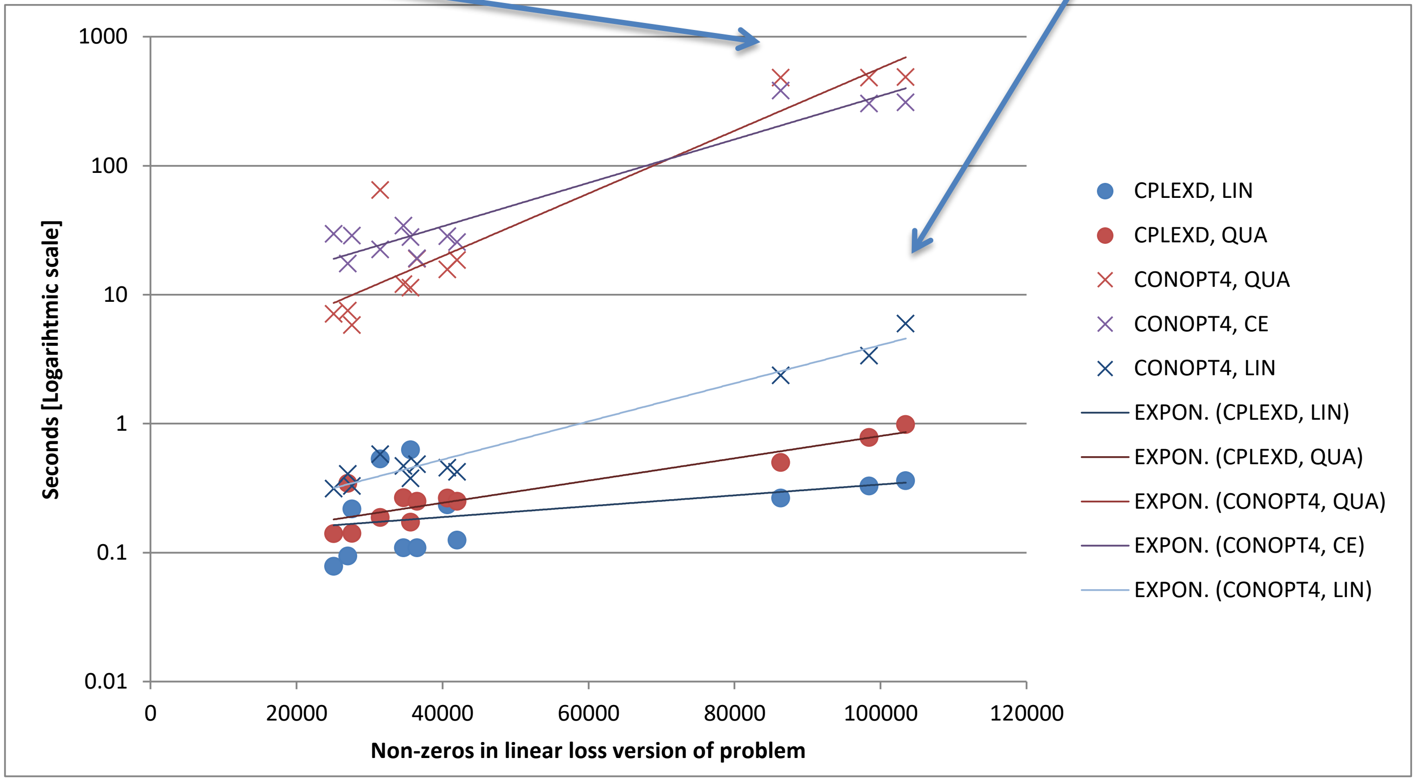
# Split example: some observations

- CE supports imply upper and lower bounds on estimates, here large *a priori* standard error required for feasibility
- Problems comprise between ~25k and ~100k non-zeros: moderately sized compared to some other split exercises
- All solvers had at least some cases where they did not find a fully optimal solution
- Some tweaks such as turning scaling off required to make solvers work on certain model classes

# Split examples: Scaling behavior

QL and CE in CONOPT4
require far more time
(close to 1.000 instead of 1 sec)

CPLEXD scales
extremely well
for LP and QP case



# Average relative diff to best solver/algorithm combination

Specialized LP/QP solvers clearly outperform the general purpose NLP solver for LL/QP and offer extremely high accuracy for LL.

CONOPT4 requires 500 times longer in average on QP

| | | Time | | Max. Imbalance | | Sum of Imbalances | |
|---|---|---|---|---|---|---|---|
| | | Solver | Total | Solver | Total | Solver | Total |
| LL | CPLEXD | 0,81 | 0,46 | 0,06 | 0,27 | 0,00 | 0,17 |
| LL | GUROBI | 0,95 | 0,16 | 0,18 | 0,60 | 0,04 | 0,32 |
| LL | CONOPT4 | 5,16 | 0,94 | 9,12 | 2,04 | 0,76 | 1,61 |
| QL | CPLEXD | 1,32 | 2,18 | 101,89 | 3,97 | 21,68 | 6,22 |
| QL | GUROBI | 6,90 | 12,27 | 5,3E6 | 8,3E4 | 3,0E6 | 2,4E5 |
| QL | CONOPT4 | 505 | 26,83 | 44,03 | 6,51 | 9,00 | 4,09 |
| CE | CONOPT4 | 466 | 24,81 | 350 | 3,82 | 105 | 7,04 |

# Take home messages

- CE and QL give identical results under normally distributed errors, QL easier to code and faster
- Specialized LP/QP solver outperform by far general purpose solvers on LP/QP problems
- If errors not known to be normally distributed, LL good choice: high accuracy and extremely fast to solve (also with NLP solvers)
- Compared to RAS, constrained optimization gives far more control (bounds, inequalities)

# And finally …

- CGEBox is open-source and open-access https://www.ilr.uni-bonn.de/em/rsrch/cgebox/cgebox_e.htm (including code for filter and split)
- CGEBox download includes the FABIO data base by Bruckner et al. to split up GTAP to more agri-food detail
- Code for SAM balancing available from the author for own experiments